Elektromagnetické pole

$$\eta = -dt^2 + dx^2 + dy^2 + dz^2$$

∇

$c = 1$

$$A_\alpha = [-\varphi, \vec{A}] \qquad \nabla_\alpha J^\alpha_E = 0$$

$$F_{\alpha\beta} = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha$$

$$S_{EM}[A_\alpha] = -\frac{\varepsilon_0}{4}\int_\Omega F_{\mu\nu} F^{\mu\nu}\, d^4\Omega$$

$$S_{ost}[\dots, A_\alpha] = \int_\Omega J^\alpha_E A_\alpha\, d^4\Omega \qquad dt\,dx\,dy\,dz$$

princip extremální akce

$$0 = \delta S[A] = S[A + \delta A] - S[A]$$

$$= -\frac{\varepsilon_0}{2}\int_\Omega (\nabla_\mu \delta A_\nu - \nabla_\nu \delta A_\mu) F^{\mu\nu}\, d^4\Omega + \delta S_{ost}$$

$$= -\varepsilon_0 \int_\Omega (\nabla_\mu \delta A_\nu) F^{\mu\nu}\, d^4\Omega + \delta S_{ost}$$

$$= -\varepsilon_0 \int_\Omega \left(\nabla_\mu(\delta A_\nu F^{\mu\nu}) - \delta A_\nu \nabla_\mu F^{\mu\nu}\right) d^4\Omega + \delta S_{ost}$$

$$= \varepsilon_0 \int_\Omega \left(-\nabla_\mu F^{\nu\mu} + \frac{1}{\varepsilon_0}\frac{\delta S_{ost}}{\delta A_\nu}\right) \delta A_\nu\, d^4\Omega$$

$$\frac{1}{\varepsilon_0}\frac{\delta S}{\delta A_\nu} = -\nabla_\mu F^{\nu\mu} + \frac{1}{\varepsilon_0}\frac{\delta S_{ost}}{\delta A_\nu} = 0 \qquad \frac{\delta S_{ost}}{\delta A_\nu} \longrightarrow J^\nu$$

$$\nabla_\mu F^{\nu\mu} = \frac{1}{\varepsilon_0} J^\nu \qquad J^\nu = J^\nu_E$$

$$\nabla_{[\lambda} F_{\kappa\nu]} = 0 \Leftrightarrow F_{\alpha\beta} = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha$$

$$F_{\alpha\beta} = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{bmatrix}$$

$$F^{\alpha\beta} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{bmatrix}$$

$$S_{EM} = \frac{\varepsilon_0}{2}\int_\Omega (\vec{E}^2 - \vec{B}^2)\, d^4\Omega \qquad dt\,dV$$

$$\vec{E} = -\vec{\nabla}\varphi - \dot{\vec{A}}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$J^\mu = \begin{bmatrix} \varrho \\ \vec{j} \end{bmatrix} \qquad J^\mu_E A_\mu = -\varrho_E \varphi + \vec{j}_E \cdot \vec{A}$$

$$S_{EM} = \int_I \underbrace{\frac{\varepsilon_0}{2}\int_\Sigma (\vec{E}^2 - \vec{B}^2)\, dV}_{L_{EM}(\varphi, \vec{A}, \dot{\varphi}, \dot{\vec{A}})}\, dt$$

$$L_{EM} = \frac{\varepsilon_0}{2}\int_\Sigma (\vec{E}^2 - \vec{B}^2)\, dV$$

$$S_{ost} = \int_I L_{ost}(\varphi, \vec{A}, \dots)$$

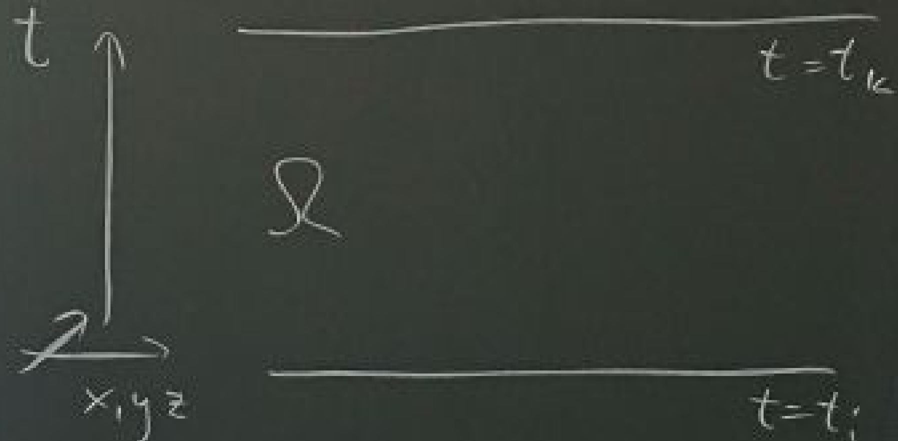
$$L_{ost} = \int_\Sigma (-\varrho_E \varphi + \vec{j}_E \cdot \vec{A})\, dV$$

