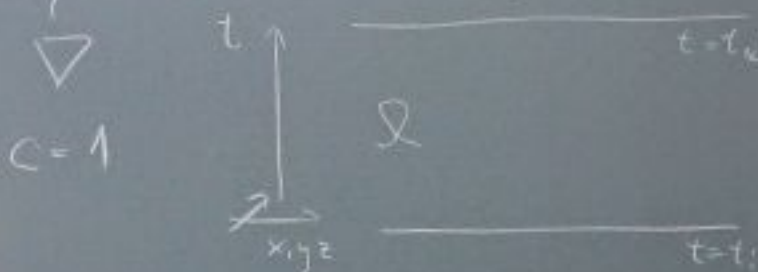


# Elektromagnetické pole

$$\eta = -dt^2 + dx^2 + dy^2 + dz^2$$



$$A_\alpha = [-\varphi, \vec{A}] \quad \nabla_\alpha J_E^\alpha = 0$$

$$F_{\alpha\beta} = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha$$

$$S_{EM}(A_\alpha) = -\frac{\epsilon_0}{4} \int_\Omega F_{\mu\nu} F^{\mu\nu} d\Omega$$

$$S_{ext}[\dots, A_\alpha] = \int_\Omega J_E^\alpha A_\alpha d\Omega \quad dt dx dy dz$$

principiálně extrémální akce

$$0 = \delta S[A] = S[A + \delta A] - S[A]$$

$$= -\frac{\epsilon_0}{2} \int_\Omega (\nabla_\mu \delta A_\nu - \nabla_\nu \delta A_\mu) F^{\mu\nu} d\Omega + \delta S_{ext}$$

$$= -\epsilon_0 \int_\Omega (\nabla_\mu \delta A_\nu) F^{\mu\nu} d\Omega + \delta S_{ext}$$

$$= -\epsilon_0 \int_\Omega (\nabla_\nu (\delta A_\nu F^{\mu\nu}) - \delta A_\nu \nabla_\nu F^{\mu\nu}) d\Omega + \delta S_{ext}$$

$$= \epsilon_0 \int_\Omega \left( -\nabla_\nu F^{\nu\mu} + \frac{1}{\epsilon_0} \frac{\delta S_{ext}}{\delta A_\nu} \right) \delta A_\nu d\Omega$$

$$\frac{1}{\epsilon_0} \frac{\delta S}{\delta A_\nu} = -\nabla_\nu F^{\nu\mu} + \frac{1}{\epsilon_0} \left( \frac{\delta S_{ext}}{\delta A_\nu} \right) = 0 \rightarrow J^\nu$$

$$\nabla_\nu F^{\nu\mu} = \frac{1}{\epsilon_0} J^\mu \quad J^\nu = J_E^\nu$$

$$\nabla_\mu F_{\mu\nu} = 0 \Leftrightarrow F_{\alpha\beta} = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha$$

$$F_{\alpha\beta} = \begin{pmatrix} 0 & E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_2 & -B_3 \\ -E_2 & -B_2 & 0 & B_1 \\ -E_3 & B_1 & -B_1 & 0 \end{pmatrix}$$

$$S_{EM} = \frac{\epsilon_0}{2} \int_\Omega (\vec{E}^2 - \vec{B}^2) dt dV$$

$$\vec{E} = -\vec{\nabla}\varphi - \dot{\vec{A}}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$J^\mu = \begin{pmatrix} \rho \\ \vec{j} \end{pmatrix} \quad J_E^\mu A_\mu = -\rho\varphi + \vec{j} \cdot \vec{A}$$

$$S_{EM} = \int_I \underbrace{\frac{\epsilon_0}{2} \int_\Sigma (\vec{E}^2 - \vec{B}^2) dV}_{L_{EM}(\varphi, \vec{A}, \dot{\varphi}, \dot{\vec{A}})} dt$$

