

# (Equivalence Principle and) Accelerating Type I Black Holes

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[2305.03744] , [2307.10534]

- Accelerating black holes with NUT parameter are puzzling, while the rotating case belongs to the Plebanski-Demianski class the non rotating metric (accelerating Taub-NUT) is not of type D.
- Solution generating techniques in GR can generate all axisymmetric and stationary solutions of the theory. In this context Einstein and Ernst equations are equivalent. Symmetries of the Ernst equations allow us to generate new non-trivial solutions starting from old ones.
- The Ehlers transformation of Ernst equations is able to add the gravitomagnetic mass to a given axisymmetric and stationary seed. Which is its action when applied to accelerating black holes?

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# Introduction: Theory and Field Equations

Theory under consideration: General Relativity coupled with Maxwell electromagnetism

$$I[g_{\mu\nu}, A_\mu] := \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[ R - \frac{G}{\mu_0} F_{\mu\nu} F^{\mu\nu} \right] .$$

Field equations for the metric  $g_{\mu\nu}$  and electromagnetic vector potential  $A_\mu$

$$\begin{aligned} R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} &= \frac{2G}{\mu_0} \left( F_{\mu\rho} F_\nu{}^\rho - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right) , \\ \partial_\mu (\sqrt{-g} F^{\mu\nu}) &= 0 . \end{aligned}$$

The Faraday tensor  $F_{\mu\nu}$  is defined from the gauge potential,  $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$ .

The most generic axisymmetric and stationary spacetime, containing two commuting killing vectors  $\partial_t$  and  $\partial_\varphi$ , can be written, for this theory, in the Lewis-Weyl-Papapetrou (LWP) form as

$$ds^2 = -f (dt - \omega d\varphi)^2 + f^{-1} \left[ \rho^2 d\varphi^2 + e^{2\gamma} (d\rho^2 + dz^2) \right] .$$

All the three structure functions appearing in the metric  $f, \omega$  and  $\gamma$  depends only on the non-Killing coordinates  $(\rho, z)$ .

A generic electromagnetic potential compatible with the spacetime symmetries, and the circularity of the LWP metric, is given by  $A = A_t(\rho, z)dt + A_\varphi(\rho, z)d\varphi$ .

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Ernst (Phys. Rev. 1968) discovered that, when the Einstein field equations are restricted to the axisymmetric and stationary LWP ansatz, they reduce to a couple of complex vectorial differential equations

## Ernst Equations

$$\begin{aligned}(\operatorname{Re} \mathcal{E} + |\Phi|^2) \nabla^2 \mathcal{E} &= (\vec{\nabla} \mathcal{E} + 2 \Phi^* \vec{\nabla} \Phi) \cdot \vec{\nabla} \mathcal{E} \quad , \\(\operatorname{Re} \mathcal{E} + |\Phi|^2) \nabla^2 \Phi &= (\vec{\nabla} \mathcal{E} + 2 \Phi^* \vec{\nabla} \Phi) \cdot \vec{\nabla} \Phi \quad .\end{aligned}$$

The complex Ernst potential are defined as

$$\Phi := A_t + i \tilde{A}_\varphi \quad , \quad \mathcal{E} := f - |\Phi \Phi^*| + i h \quad ,$$

where  $\tilde{A}_\varphi$  and  $h$  can be obtained from

$$\begin{aligned}\vec{\nabla} \tilde{A}_\varphi &:= -f r^{-1} \vec{e}_\varphi \times (\vec{\nabla} A_\varphi - \omega \vec{\nabla} A_t) \quad , \\ \vec{\nabla} h &:= -f^2 r^{-1} \vec{e}_\varphi \times \vec{\nabla} \omega - 2 \operatorname{Im}(\Phi^* \vec{\nabla} \Phi) \quad .\end{aligned}$$

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$$\begin{aligned} \partial_\rho \gamma(\rho, z) &= \frac{\rho}{4[\text{Re}(\mathcal{E}) + \Phi\Phi^*]^2} \left[ \left( \partial_\rho \mathcal{E} + 2\Phi^* \partial_\rho \Phi \right) \left( \partial_\rho \mathcal{E}^* + 2\Phi \partial_\rho \Phi^* \right) \right. \\ &\quad \left. - \left( \partial_z \mathcal{E} + 2\Phi^* \partial_z \Phi \right) \left( \partial_z \mathcal{E}^* + 2\Phi \partial_z \Phi^* \right) \right] \\ &- \frac{\rho}{\text{Re}(\mathcal{E}) + \Phi\Phi^*} \left( \partial_\rho \Phi \partial_\rho \Phi^* - \partial_z \Phi \partial_z \Phi^* \right) \quad , \end{aligned} \quad (1)$$

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# Introduction: Symmetries of Ernst Equations

Ernst equations can be derived by an effective action

$$I(\mathcal{E}, \Phi) = \int dz \int d\rho \left[ \frac{(\vec{\nabla}\mathcal{E} + 2\Phi^*\vec{\nabla}\Phi)(\vec{\nabla}\mathcal{E}^* + 2\Phi\vec{\nabla}\Phi^*)}{(\mathcal{E} + \mathcal{E}^* + 2\Phi\Phi^*)^2} - \frac{\vec{\nabla}\Phi\vec{\nabla}\Phi^*}{\mathcal{E} + \mathcal{E}^* + 2\Phi\Phi^*} \right]$$

This action has a set of Lie point symmetries which form the  $SU(2, 1)$  group. These symmetries can be written as a set of five independent transformation:

## Ernst Equations

$$\begin{aligned} (I) \quad & \mathcal{E} \longrightarrow \mathcal{E}' = \lambda\lambda^*\mathcal{E} \quad , \quad \Phi \longrightarrow \Phi' = \lambda\Phi \quad , \\ (II) \quad & \mathcal{E} \longrightarrow \mathcal{E}' = \mathcal{E} + i b \quad , \quad \Phi \longrightarrow \Phi' = \Phi \quad , \\ (III) \quad & \mathcal{E} \longrightarrow \mathcal{E}' = \frac{\mathcal{E}}{1 + ic\mathcal{E}} \quad , \quad \Phi \longrightarrow \Phi' = \frac{\Phi}{1 + ic\mathcal{E}} \quad , \\ (IV) \quad & \mathcal{E} \longrightarrow \mathcal{E}' = \mathcal{E} - 2\beta^*\Phi - \beta\beta^* \quad , \quad \Phi \longrightarrow \Phi' = \Phi + \beta \quad , \\ (V) \quad & \mathcal{E} \longrightarrow \mathcal{E}' = \frac{\mathcal{E}}{1 - 2\alpha^*\Phi - \alpha\alpha^*\mathcal{E}} \quad , \quad \Phi \longrightarrow \Phi' = \frac{\Phi + \alpha\mathcal{E}}{1 - 2\alpha^*\Phi - \alpha\alpha^*\mathcal{E}} \end{aligned}$$

where  $b, c \in \mathbb{R}$  and  $\alpha, \lambda, \beta \in \mathbb{C}$ .

