Effect of magnetic advection on spectral characteristics of thin accretion disks

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What are accretion disks?

- Infall of matter onto a graviting central object, usually with non-trivial angular momentum.
- Pre-eminent role in the formation of astrophysical structure:
  - formation of galaxies through collapse onto protogalaxies, and further enlargement,
  - formation of stars through collapse onto protostellar nebulae,
  - formation of planets from protoplanetary disks.
- What interests us: stationary accretion disks as a source of radiant power.

Accretion disks as sources of radiant power

- Matter infalls with non-trivial angular momentum.
- A process for propagation of angular momentum outward must exist to faciliate the inward motion of matter.
- Keplerian orbits exhibit differential rotation:
  - viscous stress exists,
  - angular momentum is propagated outward,
  - a portion of energy is spent on viscous dissipation.
- The energy generated by viscous dissipation is wholly (or nearly wholly) converted to radiant power.

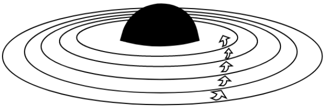
Thin disks

- Luminosity can be estimated by conversion of infalling energy to radiation at star surface

$$L_{acc} = \frac{GM\dot{M}}{R_*}\eta\dot{M}c^2. \quad (1)$$

- For thin accretion disks, we have modified Eddingtonian luminosity

$$L_{\mathrm{Edd}} = \frac{1}{16}\dot{M}_{\mathrm{Edd}}c^2, \dot{M}_{\mathrm{Edd}} = \frac{16 \times 4\pi GM}{c\kappa_T}. \quad (2)$$



X-ray black hole-star binaries

- X-ray binaries (XRB) - powerful source of radiation in the universe - can convert up to tenths of rest-mass, thermonuclear fusion only cca 0.7 percent.
- During outburst - traversal through hardness-intensity diagram - eventually settling to high/soft state.
- Emits principally thermally, can stay "soft" from days to months.
- Eventually falls back to the hard state.
- In the soft state, the disk is formed as thin and optically thick, extending down to the innermost stable circular orbit.
- Can be described by approximate models - radiant flux predicted by black hole parameters.
- Studying spectra as they change through time - obtain realisations of predicted luminosity/accretion rate.

Thin disks

1. Thinnes:
    - the vertical scale $H(R) \ll R$,
    - equivalently $c_s \ll v_\phi$.
2. Optical thickness:
    - the diffusion approximation holds, the disk surface can be ascribed a black-body spectrum.
3. Disk boundedness:
    - an inner edge R_{in} exists, physics above and below are decoupled.

Thin disk models

- Problem - how to prescribe viscosity?
- Kinematic viscosity does not suffice!
- Most likely source of viscosity - magnetically driven turbulences.
- Hard/impossible to describe analytically.
- Phenomenological prescription of viscosity based on the vertical scale of the disk

$$\nu = \alpha c_s H, \alpha \in (0, 1). \tag{3}$$

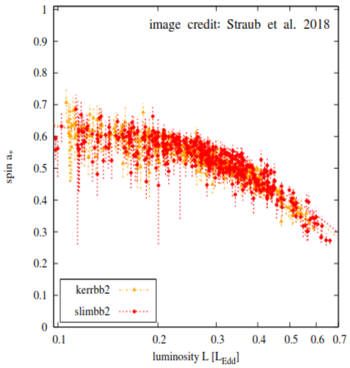
- Alternatively

$$\int_{-\infty}^{\infty} t_{\hat{R}\hat{\phi}} \mathrm{d}z = \alpha P. \tag{4}$$

Thin disk models

- Thin disk requirements
- $+$ the α-prescription
- $+$ "$Q_+ = Q_-$"
- $=>$ Newtonian Shakura-Sunyaev model (1973),
- $=>$ relativistic Novikov-Thorne model (1973).