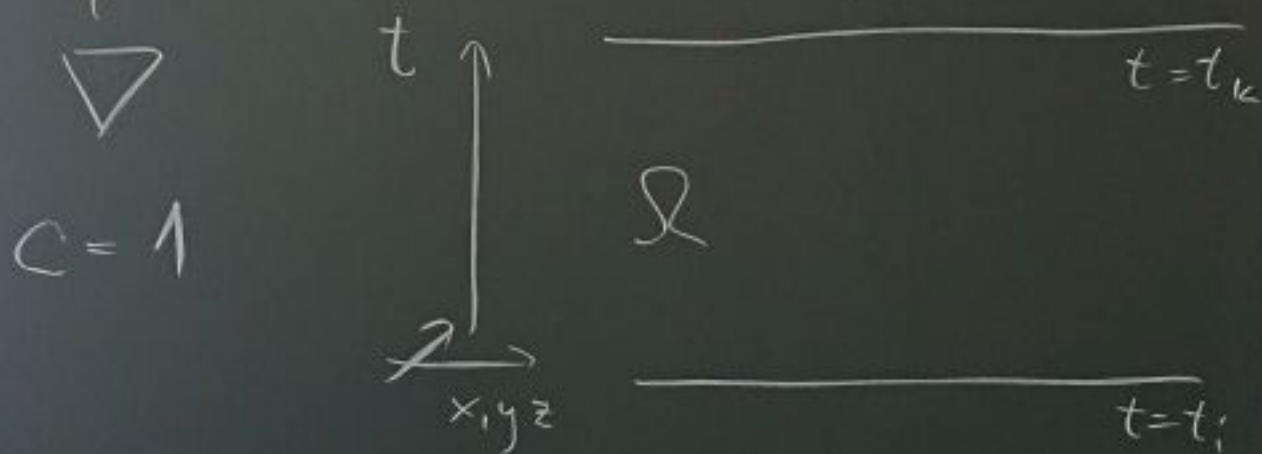
$$L_{EM} = \frac{\epsilon_0}{2} \int_\Sigma (\vec{E}^2 - \vec{B}^2) dV$$

$$S_{ext} = \int_I L_{ext}(\varphi, \vec{A}, \dots)$$

$$L_{ext} = \int_\Sigma (-\rho_E \varphi + \vec{j}_E \cdot \vec{A}) dV$$

# Elektromagnetické pole

$$\eta = -dt^2 + dx^2 + dy^2 + dz^2$$



$$A_\alpha = [-\varphi, \vec{A}] \quad \nabla_\alpha J_E^\alpha = 0$$

$$F_{\alpha\beta} = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha$$

$$S_{EM}[A_\alpha] = -\frac{\epsilon_0}{4} \int_{\Omega} F_{\mu\nu} F^{\mu\nu} d^4\Omega$$

$$S_{\text{ost}}[\dots, A_\alpha] = \int_{\mathcal{R}} J_E^\alpha A_\alpha d^4\Omega$$

princi, extréménél azse

$$0 = \delta S[A] = S[A + \delta A] - S[A]$$

$$= -\frac{\epsilon_0}{2} \int_{\Omega} (\nabla_n \delta A_\nu - \nabla_\nu \delta A_n) F^{n\nu} d^4\Omega + \delta S_{\text{ext}}$$

$$= -\epsilon_0 \int_{\Omega} (\nabla_n \delta A_\nu) F^{n\nu} d^4\Omega + \delta S_{\text{ext}}$$

$$= -\epsilon_0 \int_{\Omega} (\nabla_n (\delta A_\nu F^{n\nu}) - \delta A_\nu \nabla_n F^{n\nu}) d^4\Omega + \delta S_{\text{ext}}$$

$$= \epsilon_0 \int_{\Omega} \left( -\nabla_n F^{n\nu} + \frac{1}{\epsilon_0} \frac{\delta S_{\text{ext}}}{\delta A_\nu} \right) \delta A_\nu d^4\Omega$$

$$\frac{1}{\epsilon_0} \frac{\delta S}{\delta A_\nu} = -\nabla_n F^{n\nu} + \frac{1}{\epsilon_0} \left( \frac{\delta S_{\text{ext}}}{\delta A_\nu} \right) \xrightarrow{\quad} J^\nu = 0$$

$$\nabla_n F^{n\nu} = \frac{1}{\epsilon_0} J^\nu$$

$$J^\nu = J_E^\nu$$

$$\nabla_n F_{n\nu} = 0 \iff F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

$$F_{\alpha\beta} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

$$S_{EM} = \frac{\epsilon_0}{2} \int_{\Omega} (\vec{E}^2 - \vec{B}^2) d^4\Omega$$

dt dV

$$\vec{E} = -\vec{\nabla}\phi - \dot{\vec{A}}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$J^\mu = \begin{pmatrix} \rho \\ \vec{j} \end{pmatrix}$$

$$J^\mu A_\mu = -\rho\phi + \vec{j} \cdot \vec{A}$$

$$S_{EM} = \int_I \underbrace{\left( \frac{\epsilon_0}{2} \int_{\Sigma} (\vec{E}^2 - \vec{B}^2) dV \right)}_{L_{EM}(\varphi, \vec{A}, \dot{\varphi}, \dot{\vec{A}})} dt$$

$$L_{EM} = \frac{\epsilon_0}{2} \int_{\Sigma} (\vec{E}^2 - \vec{B}^2) dV$$

$$S_{\text{tot}} = \int_I L_{\text{tot}}(\varphi, \vec{A}, \dots)$$

$$L_{\text{tot}} = \int_{\Sigma} (-\rho_E \varphi + \vec{j}_E \cdot \vec{A}) dV$$