Some of these transformation are just gauge symmetries and can be reabsorbed by a coordinate transformation, while others actually have non-trivial physical effects.

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Some of these transformation are just gauge symmetries and can be reabsorbed by a coordinate transformation, while others actually have non-trivial physical effects.

## Example: From Schwarzschild to Reissner-Nordstrom

We use the Harrison transformation (V) to charge the Schwarzschild black hole

$$ds^2 = - \left( 1 - \frac{2m}{r} \right) dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2 \left[ \frac{dx^2}{1 - x^2} + (1 - x^2) d\varphi^2 \right]$$

By comparison with the LWP metric we deduce  $\omega = 0 \implies h = 0$ ,  $f = 1 - \frac{2m}{r}$ , so the seed Ernst potential are

$$\Phi = A_t + i\tilde{A}\varphi = 0 \quad , \quad \mathcal{E} = f - \Phi\Phi^* + ih = 1 - \frac{2m}{r} \quad .$$

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After a coordinate transformation

$$r \rightarrow \frac{\bar{r} - 2s^2m}{1 - s^2} \quad , \quad s \rightarrow \frac{\bar{m} - \sqrt{\bar{m}^2 - \bar{e}^2}}{\bar{e}} \quad , \quad m \rightarrow \frac{1}{2} \left( \bar{m} - \sqrt{\bar{m}^2 - \bar{e}^2} \right)$$

we recognise the Reissner-Nordstrom black hole

$$d\bar{s}^2 = - \left( 1 - \frac{2\bar{m}}{\bar{r}} + \frac{\bar{e}^2}{\bar{r}^2} \right) d\bar{t}^2 + \frac{d\bar{r}^2}{1 - \frac{2\bar{m}}{\bar{r}} + \frac{\bar{e}^2}{\bar{r}^2}} + \bar{r}^2 \left[ \frac{dx^2}{1 - x^2} + (1 - x^2) d\varphi^2 \right] \quad .$$



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## Example: From Schwarzschild to Reissner-Nordstrom

We use the Harrison transformation (V) to charge the Schwarzschild black hole

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There exist another non-equivalent form of the LWP metric

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The *magnetic* LWP:

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which is obtained from the standard LWP by a *conjugation* transformation

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# Action of the Harrison transformation

General fall off for an asymptotically flat axisymmetric and stationary solution

$$\begin{aligned}\mathcal{E} &\sim 1 - \frac{2(M - iB)}{r} + \frac{(z_* + 2iJ)x + \text{const}}{r^2} + O\left(\frac{1}{r^3}\right) , \\ \Phi &\sim \frac{Q_e + iQ_m}{r} + \frac{(D_e + iD_m)x + \text{const}}{r^2} + O\left(\frac{1}{r^3}\right) ,\end{aligned}$$

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# The C-metric

Accelerating Schwarzschild metric in spherical-like coordinates  $(t, r, x = \cos \theta, \varphi)$

$$ds^2 = \frac{\left(1 - \frac{2m}{r}\right) (\alpha^2 r^2 - 1) dt^2 + \frac{dr^2}{\left(1 - \frac{2m}{r}\right) (1 - \alpha^2 r^2)} + \frac{r^2 dx^2}{w(x)} + r^2 w(x) d\varphi^2}{(1 + \alpha r x)^2}$$

where  $w(x) := (1 + 2m\alpha x)(1 - x^2)$

or in Weyl cylindrical coordinates  $(t, \rho, z, \varphi)$

$$ds^2 = -\frac{\mu_1 \mu_3}{\mu_2} dt^2 + \frac{16 C_f \mu_1^3 \mu_2^3 \mu_3^3 (d\rho^2 + dz^2)}{\mu_{12} \mu_{23} W_{13}^2 W_{11} W_{22}} + \rho^2 \frac{\mu_2}{\mu_1 \mu_3} d\varphi^2,$$

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# C-metric - rod representation

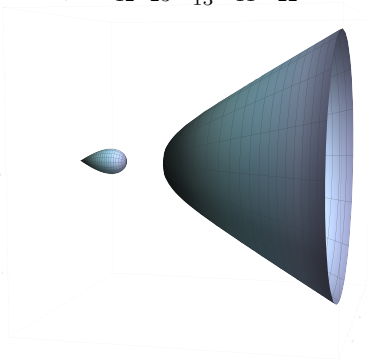
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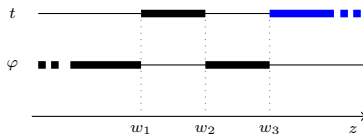
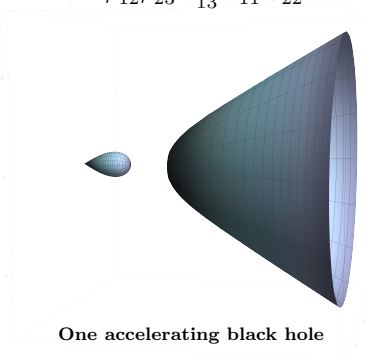
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# C-metric - rod representation

Accelerating Schwarzschild metric in Weyl coordinates

$$ds^2 = -\frac{\mu_1\mu_3}{\mu_2} dt^2 + \frac{16 C_f \mu_1^3 \mu_2^3 \mu_3^3 (d\rho^2 + dz^2)}{\mu_{12}\mu_{23}W_{13}^2 W_{11}W_{22}} + \rho^2 \frac{\mu_2}{\mu_1\mu_3} d\varphi^2,$$