Elektromagnetické pole

$$\eta = -dt^2 + dx^2 + dy^2 + dz^2$$

∇

$c = 1$



$$A_\alpha = [-\varphi, \vec{A}] \qquad \nabla_\alpha J_E^\alpha = 0$$

$$F_{\alpha\beta} = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha$$

$$S_{EM}[A_\alpha] = -\frac{\varepsilon_0}{4}\int_\Omega F_{\mu\nu}F^{\mu\nu}\, d^4\Omega$$

$$S_{ost}[\ldots, A_\alpha] = \int_\Omega J_E^\alpha A_\alpha\, d^4\Omega \qquad dt\,dx\,dy\,dz$$

princip extremální akce

$$0 = \delta S[A] = S[A+\delta A] - S[A]$$

$$= -\frac{\varepsilon_0}{2}\int_\Omega \left(\nabla_\mu \delta A_\nu - \nabla_\nu \delta A_\mu\right) F^{\mu\nu}\, d^4\Omega + \delta S_{ost}$$

$$= -\varepsilon_0 \int_\Omega \left(\nabla_\mu \delta A_\nu\right) F^{\mu\nu}\, d^4\Omega + \delta S_{ost}$$

$$= -\varepsilon_0 \int_\Omega \left(\nabla_\mu\left(\delta A_\nu F^{\mu\nu}\right) - \delta A_\nu \nabla_\mu F^{\mu\nu}\right) d^4\Omega + \delta S_{ost}$$

$$= \varepsilon_0 \int_\Omega \left(-\nabla_\mu F^{\nu\mu} + \frac{1}{\varepsilon_0}\frac{\delta S_{ost}}{\delta A_\nu}\right)\delta A_\nu\, d^4\Omega$$

$$\frac{1}{\varepsilon_0}\frac{\delta S}{\delta A_\nu} = -\nabla_\mu F^{\nu\mu} + \frac{1}{\varepsilon_0}\frac{\delta S_{ost}}{\delta A_\nu} = 0 \qquad \frac{\delta S_{ost}}{\delta A_\nu} \to J^\nu$$

$$\nabla_\mu F^{\nu\mu} = \frac{1}{\varepsilon_0} J^\nu \qquad J^\nu = J^\nu_E$$

$$\nabla_{[\mu} F_{\kappa\lambda]} = 0 \iff F_{\alpha\beta} = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha$$

$$F_{\alpha\beta} = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{bmatrix}$$

$$F^{\alpha\beta} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{bmatrix}$$

$$S_{EM} = \frac{\varepsilon_0}{2} \int_\Omega \left(\vec{E}^2 - \vec{B}^2 \right) d^4\Omega$$

$$dt\, dV$$

$$\vec{E} = -\vec{\nabla}\varphi - \dot{\vec{A}}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$J^\mu = \begin{bmatrix} \rho \\ \vec{j} \end{bmatrix} \qquad J^\mu_E A_\mu = -\rho_E \varphi + \vec{j}_E \cdot \vec{A}$$

$$S_{EM} = \int_I \underbrace{\frac{\varepsilon_0}{2} \int_\Sigma \left(\vec{E}^2 - \vec{B}^2 \right) dV}_{L_{EM}(\varphi, \vec{A}, \dot{\varphi}, \dot{\vec{A}})} \, dt$$

$$L_{EM} = \frac{\varepsilon_0}{2} \int_\Sigma \left(\vec{E}^2 - \vec{B}^2 \right) dV$$

$$S_{est} = \int_I L_{est}(\varphi, \vec{A}, \ldots)$$

$$L_{est} = \int_\Sigma \left(-\rho_E \varphi + \vec{j}_E \cdot \vec{A} \right) dV$$

