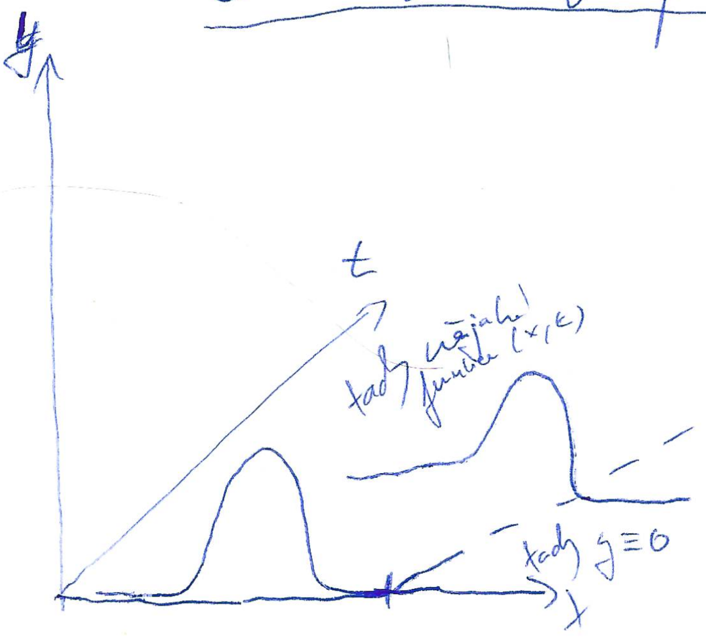
# Čela vln v polních teoriích




čelo vlny omezuje šíření informace

$y(x,t)$ musí být zjevně neanalytická

Matematický popis: charakteristické nadplochy
- nadplochy, podél kterých řešení hyperbolických PDE (vlnové rce) může být nespojité (typicky nespojitost 2. derivací, viz níže)

## Skalární pole

$\phi$ ... řešení vlnové rovnice $\Box\phi = g^{\mu\nu}\phi_{;\mu\nu} = 0$, $g^{\mu\nu}$ .. pevně zadaná geometrie

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$$\mathrm{Tr}(M^{-1}\cdot M_{,\lambda}) = \frac{(\det M)_{,\lambda}}{\det M}, \quad \Gamma^{\mu}{}_{\mu\lambda} = \frac{1}{2} g^{\mu\sigma} g_{\sigma\mu,\lambda} = \frac{1}{2}\frac{g_{,\lambda}}{g} = \frac{1}{\sqrt{-g}}\sqrt{-g}_{,\lambda}$$

$$V^{\mu}{}_{;\mu} = V^{\mu}{}_{,\mu} + \Gamma^{\mu}{}_{\mu\lambda} V^{\lambda} = V^{\mu}{}_{,\mu} + \frac{1}{\sqrt{-g}}\sqrt{-g}_{,\mu} V^{\mu} =$$

$$= \frac{1}{\sqrt{-g}}\left(\sqrt{-g}\, V^{\mu}\right)_{,\mu}$$

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$$\Rightarrow \Box\phi = \frac{1}{\sqrt{-g}}\left(\sqrt{-g}\, g^{\alpha\beta}\phi_{,\alpha}\right)_{,\beta} = g^{\alpha\beta}\phi_{,\alpha\beta} + (\text{členy bez 2. derivací } \phi) =$$

$$= g^{00}\phi_{,00} + (\text{členy bez } \phi_{,00}) = 0$$

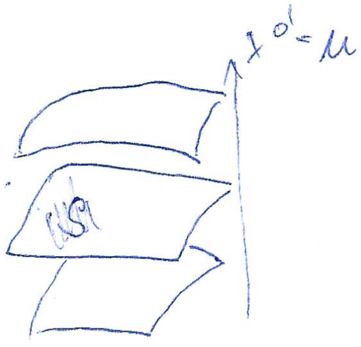
$\Rightarrow$ Pokud máme na ploše $S$ ($t=$ konst.) zadaná počáteční data $\phi$ a $\phi_{,0}$, pak lze spočíst $\phi_{,i}$ a $\phi_{,0i}$ $\Rightarrow$ pokud $\boxed{g^{00} \neq 0}$ lze vyjádřit $\phi_{,00}$

