

The city of Prague is very fine, so beautiful that it is worth a long journey for itself.

AE to his friend M. Besso, 13<sup>th</sup> May 1911

I have a magnificent institute here in which I work very comfortably. Otherwise it is less homey (Czech language, bedbugs, awful water, etc.). By the way, **Czechs are more harmless than one thinks.**

AE to M. Grossmann, 27<sup>th</sup> April 1911



## Einstein's Days and Works in Prague: Relativity Then and Now

*Einstein's*  
*Days and Works*  
*in Prague*



## Einstein's Days and Works in Prague: Relativity Then and Now

HESIOD (7<sup>th</sup> cent. BCE) W&D: "Works & Days"

"THE PRICE OF ACHIEVEMENT IS TOIL"; AND THE GODS HAVE RULED THAT YOU MUST PAY IN ADVANCE"

SUMMARY:

EINSTEIN PAID MUCH IN PRAGUE

Days: April 1911 - July 1912

Works: Principle of Equivalence; light bending (observable); dragging of inertial frames; features of a future theory of gravity

" Schinderei, Plage, Rackerei

"The price of achievement is toil<sup>†</sup>; and the gods have ruled that you must pay in advance"

Hesiod (7<sup>th</sup> cent. BC)  
Works & Days

<sup>†</sup> Schinderei, Plage, Rackerei

### Einstein paid much in Prague

Days: April 1911 – July 1912

- Works:
- Principle of Equivalence
  - light bending (observable)
  - dragging of inertial frames
  - features of a future theory of gravity



## The building of Philosophical Faculty of the German University in Prague, Viničná



University

Philipp Frank: Einstein, His Life and Times  
(A. Knopf, New York 1947)





## The building of Philosophical Faculty of the German University in Prague, Viničná



University



psychiatric asylum

Philipp Frank: Einstein, His Life and Times  
(A. Knopf, New York 1947)

Participants at the Einstein centenary celebrations in 1979 at the Charles University in Prague:

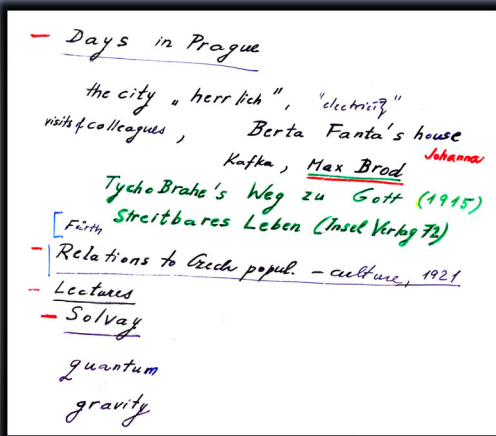


J.A. Wheeler,  
A. Traut-  
mann,  
Mrs. Melcher,  
E. Schmutzer,  
J. Langer,  
M. Bergmann,  
P. Bergmann,  
H. Melcher  
(from left to  
right)



## Why and how invited do Prague

- Charles University from 1348 – one educational centre for Czechs, Germans, Poles...
- In 1882 divided into the Czech and German parts; in 1911 4432:1844 students.
- In German part – F. Lippich math  $\rightsquigarrow$  theor. physics A. Lampa (“Machian”), G. Pick ( $g_{\mu\nu}$ ).
- Planck's letter, confession, library





## Impact on Czech culture: Karel Čapek (1890-1938)

Alan J. Friedman, Carol C. Donley:  
EINSTEIN AS MYTH AND MUSE

Cambridge Univ. Press 1985

Approaches to relativity in fiction 83

▶ Approaches to relativity in fiction  
Writers of prose fiction displayed an equally excited and wide-ranged interest in Einstein's marvels. From use of the profoundly wrong aphorism "everything is relative," to intricate, accurate analogies, the possibilities of Einstein and his theories as models for subject and form were explored by authors of novels and short stories.

A remarkable early exposition of the possibilities appeared in 1924, with Karel Čapek's novel, *Krakatit* (translated into English the following year).<sup>14</sup> Čapek's awareness of science and technology was indicated by mentions in the novel of the leading scientists of the day, including Einstein, Rutherford, Planck, Bohr, and Millikan. The plot concerns an inventor who has discovered a way to release atomic energy, and who is pursued by those who would use his discovery for their own purposes. The technical details are as accurate as they could be in the early 1920's, and atomic energy is correctly seen as a possibility emerging from the radioactivity work of Becquerel and Rutherford, and not from Einstein's theories – a distinction lacking in much subsequent fiction and popular treatments. Speculations about atomic energy were not new, even in 1924, as will be discussed in Chapter 6. A novelist's use of relativity as metaphor and form, however, may have occurred first in *Krakatit*.

The inventor, Prokop, is torn in the traditional struggle between God and the devil for his soul and his discovery. Prokop's bewilderment, in the literal form of a fever, is described by the first metaphor from relativity:

Inside his head the blows had become faster and more painful.

