Princip extremální akce

prostor historií $\mathcal{H}$

akce $S : \mathcal{H} \to \mathbb{R}$

princip extremální akce

$$\delta S[h] = 0$$

Lagrangeovský form.

$$x^a(t)$$

$$S[x(t)] = \int L(x^a, \dot{x}^a)\, dt$$

$$L(x^a, v^a)$$

$$\left(\frac{\partial L}{\partial v^a}(x, \dot{x})\right)^{\cdot} - \frac{\partial L}{\partial x^a}(x, \dot{x}) = 0$$

Hamiltonovský form

$$[x^a, p_b]$$

pozorovatelné $F(x^a, p_a)$

$$\{F, G\} = \sum_a \left[\frac{\partial F}{\partial x^a}\frac{\partial G}{\partial p_a} - \frac{\partial F}{\partial p_a}\frac{\partial G}{\partial x^a}\right]$$

$$\{x^a, p_b\} = \delta^a_b$$

$\dot{F} = \{F, H\}$ $\quad$ H Hamiltonián

$$\dot{x}^a = \{x^a, H\} = \frac{\partial H}{\partial p_a}$$

$$\dot{p}_a = \{p_a, H\} = -\frac{\partial H}{\partial x^a}$$

$$x, v \longleftrightarrow x, p$$

$$L \qquad\qquad H$$

$$p = \mu(x, v) \qquad \mu_a(x, v) = \frac{\partial L}{\partial v^a}(x, v)$$

$$v = \nu(x, p) \qquad \nu^a(x, p) = \frac{\partial H}{\partial p_a}(x, p)$$

$$\mu(x, \nu(x, p)) = p$$

$$\nu(x, \mu(x, v)) = v$$

$$H(x, p) = p_a \nu^a(x, p) - L(x, \nu(x, p))$$

$$L(x, v) = \mu_a(x, v)\, v^a - H(x, \mu(x, v))$$

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$$\delta S[h] = 0$$

Lagrangeovský form.

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Hamiltonovský form

$[x^a, p_b]$

pozorovatelné $F(x^a, p_a)$

$$\{F, G\} = \sum_a \left[\frac{\partial F}{\partial x^a}\frac{\partial G}{\partial p_a} - \frac{\partial F}{\partial p_a}\frac{\partial G}{\partial x^a}\right]$$

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$\dot{F} = \{F, H\}$ H Hamiltonián

$$\dot{x}^a = \{x^a, H\} = \frac{\partial H}{\partial p_a}$$

$$\dot{p}_a = \{p_a, H\} = -\frac{\partial H}{\partial x^a}$$

$$x, v \longleftrightarrow x, p$$

$$L \qquad\qquad H$$

$$p = \mathfrak{p}(x, v) \qquad \mathfrak{p}_a(x, v) = \frac{\partial L}{\partial v^a}(x, v)$$

$$v = \mathfrak{v}(x, p) \qquad \mathfrak{v}^a(x, p) = \frac{\partial H}{\partial p_a}(x, p)$$

$$\mathfrak{p}(x, \mathfrak{v}(x, p)) = p$$

$$\mathfrak{v}(x, \mathfrak{p}(x, v)) = v$$

$$H(x, p) = p_a \mathfrak{v}^a(x, p) - L(x, \mathfrak{v}(x, p))$$

$$L(x, v) = \mathfrak{p}_a(x, v)\, v^a - H(x, \mathfrak{p}(x, v))$$

Funkcionální prostory

prostor M

prostor funkcí na M $\quad \mathcal{F}M$

$$(\varphi + \psi)(x) = \varphi(x) + \psi(x)$$

$$(\lambda \varphi)(x) = \lambda\, \varphi(x)$$

$\varphi(x) \qquad \varphi^x \qquad x \in M$

$\vec{a} \qquad a^m \qquad m = 1 \dots 3$

$$\vec{a} \cdot \underline{\alpha} = \langle \underline{\alpha}, \vec{a} \rangle = \sum_m \alpha_m a^m$$

$$\varphi \cdot \omega = \langle \omega, \varphi \rangle = \int_M \underline{\omega_x}\, \varphi^x\, \underline{dx}$$