Confrontation with observation



- Fit the Novikov-Thorne model to the observed spectra, obtain the accretion rate/luminosity vs the BH spin.
- The spin ought to be constant - it is conserved in the time window.
- But there is spin decay!
- The model is possibly incorrect...

Equations of state - valid for spacetime flat in vertical direction

- State variables:
  1. c_s,
  2. τ,
  3. P,
  4. Σ,
  5. H,
  6. T_c.

$$
\begin{aligned}
c_s^2 &= \frac{P}{\Sigma}, \\
P(1-\beta') &= \frac{\Sigma k T_c}{\mu m_p} + H\frac{4\sigma}{3c}T_c^4, \\
\tau &= \Sigma\kappa(\Sigma, H, T_c).
\end{aligned}
\tag{5}
$$

Equations of state - magnetic pressure

- Powerful and heterogenous magnetic fields thread accretion disks.
- Uttermost importance in viscosity-creating turbulences and other accretion disks processes.
- Contribute to a fraction of total pressure, apart from gaseous and radiant fractions.
- Hard/impossible to describe analytically.
- Numerical simulation suggest a constant ratio across a stable disk - namely existence of equilibrium.
- Prescribe the magnetic pressure with a free parameter $\beta' = P_{mag}/P_{tot}$.
- Expecting an equilibrium value of 0.5.

Relativistic solution

$$ds^2 = -\frac{R^2\Delta}{A}dt^2 + \frac{A}{R^2}(d\phi - \omega dt)^2 + \frac{R^2}{\Delta}dR^2 + dz^2, \qquad (6)$$

- Kerr metric near the horizontal plane.
- $\omega = \frac{2MaR}{A}$ signifies the angular frequency of a zero-angular-momentum-observer (ZAMO) - an observer that rotates with zero angular momentum as seen from affine infinity - a courtesy of the frame-dragging effect brought about by the Kerr metric.
- We assemble the laws of conservation:
  - of rest-mass,
  - of angular momentum,
  - of energy.
- As well as the "vertical gravity".
- Complete the set of the equations of state.

Relativistic solution

$$
\begin{aligned}
\mathcal{A} &= 1 + \frac{a^2}{R^2} + 2\frac{a^2}{R^3}, \\
\mathcal{B}_+ = \mathcal{B} &= 1 + \frac{a}{R^{\frac{3}{2}}}, \\
\mathcal{C} &= 1 - \frac{3}{R} + 2\frac{a}{R^{\frac{3}{2}}}, \\
\mathcal{D} &= 1 - \frac{2}{R} + \frac{a^2}{R^2}, \\
\mathcal{E} &= 1 + 4\frac{a^2}{R^2} - 4\frac{a^2}{R^3} + 3\frac{a^4}{R^4}, \\
\mathcal{F} &= 1 - 2\frac{a}{R^{\frac{3}{2}}} + \frac{a^2}{R^2}.
\end{aligned}
\tag{7}
$$

Relativistic solution

- Stable circular orbits, describable by the 4-velocity

$$u^{\mu} = \xi_t + \Omega \xi_{\phi} = u^t \begin{pmatrix} 1 \\ 0 \\ \Omega \\ 0 \end{pmatrix}, \tag{8}$$

  with ξ_t and ξ_{ϕ} being Killing vector fields in the t and ϕ directions, which naturally exist in extremelly simple forms for a t and ϕ independent metric.

- The u^t is such that the 4-vector is normalized.

Relativistic solution

- $\Omega = \frac{\mathrm{d}\phi}{\mathrm{d}t}$ is the angular frequency seen from infinity.
- For the 4-acceleration in the R direction a_R to be zero (hence the studied object undergoing a circular orbital motion)

$$\Omega_\pm = \pm \frac{M^{\frac{1}{2}}}{R^{\frac{3}{2}} \pm a M^{\frac{1}{2}}} = \frac{1}{\mathcal{B}_\pm} \frac{M^{\frac{1}{2}}}{R^{\frac{3}{2}}}. \quad (9)$$

- We have 2 results - the $\pm$ corresponds to prograde and retrograde motion.
- Henceforth we will consider solely $\Omega_+ = \Omega$ - only prograde orbits correspond to a true stationary point in the accretion disk's phase space.