Lagrangeovský formalismus pro EM

$$L_{EM}(\varphi, \vec A, \dot\varphi, \dot{\vec A}) = \frac{\varepsilon_0}{2}\int_\Sigma \left(\vec E^2 - \vec B^2\right) dV$$

$$\vec E = -\dot{\vec A} - \vec\nabla\varphi \qquad \vec B = \vec\nabla\times\vec A$$

$$L_{ost}(\varphi, \vec A) \underset{\text{maj.}}{=} \int_\Sigma \left(-\rho_E\varphi + \vec j_E\cdot\vec A\right) dV$$

$$\left(\frac{\delta L}{\delta\dot{\vec A}}\right)^{\cdot} - \frac{\delta L}{\delta\vec A} = 0 \qquad \left(\frac{\delta L_{EM}}{\delta\dot{\vec A}}\right)^{\cdot} - \frac{\delta L_{EM}}{\delta\vec A} = \vec j$$

$$\left(\frac{\delta L}{\delta\dot\varphi}\right)^{\cdot} - \frac{\delta L}{\delta\varphi} = 0 \qquad \left(\frac{\delta L_{EM}}{\delta\dot\varphi}\right)^{\cdot} - \frac{\delta L_{EM}}{\delta\varphi} = -\rho$$

$$\vec j = \frac{\delta L_{ost}}{\delta\vec A} = \vec j_E \qquad \rho = -\frac{\delta L_{ost}}{\delta\varphi} = \rho_E$$

$$\delta\dot{\vec A} \qquad \delta L_{EM} = \varepsilon_0\int_\Sigma \left(-\delta\dot{\vec A}\right)\cdot\vec E\, dV$$

$$\vec\Pi = \frac{\delta L_{EM}}{\delta\dot{\vec A}} = -\varepsilon_0\vec E$$

$$\delta\vec A \qquad \delta L_{EM} = -\varepsilon_0\int_\Sigma \left(\vec\nabla\times\delta\vec A\right)\cdot\vec B\, dV$$

$$= \varepsilon_0\int_\Sigma \left(\vec\nabla\cdot\left(\vec B\times\delta\vec A\right) - \delta\vec A\cdot\vec\nabla\times\vec B\right) dV$$

$$\vec\nabla\cdot\left(\vec B\times\delta\vec A\right) + \vec\nabla\cdot\left(\vec B\times\delta\vec A\right)$$

$$\delta\vec A\cdot\vec\nabla\times\vec B - \vec B\cdot\left(\vec\nabla\times\delta\vec A\right)$$

$$= -\varepsilon_0\int \delta\vec A\cdot\vec\nabla\times\vec B\, dV$$

$$\frac{\delta L_{EM}}{\delta\vec A} = -\varepsilon_0\vec\nabla\times\vec B$$

$$K = \frac{\delta L_{EM}}{\delta\dot\varphi} = 0$$

$$\delta\varphi \qquad \delta L_{EM} = \varepsilon_0\int_\Sigma \left(-\vec\nabla\delta\varphi\right)\cdot\vec E\, dV$$

$$= \varepsilon_0\int_\Sigma \delta\varphi\,\vec\nabla\cdot\vec E\, dV$$

$$\frac{\delta L_{EM}}{\delta\varphi} = \varepsilon_0\vec\nabla\cdot\vec E$$

$$\left(-\varepsilon_0\vec E\right)^{\cdot} + \varepsilon_0\vec\nabla\times\vec B = \vec j$$

$$\vec\nabla\times\vec B - \dot{\vec E} = \frac{1}{\varepsilon_0}\vec j$$

$$\vec\nabla\cdot\vec E = \frac{1}{\varepsilon_0}\rho$$

$$\vec B = \vec\nabla\times\vec A \Leftrightarrow \vec\nabla\cdot\vec B = 0$$

$$-\vec\nabla\varphi = \vec E + \dot{\vec A} \Leftrightarrow \vec\nabla\times\vec E + \dot{\vec B} = 0$$

Lagrangeovský formalismus pro EM

$$L_{EM}(\varphi, \vec{A}, \dot{\varphi}, \dot{\vec{A}}) = \frac{\varepsilon_0}{2} \int_{\Sigma} (\vec{E}^2 - \vec{B}^2)\, dV$$

$$\vec{E} = -\dot{\vec{A}} - \vec{\nabla}\varphi \qquad \vec{B} = \vec{\nabla} \times \vec{A}$$

$$L_{ost}(\varphi, \vec{A}) \underset{\text{maj\'{i}}}{=} \int_{\Sigma} (-\rho_E \varphi + \vec{j}_E \cdot \vec{A})\, dV$$

$$\left(\frac{\delta L}{\delta \dot{\vec{A}}}\right)^{\cdot} - \frac{\delta L}{\delta \vec{A}} = 0 \qquad \left(\frac{\delta L_{EM}}{\delta \dot{\vec{A}}}\right)^{\cdot} - \frac{\delta L_{EM}}{\delta \vec{A}} = \vec{j}$$

$$\left(\frac{\delta L}{\delta \dot{\varphi}}\right)^{\cdot} - \frac{\delta L}{\delta \varphi} = 0 \qquad \left(\frac{\delta L_{EM}}{\delta \dot{\varphi}}\right)^{\cdot} - \frac{\delta L_{EM}}{\delta \varphi} = -\rho$$