$$\omega_x = \omega(x) \qquad \varphi^x = \varphi(x)$$

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konečná dim

$$f(x_0 + \varepsilon \vec{\Delta x}) = f(x_0) + \varepsilon\, df(x_0) + \mathcal{O}(\varepsilon^2)$$

$$df(x_0) = \sum_n \underbrace{\frac{\partial f}{\partial x^n}(x_0)}_{\text{parciální der}} \Delta x^n$$

nekonečná dim

$$F(\varphi_0 + \varepsilon \delta\varphi) =$$

$$= F(\varphi_0) + \varepsilon\, \delta F(\varphi_0) + \mathcal{O}(\varepsilon^2)$$

$$\delta F(\varphi_0) = \int_x \underbrace{\frac{\delta F}{\delta \varphi^x}}_{\text{variace funkcionálu } F} \delta\varphi^x\, dx$$

Funkcionální prostory

prostor M

prostor funkcí na M $\qquad \mathcal{F}M$

$$(\varphi + \psi)(x) = \varphi(x) + \psi(x)$$

$$(\Omega \varphi)(x) = \Omega\, \varphi(x)$$

$\varphi(x) \qquad \varphi^{x} \qquad x \in M$

$\vec{a} \qquad a^{m} \qquad m = 1 \ldots 3$

$$\vec{a} \cdot \underset{\rightarrow}{\underline{\alpha}} = \langle \underset{\rightarrow}{\alpha}, \vec{a} \rangle = \sum_{m} \alpha_m a^m$$

$$\varphi \cdot \omega = \langle \omega, \varphi \rangle = \int_M \underline{\omega_x} \, \varphi^x \, \underline{dx}$$

$$\omega_x = \omega(x) \qquad \varphi^x = \varphi(x)$$

konečná dim

$$f(x_0 + \varepsilon \vec{\Delta x}) = f(x_0) + \varepsilon\, df(x_0) + \mathcal{O}(\varepsilon^2)$$

$$df(x_0) = \sum_{a} \underbrace{\frac{\partial f}{\partial x^a}(x_0)}_{\text{parciální der}} \Delta x^a$$

nekonečné di—

$$F(\varphi_0 + \varepsilon\,\delta\varphi) =$$

$$= F(\varphi_0) + \varepsilon\,\delta F(\varphi_0) + \mathcal{O}(\varepsilon^2)$$

$$\delta F(\varphi_0) = \int_x \underbrace{\frac{\delta F}{\delta \varphi^x}}_{\text{variace funkcionálu } F} \delta\varphi^x \, dx$$

Skalární pole

akce

$\Psi + \varepsilon\delta\Psi$

$$S_{sp}[\Psi] = -\frac{1}{2c}\int_\Omega \left[\nabla_\mu \Psi \nabla_\nu \Psi \, \eta^{\mu\nu} + M^2\Psi^2\right] d^4\Omega$$

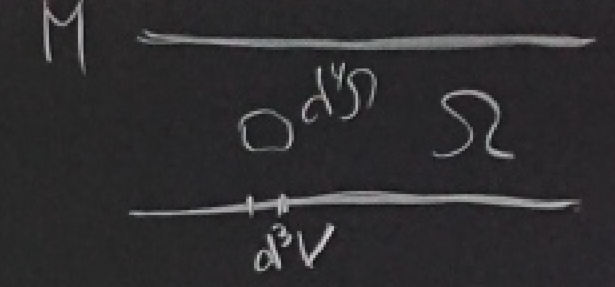
$$\Psi \to \Psi + \varepsilon\delta\Psi$$

$$\delta S[\Psi] = 0 \Rightarrow \frac{\delta S_{sp}}{\delta\Psi} + \frac{\delta S_{ost}}{\delta\Psi} = 0$$

$$\Box\Psi - M^2\Psi = j$$

Klein. Gordonova rov.