Vertical correctional force

- A rest-mass element displaced in the vertical direction feeld a "gravitational force".
- In the first order of expansion in the vertical displacement z, the acceleration is (RT in co-rotating coordinates)

$$\tilde{g} := zR^{z}_{0z0}, \tag{10}$$

- The law of hydrostatic equilibrium

$$\frac{\partial p}{\partial z} = \rho z R^{z} 0z0 = \rho \frac{z}{R^3} \frac{\mathcal{B}^2 \mathcal{D} \mathcal{E}}{\mathcal{A}^2 \mathcal{C}}. \tag{11}$$

- Vertically integrate the equation of hydrostatic equilibrium - get another equation of state

$$({c_s}')^2 \cong \frac{1}{R^3} \frac{\mathcal{B}^2 \mathcal{D} \mathcal{E}}{\mathcal{A}^2 \mathcal{C}} H^2 = \Xi^2 H^2. \tag{12}$$

Conservation of rest-mass

- Continuity equation ($\partial_t = 0$)

$$\nabla \cdot (\rho u) = 0. \tag{13}$$

- Velocity field of the form $U^\mu = (u^t, u^r, 0, \Omega u^t)$.
- Ω is the fluid angular velocity with respect to the stationary observer.
- In the frame co-rotating with the fluid, specific angular momentum is u_ϕ.
- An observer at fixed r who co-rotates with the fluid has 4-momentum $U^{(a)} = (1 - V)^{-1/2}(1, V, 0, 0)$.

$$v^r = V/\sqrt{1 - V^2} = u^r \sqrt{g_{rr}}. \tag{14}$$

- After vertical integration

$$-2\pi r \Sigma \mathcal{D}^{\frac{1}{2}} v^r = \dot{M}, \tag{15}$$

where the integration constant $\dot{M}$ is the mass accretion rate - constant across the disk.

Conservation of angular momentum

- Proceeding from the divergence-less-ness of the angular momentum vector density

$$\left(-\frac{\dot{M}}{2\pi}\widetilde{L} + R^2\mathcal{B}\mathcal{C}^{-\frac{1}{2}}\mathcal{D}\alpha P\right)_{,R} + 2R\widetilde{L}F = 0, \qquad (16)$$

- $\widetilde{L}$ signifies the specific angular momentum.
- This equations couples the radiant flux F with the vertically integrated (v. i.) pressure P.
- This pressure is merely a state variable describing the scale of the v. i. stress tensor.
- We do not derive it from first principles, rather we proceed in the opposite direction.

Conservation of energy

- Proceeding from orthogonality of the SE tensor's divergence to the 4-momentum

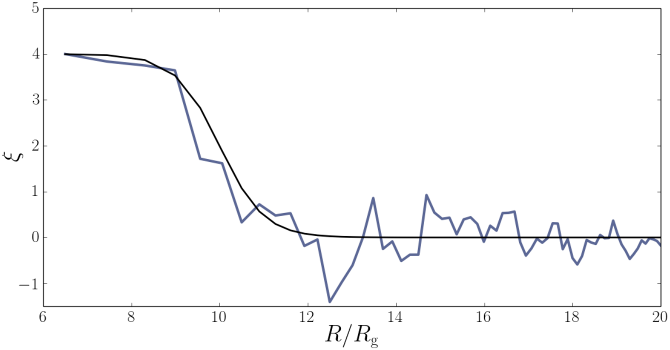
$$
\begin{aligned}
-\frac{\dot{M}}{2\pi R} T_c \frac{\partial s}{\partial R} + 2F + 2\sigma_{\widehat{R}\widehat{\phi}} \alpha P &= 0, \\
Q_{\mathrm{adv}} + Q_- - Q_+ &= 0,
\end{aligned}
\tag{17}
$$

- the individual positive Q terms correspond to heat advection, energy being radiated away and energy production by viscous dissipation.
- $\sigma_{\widehat{R}\widehat{\phi}}$ signifies the planar shear rate.