$$\vec{j} = \frac{\delta L_{ost}}{\delta \vec{A}} = \vec{j}_E \qquad \rho = -\frac{\delta L_{ost}}{\delta \varphi} = \rho_E$$

$$\delta\dot{\vec{A}} \qquad \delta L_{EM} = \varepsilon_0 \int_\Sigma \left(-\delta\dot{\vec{A}}\right)\cdot\vec{E}\; dV$$

$$\vec{\Pi} = \frac{\delta L_{EM}}{\delta\dot{\vec{A}}} = -\varepsilon_0 \vec{E}$$

$$\delta\vec{A} \qquad \delta L_{EM} = -\varepsilon_0 \int_\Sigma \left(\vec{\nabla}\times\delta\vec{A}\right)\cdot\vec{B}\; dV$$

$$= \varepsilon_0 \int_\Sigma \left(\vec{\nabla}\cdot\left(\vec{B}\times\delta\vec{A}\right) - \delta\vec{A}\cdot\vec{\nabla}\times\vec{B}\right) dV$$

$$\vec{\nabla}\cdot\left(\vec{B}\times\delta\vec{A}\right) + \vec{\nabla}\cdot\left(\vec{B}\times\delta\vec{A}\right)$$

$$\delta\vec{A}\cdot\vec{\nabla}\times\vec{B} - \vec{B}\cdot\left(\vec{\nabla}\times\delta\vec{A}\right)$$

$$= -\varepsilon_0 \int \delta\vec{A}\cdot\vec{\nabla}\times\vec{B}\; dV$$

$$\frac{\delta L_{EM}}{\delta\vec{A}} = -\varepsilon_0 \vec{\nabla}\times\vec{B}$$

$$\delta\varphi \qquad \delta L_{EM} = \varepsilon_0 \int_\Sigma (-\vec{\nabla}\delta\varphi)\cdot\vec{E}\, dV$$

$$= \varepsilon_0 \int_\Sigma \delta\varphi\, \vec{\nabla}\cdot\vec{E}\, dV$$

$$\frac{\delta L_{EM}}{\delta\varphi} = \varepsilon_0 \vec{\nabla}\cdot\vec{E}$$

$$(-\varepsilon_0 \vec{E})^{\cdot} + \varepsilon_0 \vec{\nabla}\times\vec{B} = \vec{j}$$

$$\vec{\nabla}\times\vec{B} - \dot{\vec{E}} = \frac{1}{\varepsilon_0}\vec{j}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho$$

$$\delta\varphi \qquad \delta L_{EM} = \varepsilon_0 \int_{\Sigma} (-\vec{\nabla}\delta\varphi)\cdot\vec{E}\, dV$$

$$= \varepsilon_0 \int_{\Sigma} \delta\varphi\, \vec{\nabla}\cdot\vec{E}\, dV$$

$$\frac{\delta L_{EM}}{\delta\varphi} = \varepsilon_0 \vec{\nabla}\cdot\vec{E}$$

$$(-\varepsilon_0 \vec{E})^{\cdot} + \varepsilon_0 \vec{\nabla}\times\vec{B} = \vec{j}$$

$$\vec{\nabla}\times\vec{B} - \dot{\vec{E}} = \frac{1}{\varepsilon_0}\vec{j}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho$$

$$\vec{B} = \vec{\nabla}\times\vec{A} \iff \vec{\nabla}\cdot\vec{B} = 0$$

$$-\vec{\nabla}\varphi = \vec{E} + \dot{\vec{A}} \iff \vec{\nabla}\times\vec{E} + \dot{\vec{B}} = 0$$

Hamiltonovský formalismus pro EM

$$L_{EM}(\varphi,\vec{A},\dot{\varphi},\dot{\vec{A}}) = \frac{\varepsilon_0}{2}\int_\Sigma (\vec{E}^2-\vec{B}^2)\,dV$$

$$\vec{E} = -\dot{\vec{A}} - \vec{\nabla}\varphi \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

$$L_{ext}(\varphi,\vec{A}) \underset{\text{mají}}{=} \int_\Sigma \left(-\varrho_E\varphi + \vec{j}_E\cdot\vec{A}\right)dV$$

$$\vec{\Pi} = \frac{\delta L}{\delta \dot{\vec{A}}} = -\varepsilon_0\vec{E} = \varepsilon_0(\dot{\vec{A}}+\vec{\nabla}\varphi)$$

$$K = \frac{\delta L}{\delta\dot{\varphi}} = 0 \qquad K(x)=0 \qquad K_\nu = \int_\Sigma \nu K\,dV$$