princip extremální akce



$$\delta S_{sp}[\Psi] = S_{sp}[\Psi+\varepsilon\delta\Psi] - S_{sp}[\Psi]$$

$$= -\frac{1}{c}\int_\Omega \left(\nabla_\mu\Psi \, \nabla_\nu\delta\Psi \, \eta^{\mu\nu} + M^2\Psi\delta\Psi\right) d^4\Omega + \delta S_{ostatní}$$

$$= -\frac{1}{c}\int_\Omega \left(\nabla_\mu\left((\nabla^\mu\Psi)\delta\Psi\right) - (\nabla_\mu\nabla^\mu\Psi)\delta\Psi + M^2\Psi\delta\Psi\right) d^4\Omega + \delta S_{ostatní}$$

$$= \underbrace{-\frac{1}{c}\int_{\partial\Omega}(\nabla_\mu\Psi)\underbrace{\delta\Psi}_{\text{na } \partial\Omega \to 0} n^\mu d^3V}_{0} + \frac{1}{c}\int_\Omega \left(\nabla_\mu\nabla^\mu\Psi - M^2\Psi\right)\delta\Psi \, d^4\Omega + \int_\Omega \frac{\delta S_{ost}}{\delta\Psi}\delta\Psi \, d^4\Omega$$

$$\frac{\delta S_{sp}}{\delta\Psi} = \frac{1}{c}\left(\overbrace{\nabla_\mu\nabla^\mu}^{\Box}\Psi - M^2\Psi\right) \qquad j = -c\,\frac{\delta S_{ost}}{\delta\Psi}$$

Skalární pole

akce

$$S_{SP}[\Psi] = -\frac{1}{2c}\int_{\Omega}\left[\nabla_{\mu}\Psi\,\nabla_{\nu}\Psi\,\eta^{\mu\nu} + M^{2}\Psi^{2}\right] d^{4}\Omega$$

$$\Psi \to \Psi + \varepsilon\,\delta\Psi$$

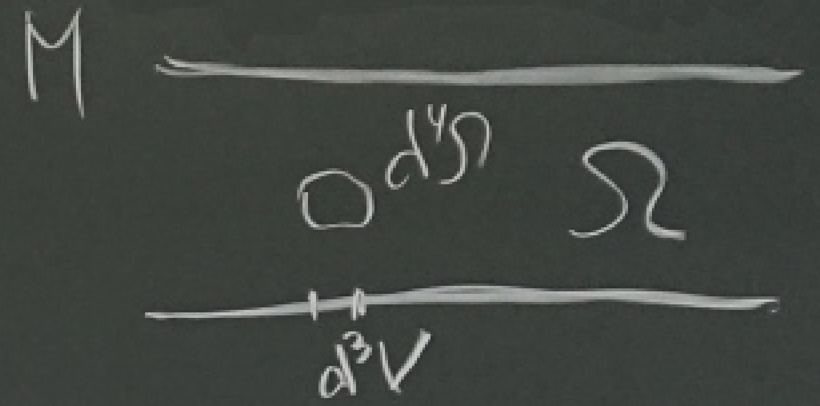
princip extremální akce

$$\delta S_{sp}[\psi] = S_{sp}[\psi + \varepsilon\delta\psi] - S_{sp}[\psi]$$

$$= -\frac{1}{c}\int_{\Omega} \left(\nabla_\mu \psi \, \nabla_\nu \delta\psi \, \eta^{\mu\nu} + M^2 \psi \, \delta\psi \right) d^4\Omega \; + \delta S_{ostatní}$$

$$S_{sp}[\psi] = -\frac{1}{2c}\int_{\Omega} \left[\nabla_\mu \overset{\overset{\psi+\varepsilon\delta\psi}{\downarrow}}{\psi} \, \nabla_\nu \psi \, \eta^{\mu\nu} + M^2 \psi^2 \right] d^4\Omega$$

princip extremální akce



$$\delta S_{sp}[\psi] = S_{sp}[\psi+\varepsilon\delta\psi] - S_{sp}[\psi]$$

$$= -\frac{1}{c}\int_\Omega \left(\nabla_\mu\psi\, \nabla_\nu\delta\psi\,\eta^{\mu\nu} + M^2\psi\,\delta\psi\right) d^4\Omega + \delta S_{ostatní}$$

$$= -\frac{1}{c}\int_\Omega \left(\nabla_\mu\left((\nabla^\mu\psi)\delta\psi\right) - (\nabla_\mu\nabla^\mu\psi)\delta\psi + M^2\psi\,\delta\psi\right) d^4\Omega + \delta S_{ostatní}$$