$\rightsquigarrow$ derivováním $\phi_{,00} = \ldots$ lze vyjádřit $\forall$ časové derivace

$\Rightarrow$ v blízkosti $S$ lze rozvinout do Taylora = jednoznačné řešení

tj. pro data na prostorupodobné nadploše nejsou čela vln

Přizpůsobení souřadnic:




$$S: u(x^\mu) = 0 \quad \text{(nebo daná konst.)}$$

Požadavek $g'^{00} \neq 0$:

$$\frac{\partial x^{0'}}{\partial x^\alpha}\frac{\partial x^{0'}}{\partial x^\beta} g^{\alpha\beta} = u_{,\alpha} u_{,\beta} g^{\alpha\beta} \neq 0$$

$\Rightarrow$ pokud je gradient $S$ nulový vektor $\boxed{g^{\alpha\beta} u_{,\alpha} u_{,\beta} = 0}$,

nelze $\Box\phi = 0$ jednoznačně vyřešit $\quad\uparrow\quad$ $S$ = nulová nadplocha

(Cauchyho data ani nejde zadat libovolně, jelikož musí být splněno $\Box\phi = (\text{členy bez } \phi_{,00}) = 0$)

$\Rightarrow$ $S$ = charakteristika

Ukažme, že při přechodu $S$ se $\phi_{,\alpha\beta}$ může měnit nespojitě:

p.p. $\Delta\phi = 0$, $\Delta\phi_{,\alpha} = 0$; $\Delta\phi_{,\alpha\beta} =: \psi_{\alpha\beta}$

Zvol $x^\mu$ a $x^\mu + dx^\mu \in S \Rightarrow u_{,\mu} dx^\mu = 0$

$$\phi_{,\alpha}(x^\mu + dx^\mu) = \phi_{,\alpha}(x^\mu) + \phi_{,\alpha\beta}(x^\mu)\, dx^\beta$$

$$0 = \Delta\phi_{,\alpha}(x^\mu + dx^\mu) = \cancelto{0}{\Delta\phi_{,\alpha}(x^\mu)} + \Delta\phi_{,\alpha\beta}(x^\mu)\, dx^\beta = \psi_{\alpha\beta}\, dx^\beta$$

omezeno podmínkou $u_{,\mu} dx^\mu = 0$

$\rightarrow$ Lagrangeovy multiplikátory $\lambda_\alpha$:

$(\psi_{\alpha\beta} + \lambda_\alpha u_{,\beta})\, dx^\beta = 0$ $\quad$ lze určit $\lambda_\alpha$, aby $dx^\beta$ byly nezávislé (analogicky jako LRI $\Leftrightarrow$ d'Alembert)

$\Downarrow$

$$\psi_{\alpha\beta} + \lambda_\alpha u_{,\beta} = 0$$

$\hookrightarrow \lambda_\alpha u_{,\beta} = u_{,\alpha} \lambda_\beta \Rightarrow \lambda_\alpha = -\psi u_{,\alpha}$

$$\Rightarrow \psi_{\alpha\beta} = \psi u_{,\alpha} u_{,\beta}$$

vlnová rce splněna na obou stranách $S \Rightarrow$ superponujeme rozdíl:

$$0 = \Delta(g^{\alpha\beta}\phi_{,\alpha\beta}) = \Delta[g^{\alpha\beta}\phi_{,\alpha\beta}] + \cancelto{0}{\Delta(\text{členy bez 2. derivací})} =$$

$$= g^{\alpha\beta}\psi u_{,\alpha} u_{,\beta}$$

$\Rightarrow \psi \neq 0$ jen pokud $g^{\alpha\beta} u_{,\alpha} u_{,\beta} = 0$ tj. $S$ je charakteristika ✓

nespojitosti 2. derivací jen při přechodu char. ploch

## Elmag. pole

p.p.: $J^\alpha = 0$, $S: u(x^\mu) = 0 \leftarrow$ nespojitost v $F_{\alpha\beta}$

$$\Rightarrow F_{\alpha\beta} = f_{\alpha\beta} + \psi_{\alpha\beta}\,\theta(u)$$

spojité, diferencovatelné na S

$$F_{\alpha\beta;\gamma} = f_{\alpha\beta;\gamma} + \psi_{\alpha\beta;\gamma}\,\theta(u) + \psi_{\alpha\beta}\,u_{,\gamma}\,\delta(u)$$