$$\varphi, K, \vec{A}, \vec{\Pi} \qquad K_\nu = 0 \qquad \nu(x)$$

$$\dot{\vec{A}} = \frac{1}{\varepsilon_0}\vec{\Pi} - \vec{\nabla}\varphi$$

$$\dot{\varphi} = \lambda \qquad \lambda(x)$$

$$H_\lambda(\varphi, K, \vec{A}, \vec{\Pi})$$

$$\dot{F} = \{F, H_\lambda\} + \frac{\partial F}{\partial t}$$

$$F(\varphi, K, \vec{A}, \vec{\Pi}; t)$$

$$H_\lambda = \int_\Sigma (\dot{\varphi}K + \dot{\vec{A}}\cdot\vec{\Pi})\,dV - L$$

$$= \int_\Sigma \left(\lambda K + \left(\frac{1}{\varepsilon_0}\vec{\Pi} - \vec{\nabla}\varphi\right)\cdot\vec{\Pi} - \frac{1}{2\varepsilon_0}\vec{\Pi}^2 + \frac{\varepsilon_0}{2}\vec{B}^2 + \varrho_E\varphi - \vec{j}_E\cdot\vec{A}\right)dV$$

$$= \int_\Sigma \left(\frac{1}{2\varepsilon_0}\vec{\Pi}^2 + \frac{\varepsilon_0}{2}\vec{B}^2 + \varphi(\varrho_E + \vec{\nabla}\cdot\vec{\Pi}) - \vec{j}_E\cdot\vec{A} + \lambda K\right)dV$$

$$\frac{\delta H}{\delta\varphi} = \varrho_E + \vec{\nabla}\cdot\vec{\Pi} \qquad \frac{\delta H}{\delta\vec{A}} = \varepsilon_0(\vec{\nabla}\times\vec{B}) - \vec{j}_E$$

$$\frac{\delta H}{\delta K} = \lambda \qquad \frac{\delta H}{\delta\vec{\Pi}} = \frac{1}{\varepsilon_0}\vec{\Pi} - \vec{\nabla}\varphi$$

$$\vec{\nabla}\cdot\vec{j}_E + \dot{\varrho}_E = 0$$

$$\nabla_\mu J^\mu_E = 0$$

$$K_\nu = 0 \qquad K = 0$$

$$G_\nu = 0$$

$$\varrho_E + \vec{\nabla}\cdot\vec{\Pi} = 0$$

Zachování vazeb

$$\{K_\nu, H_\lambda\} + \frac{\partial K_\nu}{\partial t} = 0 \qquad \xrightarrow{\frac{\partial K_\nu}{\partial t}} \; K_\nu = \int \nu K\,dt$$

$$= -\int_\Sigma \frac{\delta K_\nu}{\delta K}\frac{\delta H_\lambda}{\delta\varphi}\,dV = -\int_\Sigma \nu(\varrho_E + \vec{\nabla}\cdot\vec{\Pi})\,dV$$

$$= G_\nu$$

$$\Rightarrow \varrho_E + \vec{\nabla}\cdot\vec{\Pi} = 0$$

$$G = -\varrho_E - \vec{\nabla}\cdot\vec{\Pi} = 0 \qquad G_\nu = 0$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\varrho_E$$

$$\{G_\nu, H_\lambda\} + \frac{\partial G_\nu}{\partial t} = 0$$

$$\int_\Sigma \left(-(\vec{\nabla}\nu)\cdot(\varepsilon_0\vec{\nabla}\times\vec{B} - \vec{j}_E) - \nu\dot{\varrho}_E\right)dV$$

$$= \int_\Sigma \nu\left(\cancel{\varepsilon_0\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B})} - \cancel{(\vec{\nabla}\cdot\vec{j}_E + \dot{\varrho}_E)}\right)dV$$

Hamiltonovský formalismus pro EM

$$L_{EM}(\varphi, \vec{A}, \dot{\varphi}, \dot{\vec{A}}) = \frac{\varepsilon_0}{2}\int_\Sigma (\vec{E}^2 - \vec{B}^2)\, dV$$

$$\vec{E} = -\dot{\vec{A}} - \vec{\nabla}\varphi \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

$$L_{ost}(\varphi, \vec{A}) \underset{\text{maję.}}{=} \int_\Sigma \left(-\rho_E\varphi + \vec{j}_E\cdot\vec{A}\right) dV$$

$$\vec{\Pi} = \frac{\delta L}{\delta\dot{\vec{A}}} = -\varepsilon_0\vec{E} = \varepsilon_0(\dot{\vec{A}} + \vec{\nabla}\varphi)$$