$$= \underbrace{-\frac{1}{c}\int_{\partial\Omega}(\nabla_\mu\psi)\underbrace{\delta\psi}_{\text{na } \partial\Omega \to 0}\, n^\mu\, d^3V}_{0} + \frac{1}{c}\int_\Omega \left(\nabla_\mu\nabla^\mu\psi - M^2\psi\right)\delta\psi\, d^4\Omega + \int_\Omega \frac{\delta S_{ost}}{\delta\psi}\delta\psi\, d^4\Omega$$

$$\frac{\delta S_{sp}}{\delta\psi} = \frac{1}{c}\left(\overbrace{\nabla_\mu\nabla^\mu}^{\Box}\psi - M^2\psi\right)$$

$$j = -c\,\frac{\delta S_{ost}}{\delta\psi}$$

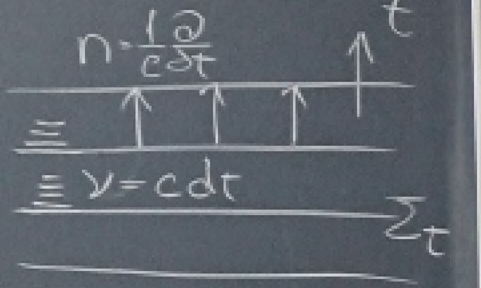
$$\delta S[\psi] = 0 \Rightarrow \frac{\delta S_{sp}}{\delta \psi} + \frac{\delta S_{int}}{\delta \psi} = 0$$

$$\Box \psi - M^2 \psi = j$$

Klein-Gordonova rov.

Skalární pole

$\psi(x)$ $\qquad x \in M$



∇_μ der na M

$\vec{\nabla}$ der na Σ_t

- čas. der. $\frac{\partial}{\partial t}$ $\qquad n \cdot \nu = 1$

$\nabla_\mu \psi \nabla_\nu \psi\, \eta^{\mu\nu} = -\frac{1}{c^2}\dot{\psi}^2 + (\vec{\nabla}\psi)^2$

$\nabla \psi = \nu\, n \cdot \nabla \psi + P \cdot \nabla \psi =$

$= \nu \frac{1}{c}\dot{\psi} + \vec{\nabla}\psi \qquad [\frac{1}{c}\dot{\psi}, \vec{\nabla}\psi]$

Lagrangeovský formalismus

$$S_{sp}[\psi] = \int_{I \subset \mathbb{R}} \int_{\Sigma_t} \left(\frac{1}{2c^2}\dot{\psi}^2 - \frac{1}{2}(\vec{\nabla}\psi)^2 - \frac{1}{2}M^2\psi^2 \right) dV\, dt$$

$\Theta + \delta\Theta \qquad \psi + \delta\psi$

$$\underset{\text{poloha}}{L(\psi}, \underset{\text{rychlost}}{\Theta)} = \int_{\Sigma_t} \Big(\underbrace{\frac{1}{2c^2}\Theta^2}_{\text{kinetická část}} - \underbrace{\frac{1}{2}(\vec{\nabla}\psi)^2 - \frac{1}{2}M^2\psi^2}_{\text{potenciál část}} \Big) dV \qquad S[\psi] = \int_I L(\psi, \dot{\psi})\, dt$$

$$\left(\frac{\delta L}{\delta \Theta}(\psi, \dot{\psi})\right)^{\cdot} - \frac{\delta L}{\delta \psi}(\psi, \dot{\psi}) = 0$$

$\frac{\delta L}{\delta \Theta}$ $\qquad \delta L = L(\psi, \Theta + \delta\Theta) - L(\psi, \Theta)$

$= \int_{\Sigma_t} \frac{1}{c^2}\Theta\, \delta\Theta\, dV$

$$\underbrace{\frac{1}{c^2}\ddot{\psi} - \vec{\nabla}^2\psi}_{-\Box} + M^2\psi = -j$$

$\frac{\delta L}{\delta \Theta} = \frac{1}{c^2}\Theta \qquad j = -\frac{\delta L_{int}}{\delta \psi}$

$\frac{\delta L}{\delta \psi}$ $\qquad \delta L = L(\psi + \delta\psi, \Theta) - L(\psi, \Theta)$

$= \int_\Sigma \left(-(\vec{\nabla}\psi)(\vec{\nabla}\delta\psi) - M^2 \psi\, \delta\psi \right) dV$