# Regularised C-metric

Accelerating Schwarzschild metric in external gravitational multipolar expansion

$$ds^2 = -\frac{\mu_1\mu_3}{\mu_2} e^{2b_1z+2b_2z^2-b_2\rho^2} dt^2 + \frac{16 C_f \mu_1^3 \mu_2^3 \mu_3^3 k(\rho, z) (d\rho^2 + dz^2)}{\mu_{12}\mu_{23}W_{13}^2 W_{11}W_{22}} + \frac{\rho^2 \mu_2}{\mu_1\mu_3} e^{-2b_1z-2b_2z^2+b_2\rho^2} d\varphi^2 ,$$

$$k(\rho, z) = \exp\left\{ -b_1^2\rho^2 + 2b_1(z - 2b_2z\rho^2 + \mu_1 - \mu_2 + \mu_3) + \frac{b_2}{2}[b_2\rho^4 + z^2(4 - 8b_2\rho^2) + 2(\rho^2 + 4w_1 - \mu_1^2 - 4w_2\mu_2 + \mu_2^2 + 4w_3\mu_3 - \mu_3^2)] \right\} .$$

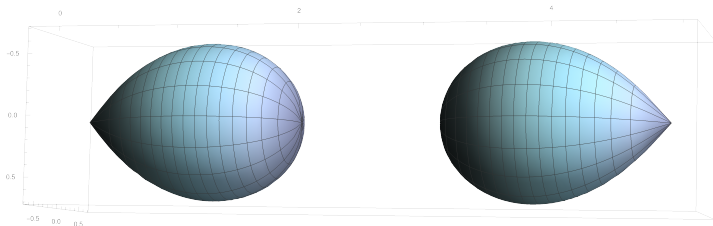


# Binary systems

Double Schwarzschild metric (Bach-Weyl 1922)

$$ds^2 = -\frac{\mu_1\mu_3}{\mu_2\mu_4} dt^2 + \frac{16 \tilde{C}_f \mu_1^3 \mu_2^5 \mu_3^3 \mu_4^5}{\mu_{12}\mu_{14}\mu_{23}\mu_{34} W_{13}^2 W_{24}^2 W_{11} W_{22} W_{33} W_{44}} (d\rho^2 + dz^2) + \rho^2 \frac{\mu_2\mu_4}{\mu_1\mu_3} d\tilde{\varphi}^2,$$

$z - M_0$

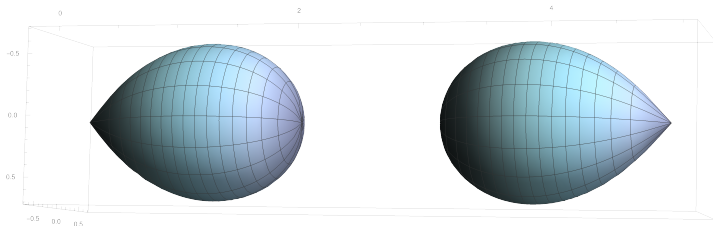


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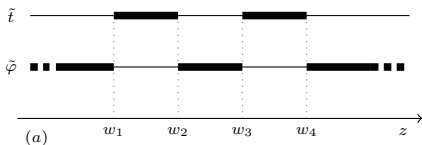
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Two black holes



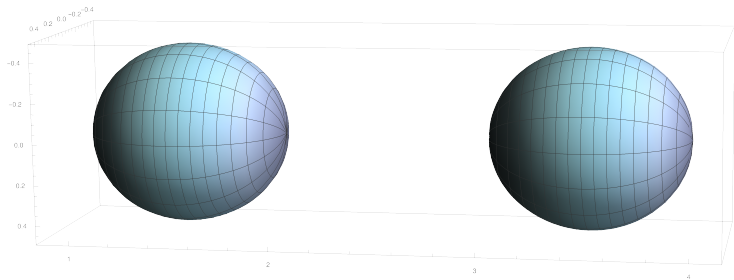
# Introduction: Binary systems - Regular

Double Schwarzschild metric in external gravitational field [arXiv:2104.07686]

$$ds^2 = -\frac{\mu_1\mu_3}{\mu_2\mu_4} e^{2b_1z+2b_2z^2-b_2\rho^2} dt^2 + \frac{16 \tilde{C}_f \mu_1^3\mu_2^5\mu_3^3\mu_4^5 \tilde{k}(\rho, z) (d\rho^2 + dz^2)}{\mu_{12}\mu_{14}\mu_{23}\mu_{34}W_{13}^2W_{24}^2W_{11}W_{22}W_{33}W_{44}}$$

$$+ \rho^2 \frac{\mu_2\mu_4}{\mu_1\mu_3} e^{-2b_1z-2b_2z^2+b_2\rho^2} d\tilde{\varphi}^2,$$

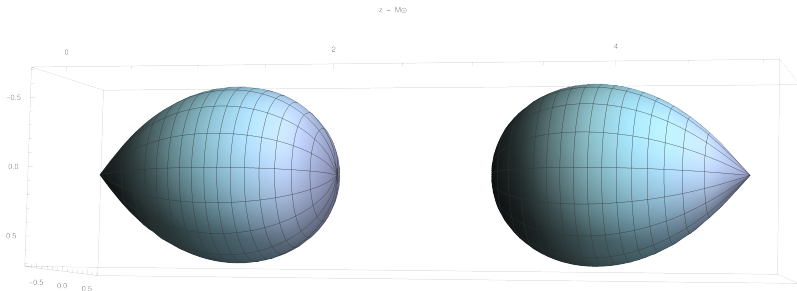
$$\tilde{k}(\rho, z) := \exp\left\{ -b_1^2\rho^2 - 2b_1(z + 2b_2z\rho^2 - \mu_1 + \mu_2 - \mu_3 + \mu_4) + b_2\left[\frac{b_2}{2}\rho^4 - 2z^2(1 + 2b_2\rho^2) + 4w_1\mu_1 - \mu_1^2 - 4w_2\mu_2 + \mu_2^2 + 4w_3\mu_3 - \mu_3^2 - 4w_4\mu_4 + \mu_4^2\right] \right\}.$$