# Lagrangeovský formalismus pro EM

$$L_{EM}(\varphi, \vec{A}, \dot{\varphi}, \dot{\vec{A}}) = \frac{\epsilon_0}{2} \int_{\Sigma} (\vec{E}^2 - \vec{B}^2) dV$$

$$\vec{E} = -\dot{\vec{A}} - \vec{\nabla}\varphi \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

$$L_{\text{ost}}(\varphi, \vec{A}) = \text{maxi.} \int_{\Sigma} (-\rho_E \varphi + \vec{j}_E \cdot \vec{A}) dV$$

$$\left( \frac{\delta L}{\delta \vec{A}} \right) - \frac{\delta L}{\delta \dot{\vec{A}}} = 0 \quad \left( \frac{\delta L_{EM}}{\delta \vec{A}} \right) - \frac{\delta L_{EM}}{\delta \dot{\vec{A}}} = \vec{j}$$

$$\left( \frac{\delta L}{\delta \varphi} \right) - \frac{\delta L}{\delta \dot{\varphi}} = 0 \quad \left( \frac{\delta L_{EM}}{\delta \varphi} \right) - \frac{\delta L_{EM}}{\delta \dot{\varphi}} = -\rho$$

$$\vec{j} = \frac{\delta L_{\text{ost}}}{\delta \vec{A}} = \vec{j}_E \quad \rho = -\frac{\delta L_{\text{ost}}}{\delta \varphi} = \rho_E$$

$$\delta \vec{A} \quad \delta L_{EM} = \epsilon_0 \int_{\Sigma} (-\delta \vec{A}) \cdot \vec{E} dV$$

$$\vec{T} = \frac{\delta L_{EM}}{\delta \vec{A}} = -\epsilon_0 \vec{E}$$

$$\delta \vec{A} \quad \delta L_{EM} = -\epsilon_0 \int_{\Sigma} (\vec{\nabla} \times \delta \vec{A}) \cdot \vec{B} dV$$

$$= \epsilon_0 \int_{\Sigma} (\vec{\nabla} \cdot (\vec{B} \times \delta \vec{A}) - \delta \vec{A} \cdot \vec{\nabla} \times \vec{B}) dV$$

$$= \epsilon_0 \int_{\Sigma} \vec{\nabla} \cdot (\vec{B} \times \delta \vec{A}) + \vec{\nabla} \cdot (\vec{B} \times \delta \vec{A})$$

$$\delta \vec{A} \cdot \vec{\nabla} \times \vec{B} - \vec{B} \cdot (\vec{\nabla} \times \delta \vec{A})$$

$$= -\epsilon_0 \int_{\Sigma} \delta \vec{A} \cdot \vec{\nabla} \times \vec{B} dV$$

$$\frac{\delta L_{EM}}{\delta \vec{A}} = -\epsilon_0 \vec{\nabla} \times \vec{B}$$

$$K = \frac{\delta L_{EM}}{\delta \dot{\varphi}} = 0$$

$$\delta \varphi \quad \delta L_{EM} = \epsilon_0 \int_{\Sigma} (-\vec{\nabla} \delta \varphi) \cdot \vec{E} dV$$

$$= \epsilon_0 \int_{\Sigma} \delta \varphi \vec{\nabla} \cdot \vec{E} dV$$

$$\frac{\delta L_{EM}}{\delta \varphi} = \epsilon_0 \vec{\nabla} \cdot \vec{E}$$

$$(-\epsilon_0 \vec{E}) + \epsilon_0 \vec{\nabla} \times \vec{B} = \vec{j}$$

$$\vec{\nabla} \times \vec{B} - \vec{E} = \frac{1}{\epsilon_0} \vec{j}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$-\vec{\nabla} \varphi = \vec{E} + \dot{\vec{A}}$$

$$\Leftrightarrow \vec{\nabla} \cdot \vec{B} = 0$$

$$\Leftrightarrow \vec{\nabla} \times \vec{E} + \dot{\vec{B}} = 0$$

# Lagrangeovský formalismus pro EM

$$L_{EM}(\varphi, \vec{A}, \dot{\varphi}, \dot{\vec{A}}) = \frac{\epsilon_0}{2} \int_{\Sigma} (\vec{E}^2 - \vec{B}^2) dV$$

$$\vec{E} = -\dot{\vec{A}} - \vec{\nabla}\varphi \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