$= -\underbrace{\int_\Sigma \vec{\nabla} \cdot (\delta\psi \vec{\nabla}\psi)\, dV}_{0} - \int_\Sigma \left(-\vec{\nabla}^2\psi + M^2\psi\right) \delta\psi\, dV$

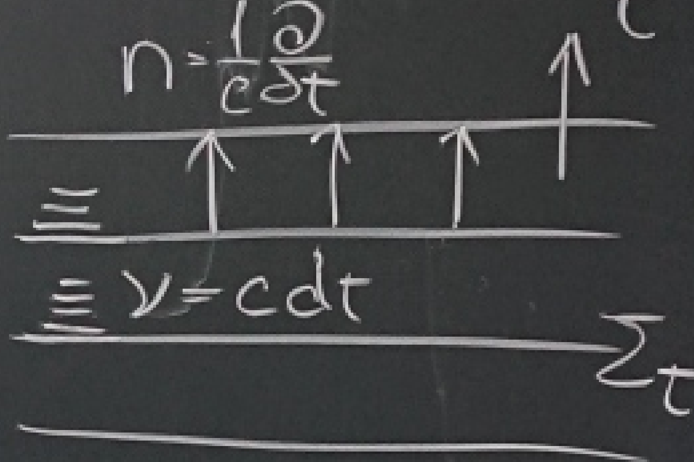
$= -\underbrace{\int_{\partial\Sigma} \delta\psi\, \vec{\nabla}\psi \cdot \vec{n}\, dS}_{0} - \int_\Sigma \underbrace{\left(-\vec{\nabla}^2\psi + M^2\psi\right)}_{\frac{\delta L}{\delta \psi}} \delta\psi\, dV$

$\frac{\delta L_{sp}}{\delta \psi} = -\left[-\vec{\nabla}^2\psi + M^2\psi\right]$

$\Box = \nabla_\mu \nabla^\mu = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \vec{\nabla}^2$

Skalární pole

$\psi(x)$ $\quad x \in M$



∇_μ der na M

$\vec{\nabla}$ der na Σ_t

• čas. der. $\frac{\partial}{\partial t}$ $\quad n \cdot \nu = 1$

$$\nabla_\mu \psi \, \nabla_\nu \psi \, g^{\mu\nu} = -\frac{1}{c^2}\dot{\psi}^2 + (\vec{\nabla}\psi)^2$$

$$\nabla \psi = \nu \, n \cdot \nabla \psi + P \cdot \nabla \psi =$$
$$= \nu \frac{1}{c}\dot{\psi} + \vec{\nabla}\psi \qquad \left[\frac{1}{c}\dot{\psi}, \vec{\nabla}\psi\right]$$

Lagrangeovský formalismus

$$S_{SP}[\psi] = \int_{I\subset\mathbb{R}} \int_{\Sigma_t} \left(\frac{1}{2c^2}\dot{\psi}^2 - \frac{1}{2}(\vec{\nabla}\psi)^2 - \frac{1}{2}M^2\psi^2 \right) dV\, dt$$

$$L(\psi, \Theta) = \int_{\Sigma_t} \Big(\underbrace{\frac{1}{2c^2}\Theta^2}_{\text{kinetické část}} - \underbrace{\frac{1}{2}(\vec{\nabla}\psi)^2 - \frac{1}{2}M^2\psi^2}_{\text{potenciál část}} \Big) dV \qquad S[\psi] = \int_I L(\psi, \dot{\psi})\, dt$$

poloha rychlost

$$L(\underset{\text{poloha}}{\psi},\underset{\text{rychlost}}{\Theta}) = \int_{\Sigma_t} \Big(\underbrace{\frac{1}{2c^2}\overset{\Theta+\delta\Theta}{\overset{\downarrow}{\Theta}}{}^2}_{\substack{\text{kinetické} \\ \text{část}}} - \underbrace{\frac{1}{2}(\vec{\nabla}\psi)^2 - \frac{1}{2}M^2\psi^2}_{\substack{\text{potenciál.} \\ \text{část}}} \Big)\, dV$$

$$\left(\frac{\delta L}{\delta\Theta}(\psi,\dot{\psi})\right)^{\cdot} - \frac{\delta L}{\delta\psi}(\psi,\dot{\psi}) = 0$$

$$\frac{\delta L}{\delta\Theta}$$

$$\delta L = L(\psi,\Theta+\delta\Theta) - L(\psi,\Theta) = \int_{\Sigma_t} \frac{1}{c^2}\Theta\,\delta\Theta\, dV$$

$$\frac{\delta L}{\delta\Theta} = \frac{1}{c^2}\Theta$$

$$\frac{\delta L}{\delta \psi}$$

$$\delta L = L(\psi + \delta\psi, \Theta) - L(\psi, \Theta)$$

$$= \int_{\Sigma} \left(-(\vec{\nabla}\psi)\cdot(\vec{\nabla}\delta\psi) - M^2 \psi \, \delta\psi \right) dV$$