$\Downarrow$ MW II

$$f_{\{\alpha\beta;\gamma\}} + \psi_{\{\alpha\beta;\gamma\}}\,\theta(u) + \psi_{\{\alpha\beta}\,u_{,\gamma\}}\,\delta(u) = 0$$

$u < 0: \; f_{\{\alpha\beta;\gamma\}} = 0 \xrightarrow{\text{spojitost, diferenc.}} f_{\{\alpha\beta;\gamma\}} = 0$ i na S

$u > 0: \; \psi_{\{\alpha\beta;\gamma\}} = 0 \longrightarrow \psi_{\{\alpha\beta;\gamma\}} = 0$ i na S

$\Rightarrow$ na S musí být $\underline{\psi_{\{\alpha\beta}\,u_{,\gamma\}} = 0}$

$g^{\beta\gamma}\cdot$, MW I: $F_\alpha{}^\beta{}_{;\beta} = 0 \Rightarrow \underline{\psi_{\alpha\beta}\,u^{,\beta} = 0}$ na S

(a díky AS $F_{\mu\nu}$ i $\psi_{\beta\alpha}u^{,\beta} = 0$)

$$\psi_{\alpha\beta}u_{,\gamma} + \psi_{\gamma\alpha}u_{,\beta} + \psi_{\beta\gamma}u_{,\alpha} = 0 \quad / u^{,\gamma}$$

$$\psi_{\alpha\beta}u_{,\gamma}u^{,\gamma} + u_{,\beta}\underbrace{\cancel{\psi_{\gamma\alpha}u^{,\gamma}}}_{0} + u_{,\alpha}\underbrace{\cancel{\psi_{\beta\gamma}u^{,\gamma}}}_{0} = 0$$

$$\psi_{\alpha\beta}u_{,\gamma}u^{,\gamma} = 0$$

$\Rightarrow \underline{\psi_{\alpha\beta} \neq 0}$ jen pokud $g^{\mu\nu}u_{,\mu}u_{,\nu} = 0$ tj. $\underline{\text{S je nulová plocha}}$

EM pole nespojité

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Matematické intermezzo:

Tvrzení: Antisymetrická matice $a_{ij}$ musí mít jedině sudou hodnost.

-Dk: označ n .. # řádků matice, h = rank $a_{ij}$

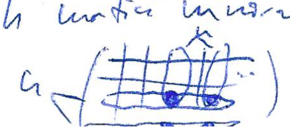
$\det(a_{ij}) = \det(a^T_{ij}) = (-1)^n \det(a_{ij}) \to$ pokud n liché .. $h < n$

p.p. $h < n$ je liché

$\exists$ h lin. nz. řádků $i_1 \dots i_h$, ostatní vyjádř. LK, díky AS má h lin. nz. sloupců stejné indexy.

Zvol h×h matici minoru $a_{m_s n_t} = \sum_{i_p} c_{m_s i_p} a_{i_p n_t} = \sum_{i_p, i_q} c_{m_s i_p} d_{i_q n_t} a_{i_p i_q}$

$s,t = 1, \dots, h$ ; LK řádků $i$ ; LK sloupců $i$



$\Rightarrow \det(a_{m_s n_t}) = \det(c_{m_s i_p}) \det(d_{i_r n_t}) \det(a_{i_p i_r}) = \lambda \det(a_{i_p i_r})$

díky stejnému pořadí LN sloupců a řádků lze vždy $a_{i_p i_r}$ vybrat jako hlavní minor tj. se symetricky „posazenými" řádky a sloupci

tj. $a_{i_p i_r}$ je AS matice s lichou hodností.