$$L_{\text{ost}}(\varphi, \vec{A}) \stackrel{\text{max.}}{=} \int_{\Sigma} (-\rho_E \varphi + \vec{j}_E \cdot \vec{A}) dV$$

$$\left( \frac{\delta L}{\delta \dot{\vec{A}}} \right) - \frac{\delta L}{\delta \vec{A}} = 0 \quad \left( \frac{\delta L_{EM}}{\delta \dot{\vec{A}}} \right) - \frac{\delta L_{EM}}{\delta \vec{A}} = \vec{j}$$

$$\left( \frac{\delta L}{\delta \dot{\varphi}} \right) - \frac{\delta L}{\delta \varphi} = 0 \quad \left( \frac{\delta L_{EM}}{\delta \dot{\varphi}} \right) - \frac{\delta L_{EM}}{\delta \varphi} = -\rho$$

$$\vec{j} = \frac{\delta L_{\text{ost}}}{\delta \vec{A}} = \vec{j}_E$$

$$\rho = -\frac{\delta L_{\text{ost}}}{\delta \varphi} = \rho_E$$

$$\delta \vec{A} \quad \delta L_{EM} = \epsilon_0 \int_{\Sigma} (-\delta \vec{A}) \cdot \vec{E} \, dV$$

$$\vec{\Pi} = \frac{\delta L_{EM}}{\delta \vec{A}} = -\epsilon_0 \vec{E}$$

$$\begin{aligned} \delta \vec{A} \quad \delta L_{EM} &= -\epsilon_0 \int_{\Sigma} (\vec{\nabla} \times \delta \vec{A}) \cdot \vec{B} \, dV \\ &= \epsilon_0 \int_{\Sigma} (\vec{\nabla} \cdot (\vec{B} \times \delta \vec{A}) - \delta \vec{A} \cdot \vec{\nabla} \times \vec{B}) \, dV \\ &= \epsilon_0 \int_{\Sigma} \underbrace{\vec{\nabla} \cdot (\vec{B} \times \delta \vec{A})}_{\delta \vec{A} \cdot \vec{\nabla} \times \vec{B}} + \underbrace{\vec{\nabla} \cdot (\vec{B} \times \delta \vec{A})}_{-\vec{B} \cdot (\vec{\nabla} \times \delta \vec{A})} \, dV \\ &= -\epsilon_0 \int_{\Sigma} \delta \vec{A} \cdot \vec{\nabla} \times \vec{B} \, dV \end{aligned}$$

$$\frac{\delta L_{EM}}{\delta \vec{A}} = -\epsilon_0 \vec{\nabla} \times \vec{B}$$



$$\delta\varphi \quad \delta L_{EM} = \epsilon_0 \int_{\Sigma} (-\vec{\nabla}\delta\varphi) \cdot \vec{E} dV$$

$$= \epsilon_0 \int_{\Sigma} \delta\varphi \vec{\nabla} \cdot \vec{E} dV$$

$$\frac{\delta L_{EM}}{\delta\varphi} = \epsilon_0 \vec{\nabla} \cdot \vec{E}$$

$$(-\epsilon_0 \vec{E})' + \epsilon_0 \vec{\nabla} \times \vec{B} = \vec{j}$$

$$\vec{\nabla} \times \vec{B} - \vec{j} = \frac{1}{\epsilon_0} \vec{j}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$

$$\delta\varphi \quad \delta L_{EM} = \epsilon_0 \int_{\Sigma} (-\vec{\nabla}\delta\varphi) \cdot \vec{E} dV$$

$$= \epsilon_0 \int_{\Sigma} \delta\varphi \vec{\nabla} \cdot \vec{E} dV$$

$$\frac{\delta L_{EM}}{\delta\varphi} = \epsilon_0 \vec{\nabla} \cdot \vec{E}$$

$$(-\epsilon_0 \vec{E}) + \epsilon_0 \vec{\nabla} \times \vec{B} = \vec{j}$$

$$\vec{\nabla} \times \vec{B} - \vec{E} = \frac{1}{\epsilon_0} \vec{j}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$

$$\vec{B} = \vec{\nabla} \times \vec{A} \Leftrightarrow \vec{\nabla} \cdot \vec{B} = 0$$

$$-\vec{\nabla}\varphi = \vec{E} + \dot{\vec{A}} \Leftrightarrow \vec{\nabla} \times \vec{E} + \dot{\vec{B}} = 0$$