Relativistic solution - advection

$$\frac{\dot{M}}{2\pi R^2}\frac{P}{\Sigma}\xi + 2F + 2\sigma_{\widehat{R}\widehat{\phi}}\alpha P = 0, \quad (18)$$

- The advection-with equation of energy conservation.
- The ξ function is phenomenologically approximated.



Relativistic solution - magnetoadvective coupling

$$\left(-\frac{\dot{M}}{2\pi}\widetilde{L} + R^2\mathcal{B}\mathcal{C}^{-\frac{1}{2}}\mathcal{D}\alpha P\right)_{,R} + 2R\tilde{L}F = 0,$$
$$\frac{\dot{M}}{2\pi R^2}\frac{P}{\Sigma}\xi + 2F + 2\sigma_{\widehat{R}\widehat{\phi}}\alpha P = 0, \tag{19}$$

- completely describes the radiant flux F and the v. i. pressure P, for non-advective case.
- No need to solve the equation of state - the flux F given without knowledge of T_c.

$$P(1-\beta') = \frac{\Sigma k_N T_c}{\mu m_p} + H\frac{4\sigma}{3c}T_c^4,$$
$$\beta' = \frac{P_{mag}}{P_{tot}}, \tag{20}$$

Novikov-Thorne solution

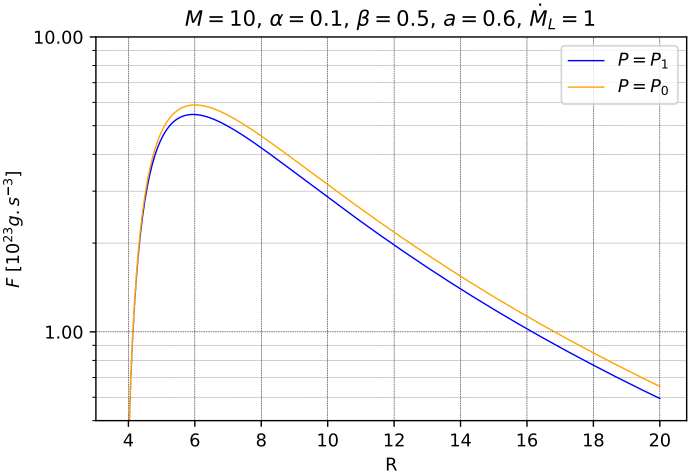
- Auxilliary function f to solve the equation (19).
  The vertically integrated total pressure and the radiant flux are

$$
\begin{aligned}
W_1 &= \frac{\dot{M}}{2\pi}\frac{\tilde{f}}{R^2\mathcal{D}},\\
P_1 &= \frac{\dot{M}}{2\pi\alpha}\frac{\tilde{f}}{R^2\mathcal{D}},\\
F_1 &= \frac{\dot{M}}{4\pi R}f.
\end{aligned}
\tag{21}
$$

How to choose pressure?

- In advection-with case, P is strongly coupled to other variables.
- Do we have to treat P as another state variable?
- The answer is no!
- The Novikov-Thorne pressure well approximates the advection-with case as well.