z 1. kroku $\det a_{i_p i_r} = 0 \Rightarrow \det(a_{m_s n_t}) = 0$ $\forall$ minory

$\Rightarrow$ hodnost nemůže být $h \Rightarrow h$ je sudé $\square$

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$\psi_{\alpha\beta} \Rightarrow \operatorname{rank} \psi_{\alpha\beta} = 4$ nebo $2$

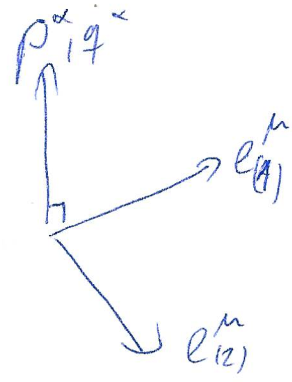
ale $\psi_{\alpha\beta} u^{,\beta} = 0 \Rightarrow$ sloupce jsou LZ $\Rightarrow \operatorname{rank} \psi_{\alpha\beta} = 2 \Rightarrow \dim \operatorname{Ker} \psi_{\alpha\beta} = 4-2=2$

$\Rightarrow \exists\, e^{\alpha}_{(A)},\ A=1,2 : \quad \psi_{\alpha\beta} e^{\beta}_{(A)} = 0$

$\operatorname{rank}=2 \Rightarrow \exists\, \underset{LN}{p_\alpha, q_\alpha} : \vec{\psi}_\alpha = \lambda_\alpha \vec{p} + \mu_\alpha \vec{q}$ pro nějaká $\lambda_\alpha, \mu_\alpha$ a

$p_\alpha e^{\alpha}_{(A)} = q_\alpha e^{\alpha}_{(A)} = 0$

díky AS ale také $\psi_{\beta\alpha} e^{\beta}_{(A)} = 0$, $\Rightarrow$ tedy $\lambda_\alpha e^{\alpha}_{(A)} = \mu_\alpha e^{\alpha}_{(A)} = 0$




$\Rightarrow$ lze napsat $\lambda_\alpha = \lambda_1 p_\alpha + \lambda_2 q_\alpha$

a $\mu_\alpha = \mu_1 p_\alpha + \mu_2 q_\alpha$

$\psi_{\alpha\beta} = \underbrace{\cancel{\lambda_1 p_\alpha p_\beta}}_{\lambda_1 = 0} + \lambda_2 q_\alpha p_\beta + \mu_1 p_\alpha q_\beta + \underbrace{\cancel{\mu_2 q_\alpha q_\beta}}_{\mu_2 = 0} =$

$= -\lambda_2 p_\alpha q_\beta - \mu_1 q_\alpha p_\beta$

$\Rightarrow \lambda_2 = -\mu_1 =: \nu \to$ absorbujeme do $p_\alpha$

$\psi_{\alpha\beta} = p_\alpha q_\beta - p_\beta q_\alpha \equiv (p \wedge q)_{\alpha\beta}$

$v_\alpha := u_{,\alpha}\ ,\ \psi_{[\alpha\beta} v_{\gamma]} = 0 \to (p \wedge q \wedge v)_{\alpha\beta\gamma} = 0$

$\Rightarrow p_\alpha, q_\alpha$ a $u_{,\alpha}$ jsou LZ

$\Rightarrow$ zvolme $p_\alpha = u_{,\alpha}$

$\psi_{\alpha\beta} = u_{,\alpha} q_\beta - u_{,\beta} q_\alpha$

$$\psi_{\alpha\beta} u^{,\beta} = u_{,\alpha}\, q_\beta u^{,\beta} - q_\alpha \cancel{u_{,\beta} u^{,\beta}}^{\,0} = 0 \quad \text{na } S$$

$\Rightarrow u_{,\alpha} q^\alpha = 0$

ve vhodné nabázení LISu: $u_{,\hat\alpha} = (u, -u, 0, 0) \Rightarrow q^{\hat\alpha} = \begin{pmatrix} u \cdot a \\ u \cdot a \\ b \\ c \end{pmatrix}$