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# Introduction: C-metric and binary systems

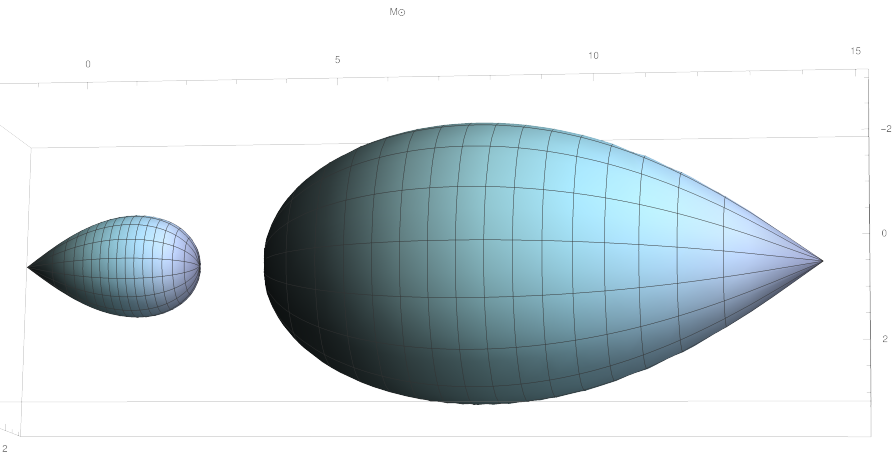
Consider the binary black hole system





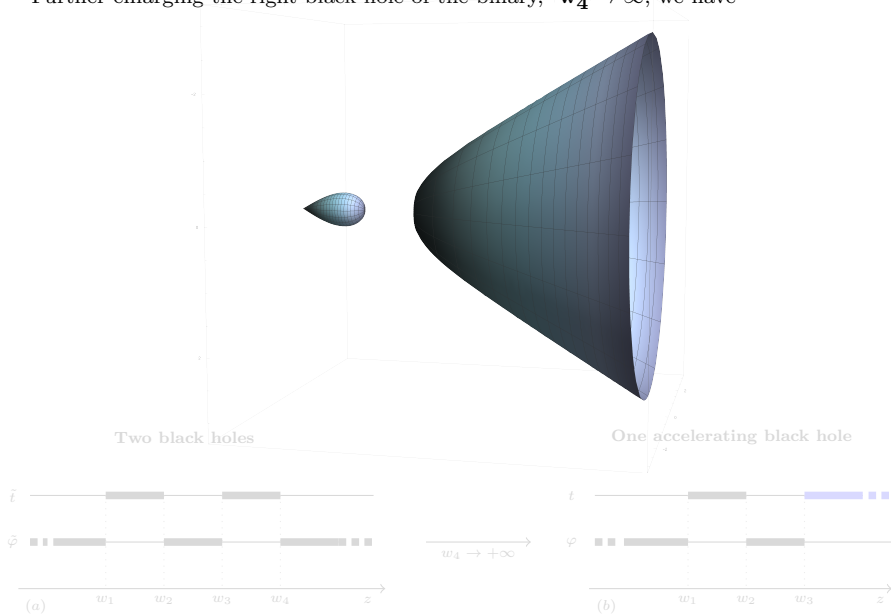
# Introduction: C-metric and binary systems

Let's enlarge the right black hole of the binary while keeping the left and the distance fixed



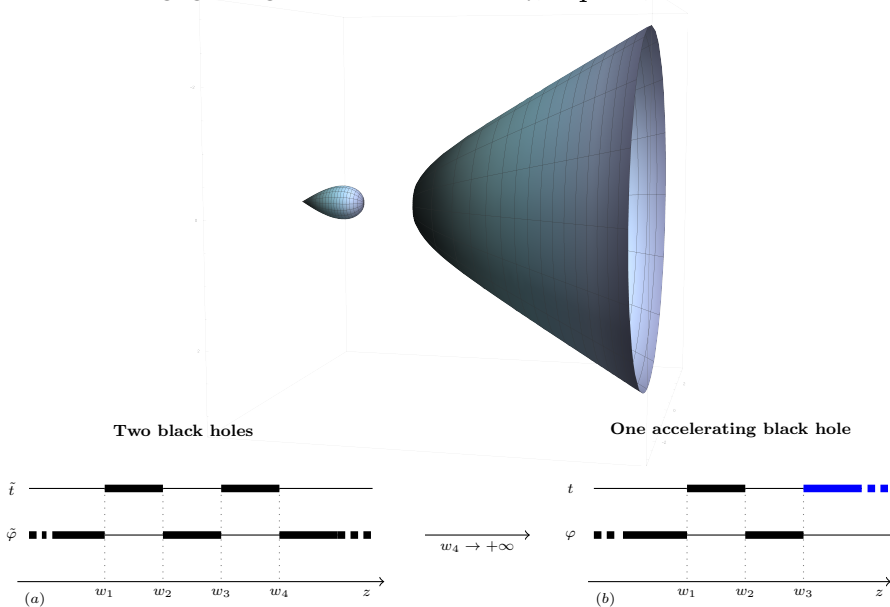
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Further enlarging the right black hole of the binary,  $w_4 \rightarrow \infty$ , we have



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The Harrison transformed Bach-Weyl metric gives a charged binary

$$\begin{aligned}
 ds^2 &= -\frac{\mu_1\mu_2\mu_3\mu_4}{(\mu_2\mu_4 - \hat{\alpha}^2\mu_1\mu_3)^2} dt^2 + \rho^2 \frac{(\mu_2\mu_4 - \hat{\alpha}^2\mu_1\mu_3)^2}{\mu_1\mu_2\mu_3\mu_4} d\hat{\varphi}^2, \\
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 A_\mu &= \left( A_{t_0} + \frac{\hat{\alpha}\mu_1\mu_3}{\mu_2\mu_4 - \hat{\alpha}^2\mu_1\mu_3}, 0, 0, 0 \right).
 \end{aligned}$$

At extremality this solution become the Majumdar-Papapetrou metric.