$$K = \frac{\delta L}{\delta\dot{\varphi}} = 0 \qquad K(x) = 0 \qquad K_\nu = \int_\Sigma \nu K\, dV$$

$$\varphi, K, \vec{A}, \vec{\Pi} \qquad K_\nu = 0 \qquad \nu(x)$$

$$\dot{\vec{A}} = \frac{1}{\varepsilon_0}\vec{\Pi} - \vec{\nabla}\varphi$$

$$\dot{\varphi} = \lambda \qquad \lambda(x)$$

$$H_\lambda(\varphi, \kappa, \vec{A}, \vec{\Pi})$$

$$H_\lambda = \int_\Sigma \left(\dot{\varphi}\kappa + \dot{\vec{A}}\cdot\vec{\Pi}\right) dV - L$$

$$= \int_\Sigma \left(\lambda\kappa + \left(\frac{1}{\varepsilon_0}\underline{\vec{\Pi}} - \underline{\vec{\nabla}\varphi\right)\cdot\vec{\Pi}} - \underline{\frac{1}{2\varepsilon_0}\vec{\Pi}^2} + \underline{\frac{\varepsilon_0}{2}\vec{B}^2} + \underline{\rho_E\varphi} - \underline{\vec{j}_E\cdot\vec{A}}\right) dV$$

$$= \int_\Sigma \left(\frac{1}{2\varepsilon_0}\vec{\Pi}^2 + \frac{\varepsilon_0}{2}\vec{B}^2 + \varphi\left(\rho_E + \vec{\nabla}\cdot\vec{\Pi}\right) - \vec{j}_E\cdot\vec{A} + \lambda\kappa\right) dV$$

$$\dot{\vec{A}} = \frac{1}{\varepsilon_0}\vec{\Pi} - \vec{\nabla}\varphi$$

$$H_\lambda(\varphi, \kappa, \vec{A}, \vec{\Pi})$$

$$\dot{\varphi} = \lambda \qquad \lambda(x)$$

$$H_\lambda = \int_\Sigma \left(\dot{\varphi}\kappa + \dot{\vec{A}}\cdot\vec{\Pi}\right) dV - L$$

$$= \int_\Sigma \left(\lambda\kappa + \left(\frac{1}{\varepsilon_0}\vec{\Pi} - \vec{\nabla}\varphi\right)\cdot\vec{\Pi} - \frac{1}{2\varepsilon_0}\vec{\Pi}^2 + \frac{\varepsilon_0}{2}\vec{B}^2 + \rho_E\varphi - \vec{j}_E\cdot\vec{A} \right) dV$$

$$= \int_\Sigma \left(\frac{1}{2\varepsilon_0}\vec{\Pi}^2 + \frac{\varepsilon_0}{2}\vec{B}^2 + \varphi\left(\rho_E + \vec{\nabla}\cdot\vec{\Pi}\right) - \vec{j}_E\cdot\vec{A} + \lambda\kappa \right) dV$$

$$\frac{\delta H}{\delta \varphi} = \rho_E + \vec{\nabla}\cdot\vec{\Pi} \qquad \frac{\delta H}{\delta \vec{A}} = \varepsilon_0\left(\vec{\nabla}\times\vec{B}\right) - \vec{j}_E$$

$$\frac{\delta H}{\delta \kappa} = \lambda \qquad \frac{\delta H}{\delta \vec{\Pi}} = \frac{1}{\varepsilon_0}\vec{\Pi} - \vec{\nabla}\varphi$$

$$\dot{F} = \{F, H_\lambda\} + \frac{\partial F}{\partial t}$$

$$F(\varphi, \kappa, \vec{A}, \vec{\pi}; t)$$

Zachování vazeb

$$\{K_\nu , H_\lambda\} + \frac{\partial K_\nu}{\partial t} = 0$$

$$K_{\dot\nu} = \int \dot\nu K \, dt$$

$$= -\int_\Sigma \frac{\delta K_\nu}{\delta K}\frac{\delta H_\lambda}{\delta \varphi} \, dV = -\int_\Sigma \nu\left(\rho_E + \vec\nabla\cdot\vec\Pi\right) dV$$

$$= G_\nu$$

$$\Rightarrow \rho_E + \vec\nabla\cdot\vec\Pi = 0 \qquad G_\nu = 0$$

$$G = -\rho_E - \vec\nabla\cdot\vec\Pi = 0$$

$$\vec\nabla\cdot\vec E = \frac{1}{\varepsilon_0}\rho_E$$

$$\{G_\nu , H_\lambda\} + \frac{\partial G_\nu}{\partial t} = 0$$

$$\int_\Sigma \left(-(\vec\nabla\nu)\cdot\left(\varepsilon_0 \vec\nabla\times\vec B - \vec j_E\right) - \nu\dot\rho_E\right) dV$$