$\rightarrow q \propto \partial u$ nebo $q$ <u>prostorupodobný</u> na $S$

Invarianty EM pole v libovolné IS:

$$F_{\alpha\beta} F^{\alpha\beta} = 2\left(\vec{B}^2 - \vec{E}^2\right), \quad F_{\alpha\beta} {}^*F^{\alpha\beta} = 4 \vec{E} \cdot \vec{B}$$

$$\left( {}^*\omega_{a_{p+1} \dots a_d} = \frac{1}{p!}\, \omega^{a_1 \dots a_p}\, \varepsilon_{a_1 \dots a_p a_{p+1} \dots a_d} \right)$$

$$\Rightarrow \Delta F_{\alpha\beta} \Delta F^{\alpha\beta} = 2\left[ (\Delta\vec{B})^2 - (\Delta\vec{E})^2 \right],$$

$$\Delta F_{\alpha\beta} \Delta {}^*F^{\alpha\beta} = 4 \Delta\vec{E} \cdot \Delta\vec{B}$$

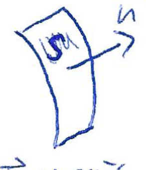
ale: $(\psi_{\alpha\beta} = u_{,\alpha} q_\beta - u_{,\beta} q_\alpha)$

$\underline{\Delta F_{\alpha\beta} \Delta F^{\alpha\beta}} = \psi_{\alpha\beta} \psi^{\alpha\beta} = [\text{členy s } u_{,\alpha} u^{,\alpha} \text{ nebo } u_{,\alpha} q^\alpha] = \underline{0}$

$\underline{\psi_{\alpha\beta} {}^*\psi^{\alpha\beta}} = [\text{členy typu } \varepsilon^{\alpha\beta\gamma\delta} u_{,\alpha} u_{,\gamma} q_\beta q_\delta] = \underline{0}$

$\Rightarrow \underline{|\Delta\vec{E}| = |\Delta\vec{B}|}_{(1)}, \quad \underline{\Delta\vec{E} \cdot \Delta\vec{B} = 0}_{(2)}$

$S: u(x^\mu) = 0 \leadsto$ přepíšeme do tvaru $t - f(x^i) = 0 \Rightarrow S: u(\overset{t}{f(x^i)}, x^j) = 0$




E-hodnost:

$\Rightarrow n_i := \dfrac{\partial f}{\partial x^i} \quad \Rightarrow \dfrac{\partial u}{\partial x^j} = u_{,0} n_j + u_{,j} = 0, \quad u_{,i} = -u_{,0} n_i$

$\psi_{\alpha\beta} u^{,\beta} = \underline{\Delta F_{\alpha\beta} u^{,\beta} = 0}, \quad \alpha = 0: \Delta E_i u^{,i} = -\Delta E_i u_{,0} n^i = 0$

$\underline{\Delta {}^*F_{\alpha\beta} u^{,\beta} = 0}, \quad \alpha = 0: \Delta B_i u^{,i} = -\Delta B_i u_{,0} n^i = 0$

$\vec{n}$ je směr šíření vlny

$\Rightarrow \underline{\Delta\vec{E} \cdot \vec{n} = \Delta\vec{B} \cdot \vec{n} = 0}_{(3)}$

Vztahy (1–3) přesně splňují vektory $\vec{E}, \vec{B}$ v případě <u>rovinné EM vlny</u>

- EM pole splňující (1–3) naz. <u>nulové pole</u> či <u>pole typu N</u>

# Gravitační pole

$\Gamma$ ... lze vynulovat podobně do LIS $\times$ $R$ ... nejde vynulovat

$\rightsquigarrow$ p.p. na S: $g_{\mu\nu}$ a $g_{\mu\nu,\lambda}$ spojité, $g_{\mu\nu,\kappa\lambda}$ může mít skok.