Hamiltonovský formalismus pro EM

$$L_{EM}(\varphi, \vec{A}, \dot{\varphi}, \dot{\vec{A}}) = \frac{\epsilon_0}{2} \int_V (\vec{E}^2 - \vec{B}^2) dV$$

$$\vec{E} = -\dot{\vec{A}} - \nabla\varphi \quad \vec{B} = \nabla \times \vec{A}$$

$$L_{EM}(\varphi, \vec{A}) = \int_V (-\rho_E \varphi + \vec{j}_E \cdot \vec{A}) dV$$

$$\vec{\Pi} = \frac{\partial L}{\partial \dot{\vec{A}}} = -\epsilon_0 \vec{E} = \epsilon_0 (\dot{\vec{A}} + \nabla\varphi)$$

$$K = \frac{\partial L}{\partial \dot{\varphi}} = 0 \quad K(x) = 0 \quad K_{\nu} = \int_V \nu K dV$$

$$\varphi, \kappa, \vec{A}, \vec{\Pi} \quad K_{\nu} = 0 \quad \nu(x)$$

$$\dot{\vec{A}} = \frac{1}{\epsilon_0} \vec{\Pi} - \nabla\varphi \quad H_{\lambda}(\varphi, \kappa, \vec{A}, \vec{\Pi})$$

$$\dot{\varphi} = \lambda \quad \lambda(x)$$

$$H_{\lambda} = \int_V (\dot{\varphi} \kappa + \dot{\vec{A}} \cdot \vec{\Pi}) dV - L$$

$$= \int_V \left( \lambda \kappa + \left( \frac{1}{\epsilon_0} \vec{\Pi} - \nabla\varphi \right) \cdot \vec{\Pi} - \frac{1}{2\epsilon_0} \vec{\Pi}^2 + \frac{\epsilon_0}{2} \vec{B}^2 + \rho_E \varphi - \vec{j}_E \cdot \vec{A} \right) dV$$

$$= \int_V \left( \frac{1}{2\epsilon_0} \vec{\Pi}^2 + \frac{\epsilon_0}{2} \vec{B}^2 + \varphi (\rho_E + \nabla \cdot \vec{\Pi}) - \vec{j}_E \cdot \vec{A} + \lambda \kappa \right) dV$$

$$\frac{\delta H}{\delta \varphi} = \rho_E + \nabla \cdot \vec{\Pi} \quad \frac{\delta H}{\delta \vec{A}} = \epsilon_0 (\nabla \times \vec{B}) - \vec{j}_E$$

$$\frac{\delta H}{\delta \kappa} = \lambda \quad \frac{\delta H}{\delta \vec{\Pi}} = \frac{1}{\epsilon_0} \vec{\Pi} - \nabla\varphi$$

$$\dot{F} = \{F, H_{\lambda}\} + \frac{\partial F}{\partial t}$$

$$F(\varphi, \kappa, \vec{A}, \vec{\Pi}; t)$$

$$\nabla \cdot \vec{j}_E + \dot{\rho}_E = 0$$

$$\nabla_{\mu} J_{\mu}^m = 0$$

$$K_{\nu} = 0 \quad \kappa = 0$$

$$G_{\nu} = 0$$

$$\rho_E + \nabla \cdot \vec{\Pi} = 0$$

Zachování nabití

$$\{K_{\nu}, H_{\lambda}\} + \frac{\partial K_{\nu}}{\partial t} = 0$$

$$= - \int_V \frac{\delta K_{\nu}}{\delta \kappa} \frac{\delta H_{\lambda}}{\delta \varphi} dV = - \int_V \nu (\rho_E + \nabla \cdot \vec{\Pi}) dV$$

$$\Rightarrow \rho_E + \nabla \cdot \vec{\Pi} = 0 \quad G_{\nu}$$

$$G = -\rho_E - \nabla \cdot \vec{\Pi} = 0 \quad G_{\nu} = 0$$

$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho_E$$

$$\{G_{\nu}, H_{\lambda}\} + \frac{\partial G_{\nu}}{\partial t} = 0$$

$$\int_V \left( (-\nabla \nu) \cdot (\epsilon_0 \nabla \times \vec{B} - \vec{j}_E) - \nu \dot{\rho}_E \right) dV$$

$$= \int_V \nu (\epsilon_0 \nabla \cdot (\nabla \times \vec{B}) - (\nabla \cdot \vec{j}_E + \dot{\rho}_E)) dV$$

