Three-Dimensional Gravity - review and some recent developments

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Introduction

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(i) Explain why three-dimensional gravity is interesting in spite of its apparent triviality ;

(ii) Discuss one recent development related to the "factorization problem" in the presence of many (≥ 2) asymptotic regions.

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6. Conclusions

Einstein theory in lower dimensions

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The Einstein equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ in three dimensions are perfectly consistent and share the same physical concepts as in four dimensions. "Spacetime tells matter how to move, matter tells space-time how to curve."

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But perhaps too simple ! In 3D, the Weyl tensor identically vanishes and the Riemann tensor reads

$$R_{\alpha\beta\gamma\delta} = R_{\alpha\gamma} g_{\beta\delta} - R_{\alpha\delta} g_{\beta\delta} - R_{\beta\gamma} g_{\alpha\delta} + R_{\beta\delta} g_{\alpha\gamma} - \frac{R}{2}\left(g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta\delta}\right)$$

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Using the field equations, this implies that the Riemann tensor is given by

$$R_{\alpha\beta\gamma\delta} = \Lambda\left(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\delta}\right),$$

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No, there exist black hole solutions ("BTZ black holes"), and these appear therefore in the simplest possible context ! (Nothing else !)

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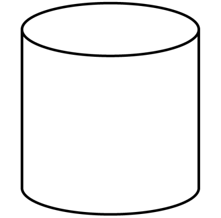
An example from plat space



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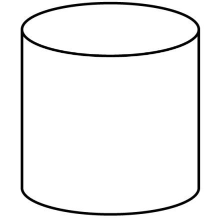

The cylinder has no curvature, like the plane.

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The metric reads (BTZ) :

$$ds^2 = -\left(-M + \left(\frac{r}{\ell}\right)^2\right) dt^2 + \left(-M + \left(\frac{r}{\ell}\right)^{-2}\right)^{-1} dr^2 + r^2 d\varphi^2$$

Horizon at $r_+ = \ell (M)^{\frac{1}{2}}$ $(J = 0)$.

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Furthermore, the BTZ black hole is at the same time very simple (few macroscopic parameters, M, J), but also very "complicated". Like its higher-dimensional cousins, it has Hawking temperature $\frac{r_+}{2\pi\ell^2}$ and possesses an entropy $S = \frac{2\pi r_+}{4G}$.

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Could one have dreamed of a simpler context in which to investigate black hole entropy ?

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One striking feature of the black hole entropy is that it is "holographic" (proportional to the area rather than the volume).

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The correspondence is useful in other areas of physics (condensed matter, quark- gluon plasmas).

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I.e., one gets two copies of the Virasoro algebra.

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Explicitly, in terms of Fourier modes,

$$[L_n, L_m] = -i\left((n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}\right),$$

$$[\widetilde{L}_n, \widetilde{L}_m] = -i\left((n-m)\widetilde{L}_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}\right),$$

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(Brown-Henneaux 1986)

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Hence, one gets a microscopic counting 'a la Boltzmann' of the BTZ black hole entropy.

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Triumph should be tempered, but this result is extremely suggestive !

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Does one get independent dual theories at the boundaries ("factorization"),

or are these theories "entangled" ("non-factorization") ?

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Analysis particularly transparent in the Chern-Simons formulation of $2 + 1$ gravity.

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Note : one has also $sl(2,\mathbb{R}) \simeq sp(2,\mathbb{R}) \simeq su(1,1)$ and thus the chain of isomorphisms $so(2,1) \simeq sl(2,\mathbb{R}) \simeq sp(2,\mathbb{R}) \simeq su(1,1)$

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$$I_{CS}[A] = \frac{k}{4\pi}\int_M Tr\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right).$$

- The parameter k is related to the (2+1)-dimensional Newton constant G as $k = \ell/4G$, where ℓ is the AdS radius of curvature.