... podobný setup jako u skalárního pole – tam: $\Delta\phi_{,\nu\rho} = \psi u_{,\nu} u_{,\rho}$

zde analogicky: $\underline{\Delta g_{\mu\nu,\kappa\lambda} = H_{\mu\nu} u_{,\kappa} u_{,\lambda}}$

$$R_{\mu\nu\kappa\lambda} = -\left( g_{\mu[\kappa|,\nu|\lambda]} + g_{\nu[\lambda|,\mu|\kappa]} + 2 g_{\iota\rho} \Gamma^{\iota}{}_{\mu[\kappa|} \Gamma^{\rho}{}_{\nu|\lambda]} \right)$$

$g$, $\partial g$ spojité

$$\Rightarrow \underline{\Delta R_{\mu\nu\kappa\lambda}} = u_{,\mu} H_{\nu[\kappa} u_{,\lambda]} - u_{,\nu} H_{\mu[\kappa} u_{,\lambda]} = \underline{2 u_{,[\mu} H_{\nu][\kappa} u_{,\lambda]}}$$

Skalární pole ... vlnová rce $\Rightarrow$ S nulová nadplocha

gravitační pole ... EFE $\overset{?}{\Rightarrow}$ S nulová nadplocha

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ve vakuu: $T_{\mu\nu} = 0$ ⊕ $\Lambda = 0$: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0$ / Tr

$$R - 2R = 0, \quad R = 0 \xrightarrow{EFE} R_{\mu\nu} = 0$$

- - - - -

ve vakuu:

$$0 = \Delta R_{\nu\lambda} = g^{\mu\kappa} \Delta R_{\mu\nu\kappa\lambda} = \frac{1}{2}\Big[ u_{,\mu} H_{\nu}{}^{\mu} u_{,\lambda} + u_{,\nu} H_{\mu\lambda} u^{,\mu} - u_{,\nu} H_{\mu}{}^{\mu} u_{,\lambda} - u_{,\mu} H_{\nu\lambda} u^{,\mu} \Big] \underset{(*)}{=} 0 \quad / \cdot u_{,\rho} u_{,\sigma}, \ [\nu\rho], [\lambda\sigma]$$

$$0 = u_{,\mu} u_{,[\rho} H_{\nu]}{}^{\mu} \cancel{u_{,[\lambda} u_{,\sigma]}}^{\,0} + \cancel{u_{,[\nu} u_{,\rho]}}^{\,0} \cdots 0 - \cancel{u_{,[\nu} u_{,\rho]}} - \underbrace{u_{,[\rho} H_{\nu][\kappa} u_{,\lambda]}}_{\frac{1}{2}\Delta R_{\rho\nu\kappa\lambda}} u_{,\mu} u^{,\mu}$$

$\Rightarrow (\Delta R_{\mu\nu\kappa\lambda})(u_{,\rho} u^{,\rho}) = 0$

... opět: pokud $\Delta R_{\mu\nu\kappa\lambda} \neq 0$, pak už musí $u_{,\rho} u^{,\rho} = 0$ $\to$ <u>S je nulová</u>

$(*)$ / $\cdot u_{,\rho}$, $[\lambda\rho]$

$$H_{\mu\lambda} u^{,\mu} u_{,\rho} u_{,\nu} - H_{\mu\rho} u^{,\mu} u_{,\lambda} u_{,\nu} = 0, \quad \mathcal{H}_{\lambda} := H_{\mu\lambda} u^{,\mu}$$

$$\mathcal{H}_{\lambda} u_{,\rho} = \mathcal{H}_{\rho} u_{,\lambda} \to \mathcal{H}_{\mu} = \kappa u_{,\mu}; \quad H_{\mu\lambda} u^{,\mu} = u_{,\lambda} \kappa$$

$\rightsquigarrow$ dosazení do $(*)$:

$$\kappa u_{,\nu} u_{,\lambda} + \kappa u_{,\nu} u_{,\lambda} - H_{\mu}{}^{\mu} u_{,\nu} u_{,\lambda} = 0 \Rightarrow \kappa = \frac{1}{2} H_{\iota}{}^{\iota}$$

$$\underline{H_{\iota\mu} u^{,\iota} = \frac{1}{2} H_{\iota}{}^{\iota} u_{,\mu}}$$

## Matematické intermezzo:

(skripta Bičák, Semerák)