$$= \int_\Sigma \nu\left(\varepsilon_0 \cancel{\vec\nabla\cdot(\vec\nabla\times\vec B)} - \cancel{\left(\vec\nabla\cdot\vec j_E + \dot\rho_E\right)}\right) dV$$

$$\vec{\nabla}\cdot\vec{j}_E + \dot{\rho}_E = 0$$

$$\nabla_\mu J^\mu_E = 0$$

$$K_\nu = 0 \qquad K = 0$$

$$G_\nu = 0$$

$$\rho_E + \vec{\nabla}\cdot\vec{\Pi} = 0$$

Zachování vazeb

$$\{K_\nu, H_\lambda\} + \frac{\partial K_\nu}{\partial t} = 0 \qquad \frac{\partial K_\nu}{\partial t} \longrightarrow K_{\dot{\nu}} = \int \dot{\nu} K \, dt$$

$$= -\int_\Sigma \frac{\delta K_\nu}{\delta K}\frac{\delta H_\lambda}{\delta \varphi}\, dV = -\int_\Sigma \nu\left(\rho_E + \vec{\nabla}\cdot\vec{\Pi}\right) dV$$

$$= G_\nu$$

$$\Rightarrow \rho_E + \vec{\nabla}\cdot\vec{\Pi} = 0 \qquad G_\nu = 0$$

$$G = -\rho_E - \vec{\nabla}\cdot\vec{\Pi} = 0$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho_E$$

$$\{G_\nu, H_\lambda\} + \frac{\partial G_\nu}{\partial t} = 0$$

$$\int_\Sigma \left((-\vec{\nabla}\nu)\cdot\left(\varepsilon_0 \vec{\nabla}\times\vec{B} - \vec{j}_E\right) - \nu\dot{\rho}_E \right) dV$$

$$= \int_\Sigma \nu\left(\varepsilon_0 \cancel{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B})} - \cancel{\left(\vec{\nabla}\cdot\vec{j}_E + \dot{\rho}_E\right)}\right) dV$$

Hamiltonovský formalismus pro EM

$$H_\lambda = \int_\Sigma \left(\frac{1}{2\varepsilon_0}\vec{\Pi}^2 + \frac{\varepsilon_0}{2}\vec{B}^2 - \vec{j}_E\cdot\vec{A} - \varphi G + \lambda K\right) dV$$

$$= \int_\Sigma \left(\frac{1}{2\varepsilon_0}\vec{\Pi}^2 + \frac{\varepsilon_0}{2}\vec{B}^2 - \vec{j}_E\cdot\vec{A}\right) dV - G_\varphi + K_\lambda$$

$K = 0 \qquad K_\nu = \int_\Sigma \nu K \, dV = 0$

$G = -\vec{\nabla}\cdot\vec{\Pi} - \varrho_E = 0$

$G_\nu = -\int_\Sigma \nu(\vec{\nabla}\cdot\vec{\Pi} + \varrho_E)\, dV = \int_\Sigma (\vec{\Pi}\cdot\vec{\nabla}\nu - \nu\varrho_E)\, dV = 0$

$\frac{\delta H_\lambda}{\delta\varphi} = \vec{\nabla}\cdot\vec{\Pi} + \varrho_E = -G \qquad \frac{\delta H_\lambda}{\delta\vec{A}} = \varepsilon_0(\vec{\nabla}\times\vec{B}) - \vec{j}_E$

$\frac{\delta H_\lambda}{\delta K} = \lambda \qquad \frac{\delta H_\lambda}{\delta\vec{\Pi}} = \frac{1}{\varepsilon_0}\vec{\Pi} - \vec{\nabla}\varphi$

$\dot{\varphi} = \frac{\delta H_\lambda}{\delta K} = \lambda$

$\dot{K} = -\frac{\delta H}{\delta\varphi} = G = 0 \qquad \vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\varrho_E$

$\dot{\vec{A}} = \frac{\delta H_\lambda}{\delta\vec{\Pi}} = \frac{1}{\varepsilon_0}\vec{\Pi} - \vec{\nabla}\varphi$

$-\frac{1}{\varepsilon_0}\vec{\Pi} = \vec{E} = -\dot{\vec{A}} - \vec{\nabla}\varphi$

$\dot{\vec{\Pi}} = -\frac{\delta H_\lambda}{\delta\vec{A}} = -\varepsilon_0(\vec{\nabla}\times\vec{B}) + \vec{j}_E$