Effect of loss of angular momentum through radiation



Advection-with disk - limits of analycality

- Put all equations of state together.
- Simplify as possible, keep P prescribed by Novikov-Thorne.
- The final equation, of only one troublesome variable T_c, is

$$
\begin{aligned}
& -\frac{32\dot{M}_L R^{\frac{3}{4}} T_c^4 \tilde{f} \sigma \mathcal{A}}{\mathcal{B}\sqrt{\mathcal{E}}\sqrt{\mathcal{D}}\sqrt{384\dot{M}_L^2 \tilde{f}\xi + R^4 T_c^4 \alpha \kappa_T \sigma \mathcal{D}}} - \frac{32\dot{M}_L \tilde{f}\beta'}{R^2 \alpha \kappa_T \mathcal{D}} \\
& + \frac{32\dot{M}_L \tilde{f}}{R^2 \alpha \kappa_T \mathcal{D}} - \frac{64\dot{M}_L T_c \tilde{k}\xi\mathcal{C}}{3\sqrt{R}\alpha\kappa_T \mu \mathcal{D}} - \frac{R^{\frac{7}{2}} T_c^5 \tilde{k}\sigma\mathcal{C}}{18\dot{M}_L \tilde{f}\mu} = 0.
\end{aligned}
\tag{22}
$$

Advection-with disk - limits of analycality

- The radiant flux is given as

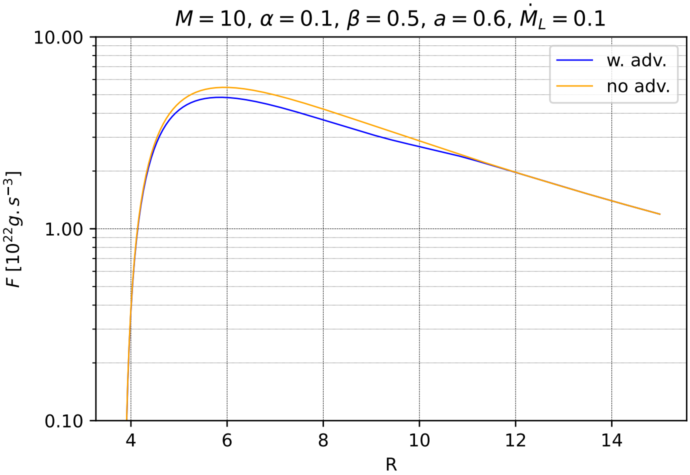
$$F = \frac{4\sigma T_c^4}{3\tau}, \tag{23}$$

- inputing the solved equations of state

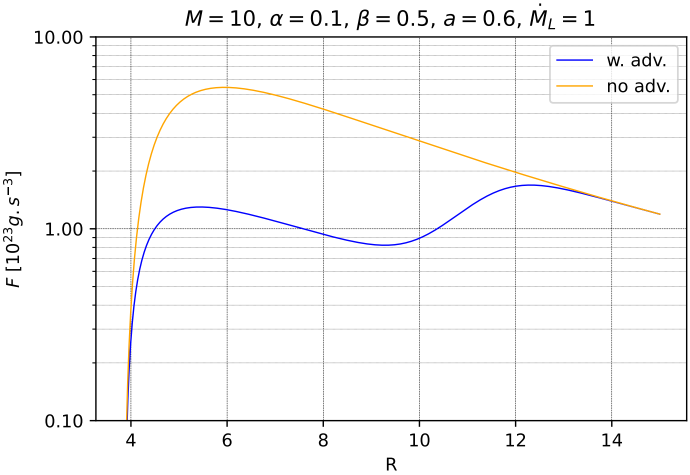
$$F = -\frac{\sigma_{\widehat{R}\widehat{\phi}}\alpha P}{1 + \frac{12\dot{M}_L \xi P}{R^2 \sigma T_c^4}}. \tag{24}$$

- This flux reduces to the viscous energy production with no advection.
- Solve the temperature equation numerically (easily by Newton/Halley method).
- Input the temperature and the NT pressure - we have the advection-with radiant flux!