Připomenutí: Weylův tenzor ... bezestopá část Riemanna

$$C^{\mu\nu}{}_{\kappa\lambda} = R^{\mu\nu}{}_{\kappa\lambda} - 2\delta^{[\mu}_{[\kappa} R^{\nu]}_{\lambda]} + \frac{1}{3}\delta^{[\mu}_{[\kappa}\delta^{\nu]}_{\lambda]} R$$

$$C_{[\mu\nu][\kappa\lambda]} = C_{\kappa\lambda\mu\nu} = C_{\mu\nu\kappa\lambda}$$

ve vakuu zřejmě ($R_{\mu\nu} = 0 = R$): $R_{\mu\nu\kappa\lambda} = C_{\mu\nu\kappa\lambda}$

Riemann, Weyl ... 2 páry AS indexů ⇝ lze zavést levý a pravý duál:

$${}^*R_{\mu\nu\kappa\lambda} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} R^{\rho\sigma}{}_{\kappa\lambda}, \quad R^*_{\mu\nu\kappa\lambda} = \frac{1}{2} R_{\mu\nu}{}^{\rho\sigma}\varepsilon_{\rho\sigma\kappa\lambda}$$

⇒ dvojitý duál:

$$\underline{{}^*R^*}{}^{\alpha\beta}{}_{\mu\nu} = \frac{1}{4}\varepsilon^{\alpha\beta\rho\sigma} R_{\rho\sigma}{}^{\kappa\lambda}\varepsilon_{\kappa\lambda\mu\nu} = -\frac{1}{4}4!\,\delta^{[\alpha}_{\kappa}\delta^{\beta}_{\lambda}\delta^{\rho}_{\mu}\delta^{\sigma]}_{\nu} R_{\rho\sigma}{}^{\kappa\lambda} =$$

$$= \dots = \underline{-R^{\alpha\beta}{}_{\mu\nu} + 2R^{[\alpha}_{\mu}\delta^{\beta]}_{\nu} + 2\delta^{[\alpha}_{\mu} R^{\beta]}_{\nu} - R\,\delta^{[\alpha}_{\nu}\delta^{\beta]}_{\nu}}$$

... Ruse–Lanczos

v případě, že stopy jsou nulové tedy:

$${}^*C^*{}_{\alpha\beta\mu\nu} + C_{\alpha\beta\mu\nu} = 0 \quad /*. \qquad ({}^{**}F_{\mu\nu} = -F_{\mu\nu})$$

$$\underline{{}^*C_{\alpha\beta\mu\nu} = C^*{}_{\alpha\beta\mu\nu}}$$

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ve vakuu: ${}^*R_{\mu\nu\kappa\lambda} = R^*_{\mu\nu\kappa\lambda}$

$$\Delta R_{\mu\nu\kappa\lambda} = 2u_{[,\mu} H_{\nu][\kappa} u_{,\lambda]}$$

$$\rightarrow \underset{(1)}{\underline{\Delta R_{\mu\nu\kappa\lambda}\Delta R^{\mu\nu\kappa\lambda}}} = \frac{1}{4}\left(2u_{,\mu} H_{\nu\kappa} u_{,\lambda} u^{,\nu} H^{\mu\lambda} u^{,\kappa}\right) = \frac{1}{2} u_{,\mu} u^{,\kappa} \frac{1}{2} H_{\nu}{}^{\nu} u_{,\kappa} \frac{1}{2} H_{\lambda}{}^{\lambda} u_{,\mu} = 0$$

podobně $\underset{(3)}{\underline{\Delta R_{\mu\nu\kappa\lambda} u^{\mu} = 0}}$

$$\Delta\, {}^*R_{\mu\nu\kappa\lambda} = \Delta R^*_{\mu\nu\kappa\lambda} = \varepsilon_{\mu\nu}{}^{\rho\sigma} u_{[,\rho} H_{\sigma][\kappa} u_{,\lambda]} = \varepsilon_{\mu\nu}{}^{\rho\sigma} u_{,\rho} H_{\sigma[\kappa} u_{,\lambda]}$$

$$\underset{(2)}{\underline{\Delta R_{\mu\nu\kappa\lambda}\Delta\, {}^*R^{\mu\nu\kappa\lambda}}} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} u_{,\mu}\underline{u_{,\rho}} H_{\nu\kappa} u_{,\lambda} H_{\sigma}{}^{\lambda} u^{,\kappa} = \underline{0}$$

$$\underset{(4)}{\underline{\Delta\, {}^*R_{\mu\nu\kappa\lambda} u^{\lambda} = 0}}$$