It appeared that he was moving with the minimum velocity of light, in some way his heart was compressed. But that was only the Fitzgerald-Lorentz contraction, he explained to himself; soon he would become as flat as a pancake. And suddenly there appeared in front of him countless glass prisms; no, they were only endless, highly polished planes which intersected at sharp angles like models of crystals. He was thrown against the edge of one of them with terrible speed.

... Prokop sobbed with fear. This was Einstein's universe and he must get there before it was too late!<sup>15</sup>

Failures of the old absolute references for morality and survival are represented in the novel by the new physics. Towards the end of the

Alan J. Friedman, Carol C. Donley: Einstein as myth and muse Cambridge University Press 1985

From use of the profoundly wrong aphorism "everything is relative", to intricate... the possibilities of Einsteins...

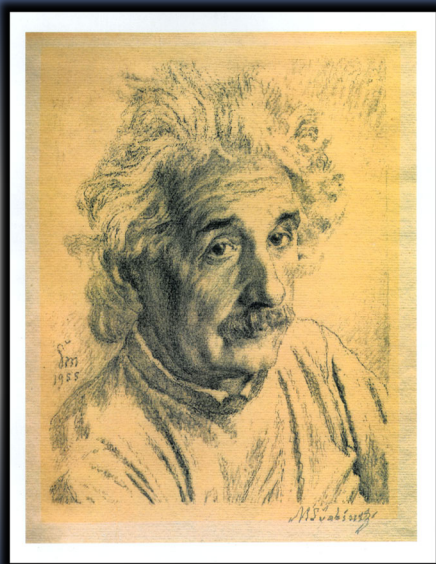
A remarkable early exposition of the possibilities appeared in 1924, with Karel Čapek's novel, **Krakatit**

Einstein, Rutherford, Planck, Borh and Millikan  
The technical details are as accurate as they could be in the early 1920's, and atomic energy is correctly seen as as a possibility emerging from the radioactivity work of Becquerel and Rutherford, and not from Einstein's theories

It appeared that he was moving with the minimum velocity of light; in some way his heart was compressed... Fitzgerald – Lorentz contraction Einstein's universe



## Max Šabinský (1873-1962)





## Impact on Czech Physics & Astronomy

- A. Dittrich, A. Žáček — popular texts on special relativity 1912 (in Czech)
- K. Nachtikal: Principle of Relativity 1922 (in Czech)
- F. Závíška: Einstein's principle of relativity and theory of gravity 1925
- F. Link – astronomer – in Comptus Rendus in March 1936  
on gravitational lenses (AE in Science December 4, 1936)
- V. Hlavatý (Louny 1894 - Bloomington 1969) – differential geometry, unified theory
- M. Brdička (1912-2007): “On gravitational waves”, Proc. Roy. Irish Acad (1951)  
see G. Gibbons, C. Rugina: Coryacher-Chaplygin, Kovalevskaya, and Brdička-Eardley-Nappi-Witten pp-waves spacetimes with higher rank Stäckel-Killing tensor, J. Math. Phys **52**, 122901 (2011)
- K. Kuchař, Canonical gravity  $\rightsquigarrow$  1968  $\rightsquigarrow$  Princeton, SLC
- Today:
  - Faculty of Mathematics and Physics (Theoretical Physics, Astronomy)
  - Academy of Science (Institute of Astronomy, Mathematical Institute)
  - Silesian University
  - Masaryk University



## Lectures and seminars of Albert Einstein in Prague

	Title	No. of students
20.4.1911 – end of July 1911	Mechanik diskreter Massenpunkte (3h)	13
	Thermodynamik (2h)	12
	Seminar	6
19.10.1911 – Wednesday before Palm Sunday 1912	Mechanik diskreter Massenpunkte (3h)	12
	Wärmelehre (2h)	13
	Seminar	7
12.4.1912 – end of July 1912	Mechanik der Kontinua (2h)	10
	Molekulartheorie der Wärme (3h)	11
	Seminar	7



## Prague's work of Albert Einstein

### On thermodynamics, radiation theory and quantum theory

- 1 Elementare Betrachtungen über die thermische Molekularbewegung in festen Körpern  
Ann. d. Phys. **35** (1911), 679-694.
- 2 État actuel du problème des chaleurs spécifiques  
Insituts Solvay, Conseil de Physique, Rapport et discussions de la Réunion Solvay, 1911; Paris, Gauthiers 1912, 407-435.  
Zum gegenwärtigen Stande des Problems der spezifischen Wärme, Deutsche Bunsengesellschaft, Abhandlungen, Nr. **7** (1914), 330-364.
- 3 Thermodynamische Begründung des photochemischen Äquivalentgesetzes,  
Ann. d. Phys. **37** (1912), 832-838.
- 4 Nachtrag zu meiner Arbeit: "Thermodynamische Begründung des photochemischen Äquivalentgesetzes",  
Ann. d. Phys. **38** (1912), 881-884.
- 5 Antwort auf eine Bemerkung von J. Stark: "Über eine Anwendung des Planckschen Aelementargesetzes",  
Ann. d. Phys. **38** (1912), 888.