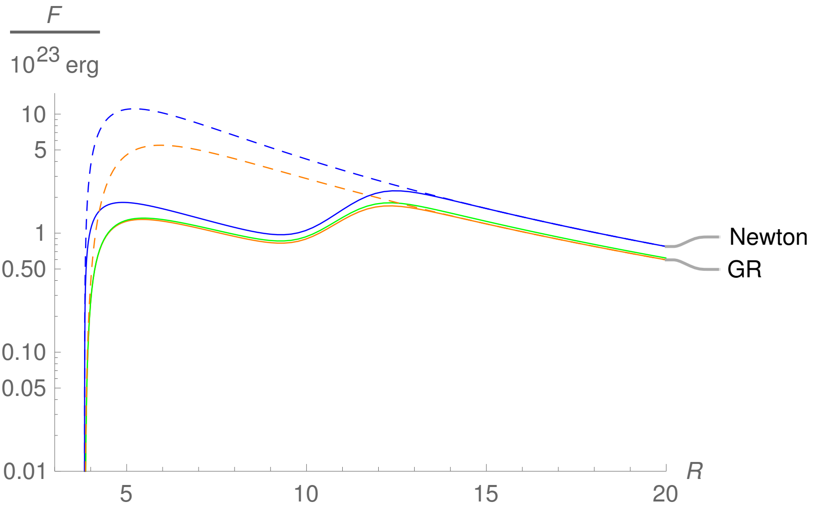
Radiant fluxes



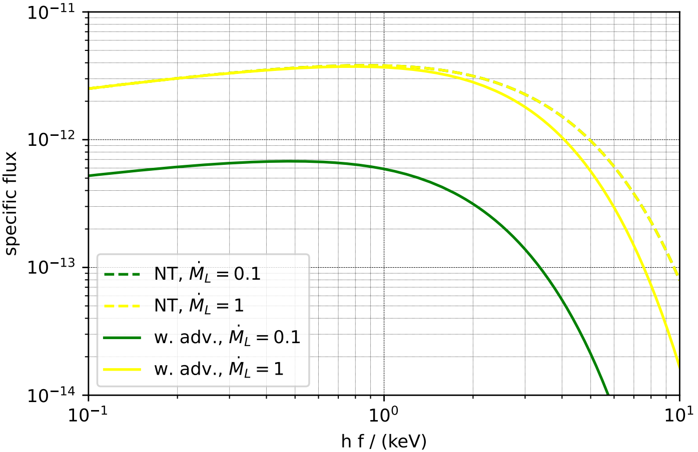
Radiant fluxes



Radiant fluxes - full solution of P



Spectra

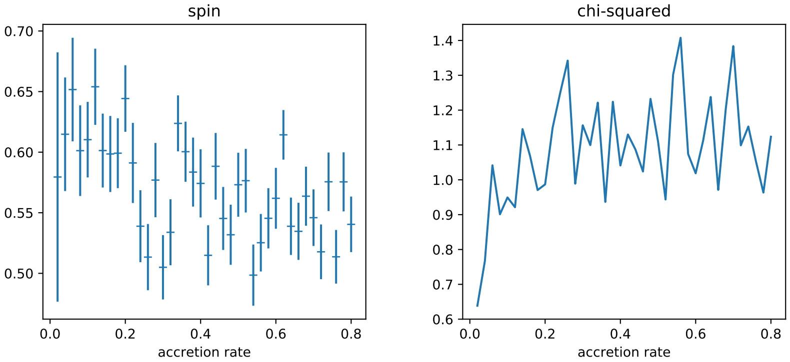


10^{-11}
10^{-12}
10^{-13}
10^{-14}
specific flux
NT, $\dot{M}_L = 0.1$
NT, $\dot{M}_L = 1$
w. adv., $\dot{M}_L = 0.1$
w. adv., $\dot{M}_L = 1$
10^{-1}
10^{0}
10^{1}
h f / (keV)