Then if you change the coordinates as follows

$$\begin{aligned}
 \rho &\rightarrow \frac{\sqrt{r(r-2m)(1-A^2r^2)(1+2Amx)(1-x^2)}}{(1+Arx)^2}, & \hat{t} &\rightarrow \sqrt{\frac{A}{2w_4}} t, \\
 z &\rightarrow z_1 + \frac{(Ar+x)[r-m(1-Arx)]}{(1+Arx)^2}, & \hat{\varphi} &\rightarrow \sqrt{\frac{2w_4}{A}} \varphi.
 \end{aligned}$$

with  $w_1 = z_1 - m$ ,  $w_2 = z_1 + m$ ,  $z_3 = z_1 + \frac{1}{2A}$ ,  $C_f = \frac{2w_4m^2}{A^3}$  and  $\hat{\alpha} = \sqrt{As}$

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# Reissner-Nordstrom in charged accelerating background

... and take the limit for  $w_4 \rightarrow \infty$ , you get the Reissner-Nordstrom spacetime in an accelerating and electric background (Type I : Harrison[C-metric])

$$ds^2 = -f(r, x)dt^2 + \frac{1}{f(r, x)} \left[ e^{2\gamma(r, x)} \left( \frac{dr^2}{\Delta_r(r)} + \frac{dx^2}{\Delta_x(x)} \right) + \rho^2(r, x)d\varphi^2 \right] ,$$

where

$$f(r, x) := \frac{r^2 \Delta_r \Omega^2}{(s^2 \Delta_r - r^2 \Omega^2)^2}$$

$$\Omega(r, x) := 1 + Arx ,$$

$$\gamma(r, x) := \frac{1}{2} \log \left( \frac{\Delta_r}{\Omega^4} \right) ,$$

$$\Delta_r(r) := (1 - A^2 r^2)(r^2 - 2mr) ,$$

$$\rho(r, x) := \frac{\sqrt{\Delta_r} \sqrt{\Delta_x}}{\Omega^2} ,$$

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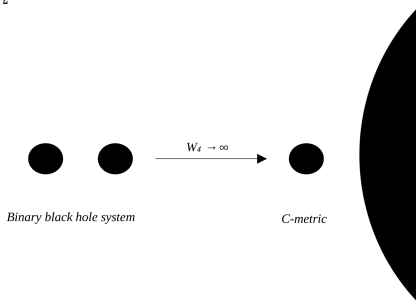
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Removing the black hole (vanishing its parameters  $m$ , ...) we have

$$ds^2 = -\hat{f}(r, x) \left[ dt - \left( \frac{2Acr^2(1-x^2)}{(1+Arx)^2} + \omega_0 \right) d\varphi \right]^2 \\ + \frac{1}{\hat{f}(r, x)} \left\{ \frac{1}{(1+Arx)^4} \left[ dr^2 + (r^2 - A^2r^4) \left( \frac{dx^2}{1-x^2} + (1-x^2)d\varphi^2 \right) \right] \right\}$$

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This can be obtained as the near-horizon limit of the RN-NUT solution ( $w_2 \rightarrow \infty$ )

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$$\begin{aligned}\rho &= \sqrt{r^2 - 2mr - q^2} \sqrt{1 - x^2}, \\ z &= z_1 + (r - m) x.\end{aligned}$$

the Reissner-Nordstrom black hole reads

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Taking  $w_2 \rightarrow \infty$  the accelerating electric background can be obtained

$$\begin{aligned}m &\rightarrow \frac{1}{2}(w_2 - w_1)(1 + 2w_2\delta^2), & q &\rightarrow (w_1 - w_2)\sqrt{2w_2}\delta, & z_1 &\rightarrow \frac{1}{2}(w_2 + w_1)(1 - 2w_2\delta^2), \\ & & & & & (4) \\ \rho &\rightarrow \rho(1 - 2w_2\delta^2), & z &\rightarrow z(1 - 2w_2\delta^2), & t &\rightarrow \frac{\sqrt{A}}{\delta^2\sqrt{2w_2}}t, & \varphi &\rightarrow \frac{\sqrt{A}}{\delta^2\sqrt{2w_2}}\varphi.\end{aligned}$$

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The Ehlers transformation acts on a solution with the following asymptotic fall-off

$$\begin{aligned}\mathcal{E} &\sim 1 - \frac{2(M - iB)}{r} + O\left(\frac{1}{r^2}\right) , \\ \Phi &\sim \frac{Q_e + iQ_m}{r} + O\left(\frac{1}{r^2}\right) ,\end{aligned}$$

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so basically is a rotation on between the mass and gravitomagnetic mass

$$\begin{pmatrix} \bar{M} \\ \bar{B} \end{pmatrix} = \begin{pmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{pmatrix} \Big|_{\psi \rightarrow 2 \arctan c} \begin{pmatrix} M \\ B \end{pmatrix} = \begin{pmatrix} \frac{(1 - c^2)M - 2Bc}{1 + c^2} \\ \frac{(1 - c^2)B + 2Mc}{1 + c^2} \end{pmatrix} .$$

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# Ehlers transformation of the C-metric ?

- Ehlers [Schwarzschild] = Taub-NUT
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- $\checkmark$  Ehlers [C-metric] ? = Accelerating Taub-NUT (Chng-Mann-Stelea 2006)  
(Type-I) (Podolsky - Vratny 2019)

$$ds^2 = -\frac{4\mu_1\mu_2\mu_3}{4\mu_2 + c^2\mu_1^2\mu_3^2} [dt - c(2z + \mu_1 - \mu_2 + \mu_3 + w_0)d\varphi]^2 + \frac{4\mu_2 + c^2\mu_1^2\mu_3^2}{4\mu_1\mu_2\mu_3} \left[ \frac{16C_f\mu_1^4\mu_2^4\mu_3^4(d\rho^2 + dz^2)}{\mu_{12}\mu_{23}W_{11}W_{22}W_{33}W_{13}^2} + \rho^2 d\varphi^2 \right].$$

This can be obtained from the Bach-Weyl-NUT binary for  $w_4 \rightarrow \infty$

$$ds^2 = -f(dt - \omega d\varphi)^2 + f^{-1} [e^{2\gamma} (d\rho^2 + dz^2) + \rho^2 d\varphi^2] . \quad (5)$$

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$$e^{2\gamma(r,x)} = \frac{16 \tilde{C}_f \mu_1^4 \mu_2^4 \mu_3^4 \mu_4^4}{\mu_{12}\mu_{14}\mu_{23}\mu_{34}W_{13}^2 W_{24}^2 W_{11}W_{22}W_{33}W_{44}} .$$

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# Ehlers transformation of the C-metric ?