$\vec{\nabla}\times\vec{B} - \dot{\vec{E}} = \frac{1}{\varepsilon_0}\vec{j}_E$

$\vec{B} = \vec{\nabla}\times\vec{A} \Leftrightarrow \vec{\nabla}\cdot\vec{B} = 0$

částečné řešení

φ – zadané fce

$K = 0$

částeč. redukov. fáz. prostor

$H_\varphi = \int_\Sigma \left(\frac{1}{2\varepsilon_0}\vec{\Pi}^2 + \frac{\varepsilon_0}{2}\vec{B}^2 - \vec{j}_E\cdot\vec{A}\right) dV - G_\varphi$

$\{\vec{A}, G_\varphi\} = \frac{\delta G_\varphi}{\delta\vec{\Pi}} = \vec{\nabla}\varphi$

$\{\vec{\Pi}, G_\varphi\} = -\frac{\delta G_\varphi}{\delta\vec{A}} = 0$

Hamiltonovský formalismus pro EM

$$H_\lambda = \int_\Sigma \left(\frac{1}{2\varepsilon_0}\vec{\Pi}^2 + \frac{\varepsilon_0}{2}\vec{B}^2 - \vec{j}_E \cdot \vec{A} - \varphi G + \lambda k \right) dV$$

$$= \int_\Sigma \left(\frac{1}{2\varepsilon_0}\vec{\Pi}^2 + \frac{\varepsilon_0}{2}\vec{B}^2 - \vec{j}_E \cdot \vec{A} \right) dV - G_\varphi + K_\lambda$$

$$k = 0 \qquad K_\nu = \int_\Sigma \nu k \, dV = 0$$

$$G = -\vec{\nabla} \cdot \vec{\Pi} - \rho_E = 0$$

$$G_\nu = -\int_\Sigma \nu (\vec{\nabla} \cdot \vec{\Pi} + \rho_E) dV = \int_\Sigma (\vec{\Pi} \cdot \vec{\nabla}\nu - \nu \rho_E) dV = 0$$

$$\frac{\delta H_\lambda}{\delta \varphi} = \vec{\nabla} \cdot \vec{\Pi} + \rho_E = -G \qquad \frac{\delta H_\lambda}{\delta \vec{A}} = \varepsilon_0 (\vec{\nabla} \times \vec{B}) - \vec{j}_E$$

$$\frac{\delta H_\lambda}{\delta k} = \lambda \qquad \frac{\delta H_\lambda}{\delta \vec{\Pi}} = \frac{1}{\varepsilon_0}\vec{\Pi} - \vec{\nabla}\varphi$$

$$\dot{\varphi} = \frac{\delta H_\lambda}{\delta \kappa} = \lambda$$

$$\dot{\kappa} = -\frac{\delta H}{\delta \varphi} = G = 0 \qquad \vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho_E$$

$$\dot{\vec{A}} = \frac{\delta H_\lambda}{\delta \vec{\pi}} = \frac{1}{\varepsilon_0}\vec{\pi} - \vec{\nabla}\varphi$$

$$-\frac{1}{\varepsilon_0}\vec{\pi} = \vec{E} = -\dot{\vec{A}} - \vec{\nabla}\varphi$$

$$\dot{\vec{\pi}} = -\frac{\delta H_\lambda}{\delta \vec{A}} = -\varepsilon_0(\vec{\nabla}\times\vec{B}) + \vec{j}_E$$

$$\vec{\nabla}\times\vec{B} - \dot{\vec{E}} = \frac{1}{\varepsilon_0}\vec{j}_E$$

$$\vec{B} = \vec{\nabla}\times\vec{A} \iff \vec{\nabla}\cdot\vec{B} = 0$$

Částečné řešení

φ – zadané fce

$K = 0$

částeč. redukov. fáz. prostor

$$H_\varphi = \int_\Sigma \left(\frac{1}{2\varepsilon_0} \vec{\Pi}^2 + \frac{\varepsilon_0}{2} \vec{B}^2 - \vec{j}_E \cdot \vec{A} \right) dV - G_\varphi$$

částečné řešení

φ – zadané fce

$K = 0$

částeč. redukov. fáz. prostor

$$H_\varphi = \int_\Sigma \left(\frac{1}{2\varepsilon_0} \vec{\Pi}^2 + \frac{\varepsilon_0}{2} \vec{B}^2 - \vec{j}_E \cdot \vec{A} \right) dV - G_\varphi$$

$$\{ \vec{A}, G_\varphi \} = \frac{\delta G_\varphi}{\delta \vec{\Pi}} = \vec{\nabla} \varphi$$

$$\{ \vec{\Pi}, G_\varphi \} = - \frac{\delta G_\varphi}{\delta \vec{A}} = 0$$