# Hamiltonovský formalismus pro EM

$$L_{EM}(\varphi, \vec{A}, \dot{\varphi}, \dot{\vec{A}}) = \frac{\epsilon_0}{2} \int_{\Sigma} (\vec{E}^2 - \vec{B}^2) dV$$

$$\vec{E} = -\dot{\vec{A}} - \vec{\nabla}\varphi \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

$$L_{\text{ost}}(\varphi, \vec{A}) = \int_{\Sigma} (-\rho_E \varphi + \vec{j}_E \cdot \vec{A}) dV$$

$$\vec{\Pi} = \frac{\partial L}{\partial \dot{\vec{A}}} = -\epsilon_0 \vec{E} = \epsilon_0 (\dot{\vec{A}} + \vec{\nabla}\varphi)$$

$$K = \frac{\partial L}{\partial \dot{\varphi}} = 0 \quad K(x) = 0 \quad K_y = \int_{\Sigma} \gamma K dV$$

$$\varphi, K, \vec{A}, \vec{\Pi} \quad K_y = 0 \quad \gamma(x)$$

$$\vec{A} = \frac{1}{\epsilon_0} \vec{\Pi} - \vec{\nabla} \varphi$$

$$H_\lambda(\varphi, \kappa, \vec{A}, \vec{\Pi})$$

$$\dot{\varphi} = \lambda \quad \lambda(x)$$

$$H_\lambda = \int_{\Sigma} (\dot{\varphi} \kappa + \vec{A} \cdot \vec{\Pi}) dV - L$$

$$= \int_{\Sigma} \left( \lambda \kappa + \left( \frac{1}{\epsilon_0} \vec{\Pi} - \vec{\nabla} \varphi \right) \cdot \vec{\Pi} - \frac{1}{2\epsilon_0} \vec{\Pi}^2 + \frac{\epsilon_0}{2} \vec{B}^2 + \rho_E \varphi - \vec{j}_E \cdot \vec{A} \right) dV$$

$$= \int_{\Sigma} \left( \frac{1}{2\epsilon_0} \vec{\Pi}^2 + \frac{\epsilon_0}{2} \vec{B}^2 + \varphi (\rho_E + \vec{\nabla} \cdot \vec{\Pi}) - \vec{j}_E \cdot \vec{A} + \lambda \kappa \right) dV$$

$$\vec{A} = \frac{1}{\epsilon_0} \vec{\Pi} - \vec{\nabla} \varphi$$

$$H_\lambda(\varphi, \kappa, \vec{A}, \vec{\Pi})$$

$$\dot{\varphi} = \lambda \quad \lambda(x)$$

$$H_\lambda = \int_{\Sigma} (\dot{\varphi} \kappa + \vec{A} \cdot \vec{\Pi}) dV - L$$

$$= \int_{\Sigma} \left( \lambda \kappa + \left( \frac{1}{\epsilon_0} \vec{\Pi} - \vec{\nabla} \varphi \right) \cdot \vec{\Pi} - \frac{1}{2\epsilon_0} \vec{\Pi}^2 + \frac{\epsilon_0}{2} \vec{B}^2 + \rho_E \varphi - \vec{j}_E \cdot \vec{A} \right) dV$$

$$= \int_{\Sigma} \left( \frac{1}{2\epsilon_0} \vec{\Pi}^2 + \frac{\epsilon_0}{2} \vec{B}^2 + \varphi (\rho_E + \vec{\nabla} \cdot \vec{\Pi}) - \vec{j}_E \cdot \vec{A} + \lambda \kappa \right) dV$$

$$\frac{\delta H}{\delta \varphi} = \rho_E + \vec{\nabla} \cdot \vec{\Pi}$$

$$\frac{\delta H}{\delta \vec{A}} = \epsilon_0 (\vec{\nabla} \times \vec{B}) - \vec{j}_E$$

$$\frac{\delta H}{\delta \vec{\Pi}} = \frac{1}{\epsilon_0} \vec{\Pi} - \vec{\nabla} \varphi$$

$$\frac{\delta H}{\delta \kappa} = \lambda$$

$$\dot{F} = \{F, H_u\} + \frac{\partial F}{\partial t}$$

$$F(\varphi, \kappa, \vec{A}, \vec{\Pi}; t)$$

Zachování vztah

$$\{K_v, H_v\} + \frac{\partial K_v}{\partial t} = 0$$

$$\rightarrow K_v = \int \dot{\gamma} k dt$$

$$= - \int_{\Sigma} \frac{\delta K_v}{\delta k} \frac{\delta H_v}{\delta \varphi} dV = - \int_{\Sigma} \gamma (\rho_E + \vec{\nabla} \cdot \vec{\Pi}) dV$$

$$\Rightarrow \rho_E + \vec{\nabla} \cdot \vec{\Pi} = 0 \quad = G_v$$

$$G = -\rho_E - \vec{\nabla} \cdot \vec{\Pi} = 0 \quad G_v = 0$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho_E$$