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$$ds^2 = -\frac{4\mu_1\mu_2\mu_3}{4\mu_2 + c^2\mu_1^2\mu_3^2} [dt - c(2z + \mu_1 - \mu_2 + \mu_3 + w_0)d\varphi]^2 + \frac{4\mu_2 + c^2\mu_1^2\mu_3^2}{4\mu_1\mu_2\mu_3} \left[ \frac{16C_f\mu_1^4\mu_2^2\mu_3^4(d\rho^2 + dz^2)}{\mu_{12}\mu_{23}W_{11}W_{22}W_{33}W_{13}^2} + \rho^2 d\varphi^2 \right].$$

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Plebanski-Demianski seed in terms of the metric and electromagnetic potential:

$$ds^2 = -f(r, x) [dt - \omega(r, x)d\varphi]^2 + \frac{1}{f(r, x)} \left[ e^{2\gamma(r, x)} \left( \frac{dr^2}{\Delta_r(r)} + \frac{dx^2}{\Delta_x(x)} \right) + \rho^2(r, x)d\varphi^2 \right].$$

$$f(r, x) := \frac{\hat{\omega}^2 \Delta_x - \Delta_r}{\hat{\omega} \Omega^2 \mathcal{R}^2}, \quad \gamma(r, x) := \frac{1}{2} \log \left( \frac{\Delta_r - \hat{\omega}^2 \Delta_x}{\Omega^4} \right),$$

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$$\mathcal{R}(r, x) := \sqrt{r^2 + \hat{\omega}^2 x^2} \quad \Omega(r, x) := 1 - \alpha r x,$$

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P-D seed in terms of Ernst complex gravitational and electromagnetic potentials

$$\mathcal{E}(r, x) = \frac{r\hat{\omega}\Delta_x + i\{\Omega^2\hat{\omega}[-ikr\hat{\omega} + x(e^2 + p^2 + k\hat{\omega}^2)] + x\Delta_y\}}{rx\Omega^2\hat{\omega}(-ir + \hat{\omega}x)},$$

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# Plebanski-Demianski with a double NUT parameter

Ehlers[Plebansky-Demianski]:

$$\bar{\mathcal{E}}(r, x) = -\frac{i[\Omega^2\hat{\omega}(-ikr\hat{\omega} + x(e^2 + p^2 + k\hat{\omega}^2))] - ir\hat{\omega}\Delta_x + x\Delta_y}{\Omega^2\hat{\omega}[-ickr\hat{\omega} + xr(ir - x\hat{\omega}) + cx(e^2 + p^2 + k\hat{\omega}^2)] - icr\hat{\omega}\Delta_x + cx\Delta_r},$$

$$\bar{\Phi}(r, x) = \frac{(p - ie)xr\Omega^2\hat{\omega}}{\Omega^2\hat{\omega}[-ickr\hat{\omega} + xr(ir - x\hat{\omega}) + cx(e^2 + p^2 + k\hat{\omega}^2)] - icr\hat{\omega}\Delta_x + cx\Delta_r}.$$

$$ds^2 = -\frac{f(r, x)}{|1 + ic\mathcal{E}|^2} [dt - \bar{\omega}(r, x)d\varphi]^2 + \frac{|1 + ic\mathcal{E}|^2}{f(r, x)} \left[ e^{2\gamma(r, x)} \left( \frac{dr^2}{\Delta_r(r)} + \frac{dx^2}{\Delta_x(x)} \right) + \rho^2(r, x)d\varphi^2 \right].$$

$\gamma(r, x)$  function remains invariant under the Ehlers transformation, while

$$\bar{\omega} = \omega(r, x) + c^2 \frac{r^2\Delta_r(\Delta_x + k\Omega^2\hat{\omega})^2 + x^2\Delta_x[\Delta_r + \Omega^2\hat{\omega}(q^2 + k\hat{\omega}^2)]^2}{x^2r^2\hat{\omega}\Omega^4(\Delta_r - \hat{\omega}^2\Delta_x)} + \frac{2cr^3\alpha\Delta_r(\Delta_x + k\Omega^2\hat{\omega})}{xr^2\alpha\hat{\omega}\Omega^2(\Delta_r - \hat{\omega}^2\Delta_x)} \\ + 2c \frac{x\Omega(\Delta_r - \Delta_x\hat{\omega}^2)[\Delta_r + \Omega\hat{\omega}(q^2 + k\hat{\omega}^2 - 2mr)] + xr\hat{\omega}\{x\alpha\Delta_r[\Delta_x\hat{\omega} + \Omega^2(q^2 + k\hat{\omega}^2)]\}}{xr^2\alpha\hat{\omega}\Omega^2(\Delta_r - \hat{\omega}^2\Delta_x)} + \omega_0.$$

The non-null component of the electromagnetic vector results

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# Removal of the Minsner string

Convenient parametrization

$$\begin{aligned}\hat{\omega} &\rightarrow \frac{a^2 + \ell^2}{a} \\ \hat{n} &\rightarrow \frac{\ell(a^2 + \ell^2)^2 + am(\ell^4 - a^4)\alpha}{(a^2 + \ell^2)^2 - 2aml(a^2 + \ell^2)\alpha + a^2\ell^2(a^2 + e^2 + p^2 - \ell^2)\alpha^2} \\ \epsilon &\rightarrow \frac{(a^2 + \ell^2)^2 + 4aml(a^2 + \ell^2)\alpha - a^2(a^2 - \ell^2)(a^2 + e^2 + p^2 - \ell^2)\alpha^2}{(a^2 + \ell^2)^2 - 2aml(a^2 + \ell^2)\alpha + a^2\ell^2(a^2 + e^2 + p^2 - \ell^2)\alpha^2} \\ k &\rightarrow \frac{a^2(a^2 - \ell^2)}{(a^2 + \ell^2)^2 - 2aml(a^2 + \ell^2)\alpha + a^2\ell^2(a^2 + e^2 + p^2 - \ell^2)\alpha^2} \\ c &\rightarrow \frac{-m + \sqrt{m^2 + n^2 - 2n\ell}}{n} .\end{aligned}$$