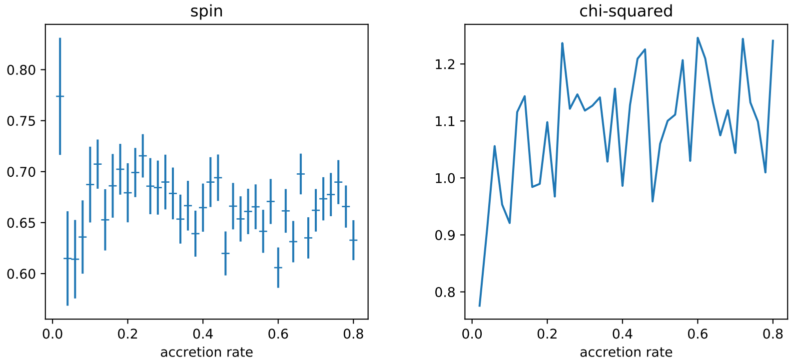
Testing our model

- How to test our model?
- Most straightforwardly - fit our model to observed spectra.
  - Not immediately viable:
  - observed spectra must be carefully selected, to be "nice" enough.
- Alternative - generate fake spectra.
  - Put a virtual accretion disk far away from the Earth.
  - Ray-trace radiation.
  - Simulate transmission through interstellar hydrogen clouds.
  - Simulate detection at a satellite's view matrix - including errors.
- Fit the Novikov-Thorne model to these spectra.
- See if the spin-against-$\dot{M}$ plot shows a spin decay.

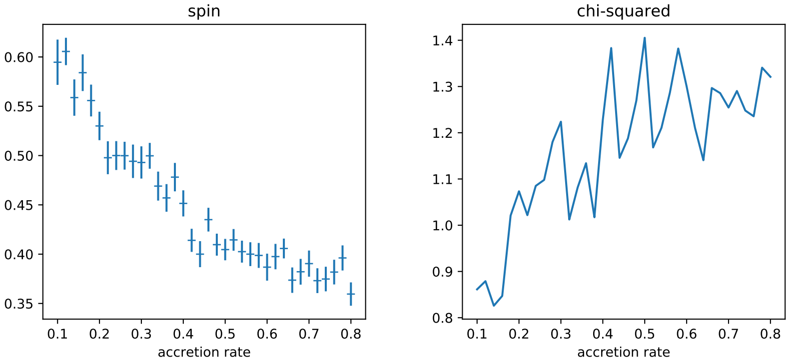
Testing our model - Newtonian, $\beta' = 0.5$



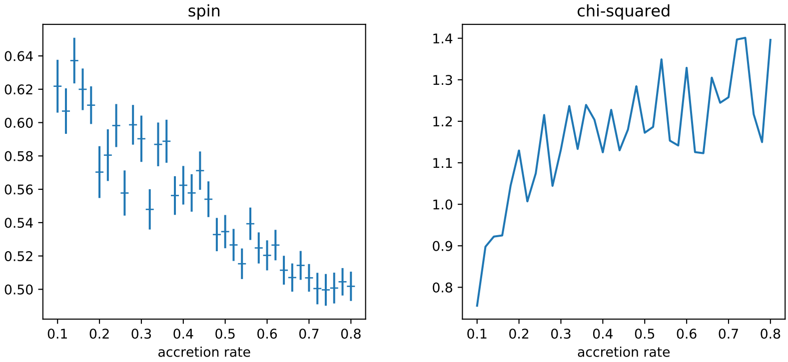
Testing our model - Newtonian, $\beta' = 0$