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in terms of which one finds indeed

$$I[e,\omega] = \frac{1}{8\pi G} \int_M d^3x \left(\frac{1}{2} eR + \frac{e}{\ell^2} \right)$$

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Note that the Chern-Simons gauge transformations enable one to go to gauges where the triad is degenerate.

$\mathfrak{sl}(2,\mathbb{R})$ conventions

The $\mathfrak{sl}(2,\mathbb{R})$ generators are noted by L_0, $L_\pm$ and they satisfy the algebra

$$[L_0, L_\pm] = \pm L_\pm\,, \qquad [L_+, L_-] = 2L_0\,.$$

We use the following matrix representation of $\mathfrak{sl}(2,\mathbb{R})$

$$L_0 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad L_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad L_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

They enjoy the property that

$$\mathrm{Tr}(L_0 L_0) = \frac{1}{2}\,, \qquad \mathrm{Tr}(L_+ L_-) = \mathrm{Tr}(L_- L_+) = 1\,.$$

(Chevalley-Serre basis, up to some obvious signs and normalizations)

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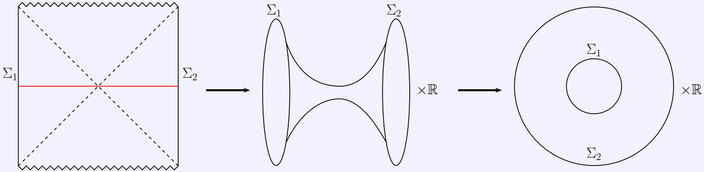

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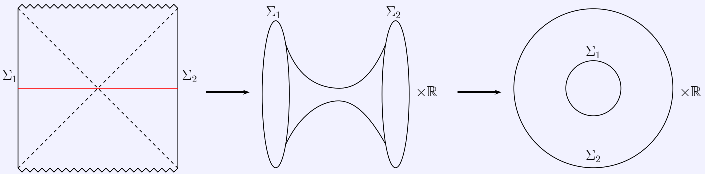

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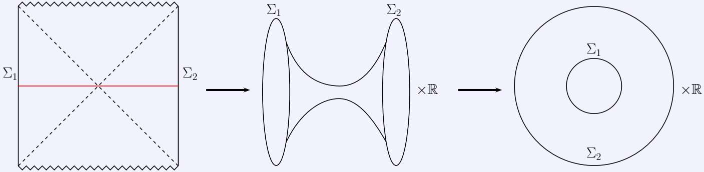

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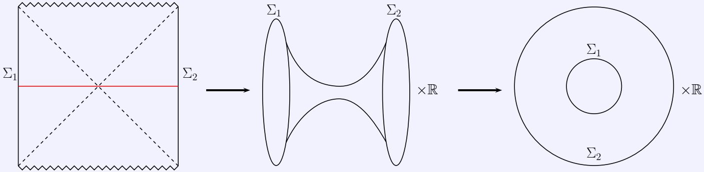

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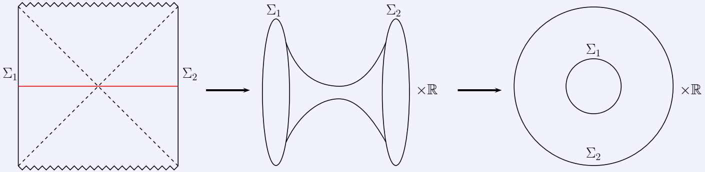

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($J = 0$ for definiteness, in which case $\mathscr{L}^+ = \mathscr{L}^-$ but same is true if $J \neq 0$.)

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An elliptic element can be written as $k_e(L_- - L_+)$ for some k_e

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Conical singularities have elliptic holonomies ($\in so(2)$).

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We consider first the disk and then the annulus.

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If we choose $A_- = A_t - A_\varphi = 0$ at the boundary, the dynamics is generated by the boundary Hamiltonian $\frac{1}{2} \frac{2\pi}{k} \oint j^2(\varphi) d\varphi$.

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