To remove the discontinuity on the axis of symmetry

$$\Delta\omega = \lim_{x \rightarrow 1} \omega(r, x) - \lim_{x \rightarrow -1} \omega(r, x) = 4(n - \ell) = 0 .$$

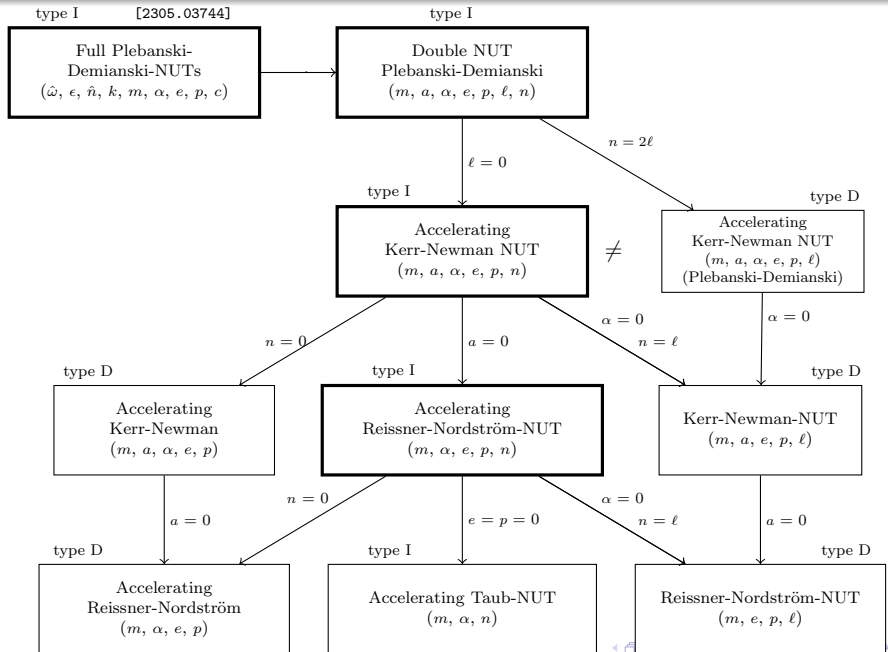
horizons from

$$Q(r) = (r - \bar{r}_+)(r - \bar{r}_-) \left[ 1 - \frac{a(a - \ell)(r - \hat{r}_-)\alpha}{a^2 + \ell^2} \right] \left[ 1 + \frac{a(a + \ell)(r - \hat{r}_-)\alpha}{a^2 + \ell^2} \right] ,$$

where

$$\bar{r}_{\pm} := m \pm \sqrt{m^2 + (n - \ell)^2 - a^2 - e^2 - p^2} \quad \text{and} \quad \hat{r}_{\pm} := m \pm \sqrt{m^2 + n^2 - 2n\ell} ;$$

# Plebanski-Demianski NUTs family



# Harrison and Ehlers transformations commute

Ehlers transformation

$$(III) : \quad \mathcal{E} \longrightarrow \bar{\mathcal{E}} = \frac{\mathcal{E}}{1 + ic\mathcal{E}} \quad , \quad \Phi \longrightarrow \bar{\Phi} = \frac{\Phi}{1 + ic\mathcal{E}} \quad .$$

Harrison transformation

$$(V) : \quad \mathcal{E} \longrightarrow \bar{\mathcal{E}} = \frac{\mathcal{E}}{1 - 2\alpha^*\Phi - \alpha\alpha^*\mathcal{E}} \quad , \quad \Phi \longrightarrow \bar{\Phi} = \frac{\Phi + \alpha\mathcal{E}}{1 - 2\alpha^*\Phi - \alpha\alpha^*\mathcal{E}} \quad .$$

Ehlers and Harrison transformations commute  $[(III) , (V)] = 0$

$$(III) \circ (V) = \begin{cases} \mathcal{E} & \longrightarrow & \bar{\mathcal{E}} = \frac{\mathcal{E}}{1 + ic\mathcal{E} - \alpha\alpha^*\mathcal{E} - 2\alpha^*\Phi} \\ \Phi & \longrightarrow & \bar{\Phi} = \frac{\Phi + \alpha\mathcal{E}}{1 + ic\mathcal{E} - \alpha\alpha^*\mathcal{E} - 2\alpha^*\Phi} \end{cases} \quad .$$

PD can thus be further generalised to the presence of extra NUT and electromagnetic field  $(\mathcal{E}, \Phi)$

$$\frac{-ir\hat{\omega}\Delta_x + x\Delta_r + \hat{\omega}[-ikr\hat{\omega} + x](q^2 + k\hat{\omega}^2)]\Omega^2}{(c + is^2)(r\hat{\omega}\Delta_x + ix\Delta_r) - \hat{\omega}[r^2x + 2(ip + e)rsx + ir(x^2 + ikc - ks^2)\hat{\omega} + (s^2 - icx)(q^2 + k\hat{\omega}^2)]\Omega^2} \cdot$$
$$\frac{-irs\hat{\omega}\Delta_x + sx\Delta_r + \hat{\omega}[(ip + e)rx + (q^2 + k\hat{\omega}^2)sx - ikrs\hat{\omega}]\Omega^2}{(c + is^2)(r\hat{\omega}\Delta_x + ix\Delta_r) - \hat{\omega}[r^2x + 2(ip + e)rsx + ir(x^2 + ikc - ks^2)\hat{\omega} + (s^2 - icx)(q^2 + k\hat{\omega}^2)]\Omega^2} \cdot$$