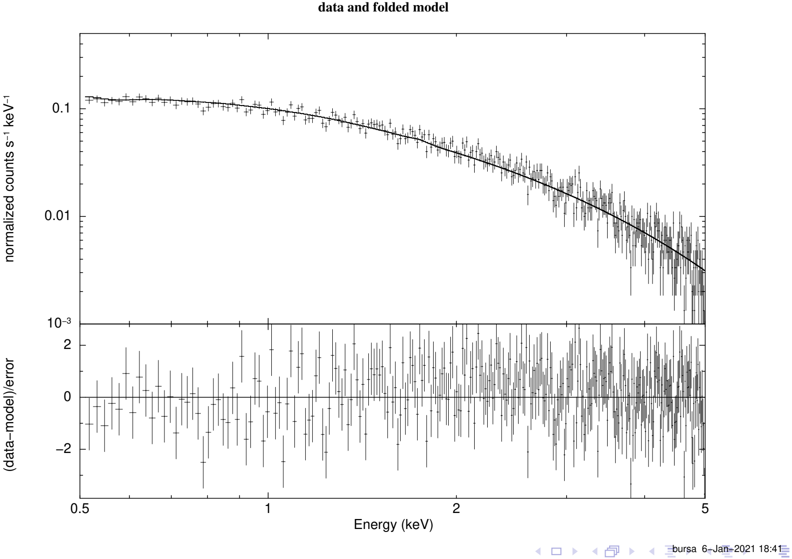
Testing our model - relativistic, $\beta' = 0.5$



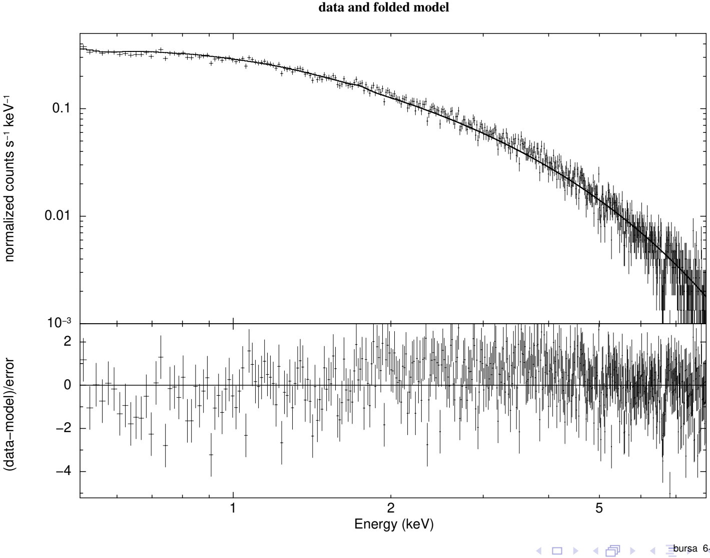
Testing our model - relativistic, $\beta' = 0$



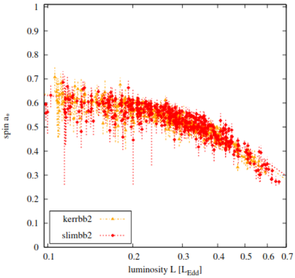
Fitting - Accretion rate = 0.2

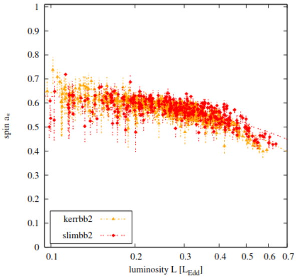


Fitting - Accretion rate = 0.8



Conclusion


spin a_*
luminosity L [L_{Edd}]
kerrbb2
slimbb2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7


spin a_*
luminosity L [L_{Edd}]
kerrbb2
slimbb2
1
0.9
0.8
0.7
0.6
0.5
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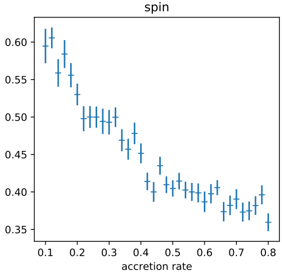
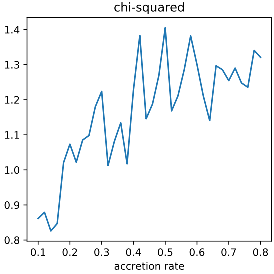
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7


spin
0.60
0.55
0.50
0.45
0.40
0.35
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
accretion rate


chi-squared
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
accretion rate