$$\{G_v, H_v\} + \frac{\partial G_v}{\partial t} = 0$$

$$\int_{\Sigma} \left( -(\vec{\nabla} \nu) \cdot (\epsilon_0 \vec{\nabla} \times \vec{B} - \dot{\gamma} \rho_E) - \nu \dot{\rho}_E \right) dV$$

$$= \int_{\Sigma} \nu \left( \epsilon_0 \vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) - (\vec{\nabla} \cdot \dot{\gamma} \rho_E + \dot{\rho}_E) \right) dV$$



$$\vec{\nabla} \cdot \vec{j}_E + \dot{\rho}_E = 0$$

$$\nabla_M J_E^M = 0$$

$$K_v = 0 \quad K = 0$$

$$G_v = 0$$

$$\rho_E + \vec{\nabla} \cdot \vec{\Pi} = 0$$

Zachování vazeb

$$\{K_v, H_v\} + \frac{\partial K_v}{\partial t} = 0$$

$$\rightarrow K_v = \int v k dt$$

$$= - \int_{\Sigma} \frac{\delta K_v}{\delta k} \frac{\delta H_v}{\delta \varphi} dV = - \int_{\Sigma} v (\rho_E + \vec{\nabla} \cdot \vec{\Pi}) dV$$

$$\Rightarrow \rho_E + \vec{\nabla} \cdot \vec{\Pi} = 0 \quad = G_v$$

$$G = -\rho_E - \vec{\nabla} \cdot \vec{\Pi} = 0 \quad G_v = 0$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho_E$$

$$\{G_v, H_v\} + \frac{\partial G_v}{\partial t} = 0$$

$$\int_{\Sigma} \left( -(\vec{\nabla} v) \cdot (\epsilon_0 \vec{\nabla} \times \vec{B} - \vec{j}_E) - v \dot{\rho}_E \right) dV$$

$$= \int_{\Sigma} v \left( \epsilon_0 \vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) - (\vec{\nabla} \cdot \vec{j}_E + \dot{\rho}_E) \right) dV$$

# Hamiltonovský formalismus pro EM

$$H_\lambda = \int_{\Sigma} \left( \frac{1}{2\epsilon_0} \vec{\Pi}^2 + \frac{\epsilon_0}{2} \vec{B}^2 - \vec{j}_E \cdot \vec{A} - \varphi G + \lambda K \right) dV$$

$$= \int_{\Sigma} \left( \frac{1}{2\epsilon_0} \vec{\Pi}^2 + \frac{\epsilon_0}{2} \vec{B}^2 - \vec{j}_E \cdot \vec{A} \right) dV - G_\varphi + K_\lambda$$

$$K=0 \quad K_\lambda = \int_{\Sigma} \lambda K dV = 0$$

$$G = -\vec{\nabla} \cdot \vec{\Pi} - \rho_E = 0$$

$$G_\lambda = -\int_{\Sigma} \lambda (\vec{\nabla} \cdot \vec{\Pi} + \rho_E) dV = -\int_{\Sigma} (\vec{\Pi} \cdot \vec{\nabla}_\nu - \lambda \rho_E) dV = 0$$

$$\frac{\delta H_\lambda}{\delta \varphi} = \vec{\nabla} \cdot \vec{\Pi} + \rho_E = -G \quad \frac{\delta H_\lambda}{\delta \vec{A}} = \epsilon_0 (\vec{\nabla} \times \vec{B}) - \vec{j}_E$$

$$\frac{\delta H_\lambda}{\delta \vec{\Pi}} = \frac{1}{\epsilon_0} \vec{\Pi} - \vec{\nabla} \varphi$$

$$\frac{\delta H_\lambda}{\delta \lambda} = \lambda$$

$$\dot{\varphi} = \frac{\delta H_\lambda}{\delta K} = \lambda$$

$$\dot{K} = -\frac{\delta H_\lambda}{\delta \varphi} = G = 0$$

$$\dot{\vec{A}} = \frac{\delta H_\lambda}{\delta \vec{\Pi}} = \frac{1}{\epsilon_0} \vec{\Pi} - \vec{\nabla} \varphi$$

$$-\frac{1}{\epsilon_0} \vec{\Pi} = \dot{\vec{E}} = -\dot{\vec{A}} - \vec{\nabla} \varphi$$

$$\dot{\vec{\Pi}} = -\frac{\delta H_\lambda}{\delta \vec{A}} = -\epsilon_0 (\vec{\nabla} \times \vec{B}) + \vec{j}_E$$

$$\vec{\nabla} \times \vec{B} - \dot{\vec{E}} = \frac{1}{\epsilon_0} \vec{j}_E$$

$$\vec{B} = \vec{\nabla} \times \vec{A} \Leftrightarrow \vec{\nabla} \cdot \vec{B} = 0$$