# Plebanski-Demianski-NUTs in electromagnetic Rindler background

In metric form

$$ds^2 = -\bar{f}(r, x) [dt - \bar{\omega}(r, x)d\varphi]^2 + \frac{1}{\bar{f}(r, x)} \left[ e^{2\gamma(r, x)} \left( \frac{dr^2}{\Delta_r(r)} + \frac{dx^2}{\Delta_x(x)} \right) + \rho^2(r, x)d\varphi^2 \right].$$

$$\bar{f} = \frac{f}{1 + (ic - s^2)\mathcal{E} - 2s\Phi},$$

where  $\bar{\omega}(r, x) =$

$$\frac{1}{r^2 x^2 \hat{\omega} (\hat{\omega}^2 \Delta_x - \Delta_r) (1 - \Omega)^4} \left\{ c^2 (\Omega - 1) \left[ r^2 \Delta_r (\Delta_x + k \hat{\omega} \Omega^2)^2 + x^2 \Delta_x (\Delta_r + \hat{\omega} (q^2 + k \hat{\omega}^2) \Omega^2)^2 \right] \right. \\ - 2crx\Omega^2 \left[ r^2 \Delta_r (\Omega - 1) (\Delta_x + k \hat{\omega} \Omega^2) + 2r \left( mx^2 \hat{\omega} (\Delta_r - \hat{\omega}^2 \Delta_x) \Omega^2 + es \Delta_r (\Omega - 1) (\Delta_x + k \hat{\omega} \Omega^2) \right) \right] \\ + x \left( -x \Delta_r^2 \Omega + \hat{\omega}^2 (q^2 + k \hat{\omega}^2) \Delta_x \Omega^2 (x \hat{\omega}^2 - 2ps + 2ps\Omega) + \hat{\omega} \Delta_r (x(q^2 + k \hat{\omega}^2) (\Omega - 2) \Omega^2 \right. \\ \left. + \Delta_x (2(ps + x\hat{\omega})\Omega - 2ps - x\hat{\omega})) \right) \left. \right] + (\Omega - 1) \left[ x^2 \hat{\omega}^2 (r^4 + 4er^3 s + 6q^2 r^2 s^2 + 4ers^3 (q^2 + k \hat{\omega}^2)) \Delta_x \Omega^4 \right. \\ \left. + s^4 x^2 \Delta_r^2 \Delta_x + \Delta_r \left( r^4 s^4 \Delta_x^2 + 2s^3 (kr^2 s \hat{\omega} - 2psr^2 x + (2er + q^2 s)x^2 \hat{\omega} + ksx^2 \hat{\omega}^3) \Delta_x \Omega^2 \right. \right. \\ \left. \left. + r^2 \omega (4psx^3 - 4kps^3 x + k^2 s^4 \hat{\omega} + x^4 \hat{\omega}) \Omega^4 \right) \right] \left. \right\}.$$

# Plebanski-Demianski-NUTs in electromagnetic Rindler background

In metric form

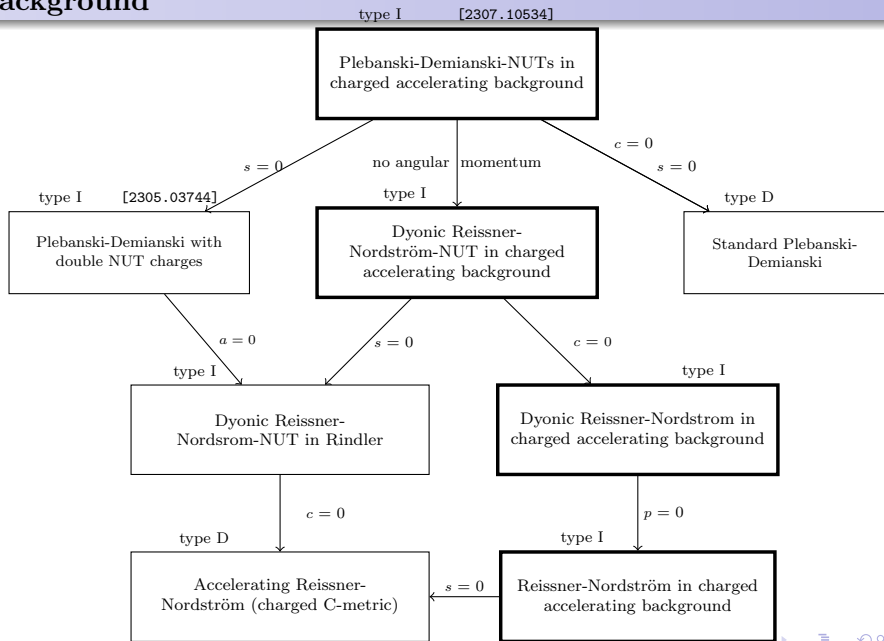
$$d\bar{s}^2 = -\bar{f}(r, x) [dt - \bar{\omega}(r, x)d\varphi]^2 + \frac{1}{\bar{f}(r, x)} \left[ e^{2\gamma(r, x)} \left( \frac{dr^2}{\Delta_r(r)} + \frac{dx^2}{\Delta_x(x)} \right) + \rho^2(r, x)d\varphi^2 \right].$$

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# Plebanski-Demianski-NUTs in electromagnetic Rindler background





# Summary, Conclusions & Perspectives

- Accelerating black holes describe a limit of a wider system of binary black holes where one of the two sources is much bigger with respect to the other. They can be thought as the near horizon limit, close to the bigger black hole, whose event horizon become the accelerating horizon of the solution.
- The Plebanski-Demianski family of solution can be generalised in order to contain an extra independent electric and magnetic charge or NUT parameter.
- The extra charges (with respect to PD) are related to the accelerating background and are reminiscent of the charges of an infinitely inflated big black hole close to the main small one.
- In case of rotating black holes the Misner string defect can be erased by fine-tuning the two independent NUT parameters (without completely eliminating the NUT parameter, in fact the metric remains of type I)
- This picture opens to further generalisations of accelerating black holes. These can be obtained by the more general Kerr-Newman-NUT binary system (Belinski-Alekseev 2019). For instance when also the big black hole of the binary is endowed with angular momentum.

[2305.03744] , [2307.10534]

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