$\psi + \delta\psi$

$$L(\psi, \Theta) = \int_{\Sigma_t} \left(\underbrace{\frac{1}{2c^2}\Theta^2}_{\text{kinetické část}} - \underbrace{\frac{1}{2}(\vec{\nabla}\psi)^2 - \frac{1}{2}M^2\psi^2}_{\text{potenciál. část}} \right) dV$$

poloha rychlost

$$\frac{\delta L}{\delta \psi} \qquad \delta L = L(\psi + \delta\psi, \theta) - L(\psi, \theta)$$

$$= \int_\Sigma \left(-(\vec{\nabla}\psi)\cdot(\vec{\nabla}\delta\psi) - M^2 \psi \,\delta\psi \right) dV$$

$$= -\underbrace{\int_\Sigma \vec{\nabla}\cdot(\delta\psi\,\vec{\nabla}\psi)\, dV}_{0} - \int_\Sigma \left(-\vec{\nabla}^2\psi + M^2\psi\right)\delta\psi\, dV$$

$$= -\underbrace{\int_{\partial\Sigma} \delta\psi\,\vec{\nabla}\psi\cdot\vec{n}\, dS}_{0} - \int_\Sigma \left(-\vec{\nabla}^2\psi + M^2\psi\right)\delta\psi\, dV$$

$$\frac{\delta L}{\delta \psi} \quad \delta L = L(\psi + \delta\psi, \theta) - L(\psi, \theta)$$

$$= \int_{\Sigma} \left((-\vec{\nabla}\psi)\cdot(\vec{\nabla}\delta\psi) - M^2 \psi \delta\psi \right) dV$$

$$= -\underbrace{\int_{\Sigma} \vec{\nabla}\cdot(\delta\psi \vec{\nabla}\psi)\, dV}_{0} - \int_{\Sigma} (-\vec{\nabla}^2\psi + M^2\psi)\delta\psi\, dV$$

$$= -\underbrace{\int_{\partial\Sigma} \delta\psi \vec{\nabla}\psi \cdot \vec{n}\, dS}_{0} - \int_{\Sigma} \underbrace{(-\vec{\nabla}^2\psi + M^2\psi)}_{\frac{\delta L}{\delta\psi}} \delta\psi\, dV$$

$$\frac{\delta L_{sp}}{\delta \psi} = -\left[-\vec{\nabla}^2\psi + M^2\psi\right]$$

$$\left(\frac{\delta L}{\delta \Theta}(\psi,\dot{\psi})\right)^{\cdot} - \frac{\delta L}{\delta \psi}(\psi,\dot{\psi}) = 0$$

$$\boxed{\frac{\delta L_{sp}}{\delta \psi} = -\left[-\vec{\nabla}^2\psi + M^2\psi\right]}$$

$$\frac{1}{c^2}\ddot{\psi} - \vec{\nabla}^2\psi + M^2\psi = -j$$

$$\frac{\delta L_{sp}}{\delta \Theta} = \frac{1}{c^2}\Theta$$

$$j = -\frac{\delta L_{Gal}}{\delta \psi}$$

$$\left(\frac{\delta L}{\delta \Theta}(\psi,\dot\psi)\right)^{\cdot} - \frac{\delta L}{\delta \psi}(\psi,\dot\psi) = 0$$

$$\boxed{\Box = \nabla_\mu \nabla^\mu = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \vec{\nabla}^2}$$

$$\underbrace{\frac{1}{c^2}\ddot\psi - \vec{\nabla}^2\psi}_{-\Box} + M^2\psi = -j$$