Částečné řešení

$\varphi$  - zadaná fce

$$K=0$$

části redukou. fce v prostoru

$$H_\varphi = \int_{\Sigma} \left( \frac{1}{2\epsilon_0} \vec{\Pi}^2 + \frac{\epsilon_0}{2} \vec{B}^2 - \vec{j}_E \cdot \vec{A} \right) dV - G_\varphi$$

$$\{ \vec{A}, G_\varphi \} = \frac{\delta G_\varphi}{\delta \vec{\Pi}} = \vec{\nabla} \varphi$$

$$\{ \vec{\Pi}, G_\varphi \} = -\frac{\delta G_\varphi}{\delta \vec{A}} = 0$$

# Hamiltonovský formalismus pro EM

$$H_\lambda = \int_{\Sigma} \left( \frac{1}{2\epsilon_0} \vec{\Pi}^2 + \frac{\epsilon_0}{2} \vec{B}^2 - \vec{j}_E \cdot \vec{A} - \rho_G \varphi + \lambda K \right) dV$$
$$= \int_{\Sigma} \left( \frac{1}{2\epsilon_0} \vec{\Pi}^2 + \frac{\epsilon_0}{2} \vec{B}^2 - \vec{j}_E \cdot \vec{A} \right) dV - G_\varphi + K_\lambda$$

$$K = 0 \quad K_\lambda = \int_{\Sigma} \lambda K dV = 0$$

$$G = -\vec{\nabla} \cdot \vec{\Pi} - \rho_E = 0$$

$$G_\lambda = -\int_{\Sigma} \lambda (\vec{\nabla} \cdot \vec{\Pi} + \rho_E) dV = \int_{\Sigma} (\vec{\Pi} \cdot \vec{\nabla}_\lambda - \lambda \rho_E) dV = 0$$

$$\frac{\delta H_\lambda}{\delta \varphi} = \vec{\nabla} \cdot \vec{\Pi} + \rho_E = -G$$

$$\frac{\delta H_\lambda}{\delta \vec{A}} = \epsilon_0 (\vec{\nabla} \times \vec{B}) - \vec{j}_E$$

$$\frac{\delta H_\lambda}{\delta K} = \lambda$$

$$\frac{\delta H_\lambda}{\delta \vec{\Pi}} = \frac{1}{\epsilon_0} \vec{\Pi} - \vec{\nabla} \varphi$$

$$\varphi = \frac{\delta H_{\lambda}}{\delta \lambda} = \lambda$$

$$\vec{K} = -\frac{\delta H_{\lambda}}{\delta \vec{\varphi}} = \vec{0} = 0$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho_E$$

$$\vec{D} = \frac{\delta H_{\lambda}}{\delta \vec{\Pi}} = \frac{1}{\epsilon_0} \vec{\Pi} - \vec{\nabla} \varphi$$

$$-\frac{1}{\epsilon_0} \vec{\Pi} = \vec{E} = -\vec{A} - \vec{\nabla} \varphi$$

$$\vec{\Pi} = -\frac{\delta H_{\lambda}}{\delta \vec{A}} = -\epsilon_0 (\vec{\nabla} \times \vec{B}) + \vec{j}_E$$

$$\vec{\nabla} \times \vec{B} - \vec{E} = \frac{1}{\epsilon_0} \vec{j}_E$$

$$\vec{B} = \vec{\nabla} \times \vec{A} \Leftrightarrow \vec{\nabla} \cdot \vec{B} = 0$$

částečné řešení

$\varphi$  - zadaná fce

$$K = 0$$

částeč. redukov. fce v prostoru

$$H_{\varphi} = \int_{\Sigma} \left( \frac{1}{2\epsilon_0} \vec{\Pi}^2 + \frac{\epsilon_0}{2} \vec{B}^2 - \vec{j}_E \cdot \vec{A} \right) dV - G_{\varphi}$$

Částečné řešení

$\varphi$  - zadaná fce

$$K = 0$$

částeč. redukov. fce v prostoru

$$H_\varphi = \int_{\Sigma} \left( \frac{1}{2\epsilon_0} \vec{\Pi}^2 + \frac{\epsilon_0}{2} \vec{B}^2 - \vec{j}_E \cdot \vec{A} \right) dV - G_\varphi$$

$$\{ \vec{A}, G_\varphi \} = \frac{\delta G_\varphi}{\delta \vec{\Pi}} = \vec{\nabla} \varphi$$

$$\{ \vec{\Pi}, G_\varphi \} = - \frac{\delta G_\varphi}{\delta \vec{A}} = 0$$