$\Rightarrow$ (1-4) naprosto analogické vztahům odvozeným pro EM pole

$\Rightarrow$ now. nulové grav. pole, pole typu N

Poznámky ke závěr

1) $u_{,\mu}$ .. normála k S ..

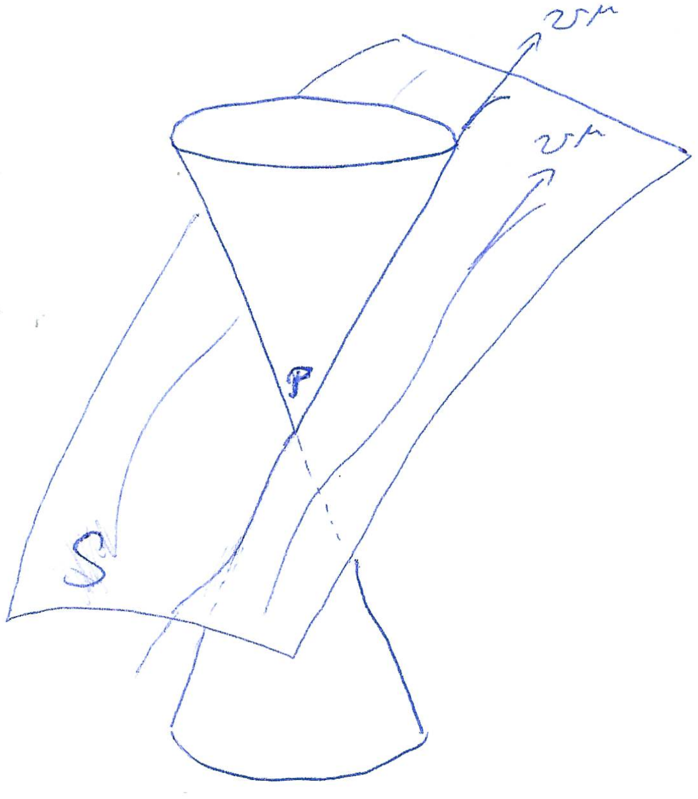
$v^\mu \parallel S \Leftrightarrow u_{,\mu} v^\mu = 0$ ale $u_{,\mu} u^{,\mu} = 0 \Rightarrow u^{,\mu} \parallel S$

integrální křivky pole $v^\mu \equiv u^{,\mu}$ ... bicharakteristiky

ozn. $\omega_\lambda := v_{\lambda;\mu} v^\mu$

$\xrightarrow[\text{(lokálně)}]{\text{LIS}}$ $\omega_\lambda = u_{,\lambda\mu} u^{,\mu} = u_{,\mu\lambda} u^{,\mu} = \frac{1}{2}(u_{,\mu} u^{,\mu})_{,\lambda} = 0$

$\Rightarrow \omega_\lambda = v_{\lambda;\mu} v^\mu = 0$ ... rce geodetiky $\Rightarrow$ bicharakteristiky jsou nulové geodetiky




- světelný kužel v P se S dotýká vždy podél nějaké bicharakteristiky

2) řád řešení vlny:
- skalární a grav. pole
  - nespojitost 2. derivací $\Rightarrow p = 2$
- EM pole
  - nespojitost $F_{\mu\nu} \Leftrightarrow$ 1. derivace $A_\mu \Rightarrow p = 1$

proč $A_\mu$ ... splňuje hyperbolickou rci

$\Box A^\alpha = -4\pi J^\alpha = 0$

$(-\cancel{R^\alpha_\mu} A^\mu)$

(pro g by to bylo asi $R_{\alpha\mu} = 0$)