$$\frac{\delta L_{sp}}{\delta \Theta} = \frac{1}{c^2}\Theta$$

$$j = -\frac{\delta L_{GDA}}{\delta \psi}$$

Skalární pole

$$\pi = \frac{\delta L}{\delta \theta}(\psi, \theta) = \frac{1}{c^2}\theta$$

$$\theta = c^2 \pi$$

$$H(\psi,\pi) = \int_{\Sigma_t} \theta^x \pi_x \, dV - L(\psi,\theta)$$

$$= \int_\Sigma \left(c^2\pi^2 - \frac{1}{2}c^2\pi^2 + \frac{1}{2}(\vec{\nabla}\psi)^2 + \frac{1}{2}M\psi^2 \right) dV$$

$$= \int_\Sigma \left(\frac{c^2}{2}\pi^2 + \frac{1}{2}(\vec{\nabla}\psi)^2 + \frac{1}{2}M^2\psi^2 \right) dV$$

Hamiltonovský formalismus

$$\dot{\psi} = \frac{\delta H}{\delta \pi} = c^2\pi \qquad \dot{\pi} = -\frac{\delta H}{\delta \psi}$$

$$\delta H = H(\psi + \delta\psi, \pi) - H(\psi,\pi) =$$

$$= \int_\Sigma \left(\vec{\nabla}\psi \cdot \vec{\nabla}\delta\psi + M^2\psi\,\delta\psi \right) dV$$

$$= \int_\Sigma \left(-\vec{\nabla}^2\psi + M^2\psi \right) \delta\psi \, dV$$

$$\frac{\delta H}{\delta \psi} = -\vec{\nabla}^2\psi + M^2\psi$$

$$\underbrace{-\frac{1}{c^2}\ddot{\psi} + \vec{\nabla}^2\psi}_{\Box\psi} - M^2\psi = 0$$

$$H(\psi,\pi)$$

$$\{F, G\} = \int_\Sigma \left[\frac{\delta F}{\delta \psi^x}\frac{\delta G}{\delta \pi_x} - \frac{\delta F}{\delta \pi_x}\frac{\delta G}{\delta \psi^x} \right] dV$$

$$F(\psi,\pi) \qquad G(\psi,\pi)$$

$$\{\psi^x, \pi_y\} = \delta^x_y$$

Skalární pole

$$\pi = \frac{\delta L}{\delta \Theta}(\psi, \Theta) = \frac{1}{c^2}\Theta$$

$$\Theta = c^2 \pi$$

$$H(\psi, \pi) = \int_{\Sigma_t} \Theta^x \pi_x \, dV - L(\psi, \Theta)$$

$$= \int_{\Sigma} \left(c^2\pi^2 - \frac{1}{2}c^2\pi^2 + \frac{1}{2}(\vec{\nabla}\psi)^2 + \frac{1}{2}M\psi^2 \right) dV$$

$$= \int_{\Sigma} \left(\frac{c^2}{2}\pi^2 + \frac{1}{2}(\vec{\nabla}\psi)^2 + \frac{1}{2}M^2\psi^2 \right) dV$$

Hamiltonovský formalismus

$$\dot{\psi} = \frac{\delta H}{\delta \pi} = c^2 \pi \qquad \dot{\pi} = -\frac{\delta H}{\delta \psi} \qquad H(\psi, \pi)$$

$$\delta H = H(\psi + \delta\psi, \pi) - H(\psi, \pi) =$$

$$= \int_\Sigma \left(\vec{\nabla}\psi \cdot \vec{\nabla}\delta\psi + M^2 \psi \delta\psi \right) dV$$

$$= \int_\Sigma \left(-\vec{\nabla}^2 \psi + M^2 \psi \right) \delta\psi \, dV$$

$$\frac{\delta H}{\delta \psi} = -\vec{\nabla}^2 \psi + M^2 \psi \qquad \underbrace{-\frac{1}{c^2}\ddot{\psi} + \vec{\nabla}^2 \psi}_{\Box\psi} - M^2 \psi = 0$$

$$\{F, G\} = \int_{\Sigma} \frac{\delta F}{\delta \psi^x} \frac{\delta G}{\delta \pi_x} - \frac{\delta F}{\delta \pi_x} \frac{\delta G}{\delta \psi^x} \Big] dV$$

$$F(\psi, \pi) \qquad G(\psi, \pi)$$

$$\{\psi^x, \pi_y\} = \delta^x_y$$