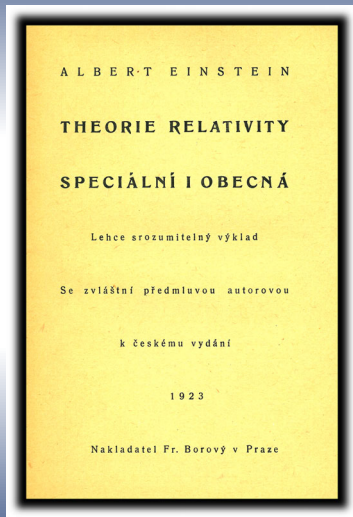
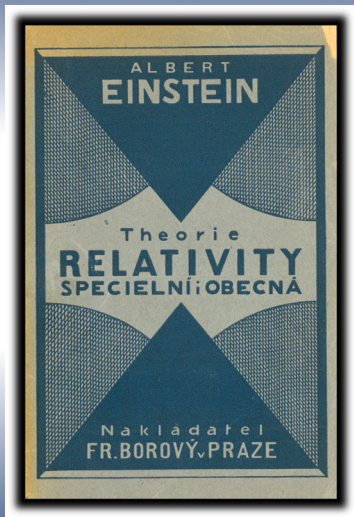
## Prague's work of Albert Einstein

### On the theory of relativity and gravitation

- 1 Die Relativitäts-Theorie,  
Vierteljahr. d. Naturf. Ges. Zürich **56** 1911, 1-14.
- 2 Zum Ehrenfest'schen Paradoxon,  
Phys. Z. **12** (1911), 509-510.
- 3 **Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes,**  
**Ann. d. Phys. 35 (1911), 898-908.**
- 4 Lichtgeschwindigkeit und Statik des Gravitationsfeldes,  
Ann. d. Phys. **38** (1912), 355-369.
- 5 **Zur theorie des statischen Gravitationsfeldes,**  
**Ann. d. Phys. 38 (1912), 443-458.**
- 6 **Gibt es eine Gravitationswirkung, die der elektrodynamischen  
Induktionswirkung analog ist?**  
Vierteljahrsschrift für gerichtliche Medizin **44** (1912), 37-40.
- 7 **Relativität und Gravitation: Erwiderung auf eine Bemerkung von  
M. Abraham,**  
**Ann. d. Phys. 38 (1912), 1059-1064.**



## About the special and general theory of relativity in plain terms (Czech translation)





## About the special and general theory of relativity in plain terms (Czech translation)

VORWORT  
DES AUTORS ZUR TSCHECHISCHEN AUSGABE.

Es freut mich, daß das kleine Büchlein, in dem die Hauptgedanken der Relativitätstheorie ohne die mathematische Durchführung dargestellt sind, nun in der Nationalsprache desjenigen Landes erscheint, in dem ich die nötige Sammlung fand, um dem schon seit 1908 gefaßten Grundgedanken der allgemeinen Relativitätstheorie allmählich eine bestimmte Form zu geben. In den stillen Räumen des Theoretisch-Physikalischen Instituts der Prager Deutschen Universität in der Viničná ulice kam ich 1911 auf die Entdeckung, daß das Äquivalenzprinzip eine Ablenkung der Lichtstrahlen an der Sonne von beobachtbarem Betrage verlangt, ohne zu wissen, daß mehr als hundert Jahre vorher eine ähnliche Konsequenz aus der Newtonschen Mechanik in Verbindung mit Newtons Emissionstheorie des Lichtes gezogen worden war. Auch die immer noch nicht einwandfrei bestätigte Konsequenz von der Rotverschiebung der Spektrallinien entdeckte ich in Prag. Den entscheidenden Gedanken von der Analogie des mit der Theorie verbundenen mathematischen Problems mit der Gaußschen Flächentheorie hatte ich allerdings erst 1912 nach meiner Rückkehr nach Zürich, ohne zunächst Riemanns und Riccis, sowie Levi-Civitas Forschungen zu kennen. Auf diese wurde ich erst durch meinen Freund Großmann in Zürich aufmerksam, als ich ihm das Problem stellte, allgemein kovariante Tensoren aufzusuchen, deren Komponenten nur von Ableitungen der Koeffizienten der quadratischen Fundamentalinvariante abhängen. Heute scheinen sich Leistungen und Leistungs-

7

grenzen der Theorie schon klar übersehen zu lassen. Sie liefert tiefe Erkenntnisse über die physikalische Natur von Raum, Zeit, Masse, Gravitation, aber kein hinreichendes Mittel zur Lösung des Problems der Quanten und der atomistischen Konstitution der elektrischen Elementargebilde, aus denen die Materie besteht.

A. Einstein.

8



## About the special and general theory of relativity in plain terms (Czech translation)

Prague stage of his journey to the general theory of relativity? First of all, there is a precious document — [Einstein's foreword to the Czech edition of 1923](#) of his famous little popular book “[About the Special and General Theory of Relativity in Plain Terms](#)” (see the opposite page for the original German text) <sup>1</sup> *“I am pleased that this small book, in which the main ideas of the theory of relativity are explained without mathematical elaboration, should now appear in the native language of the country in which I found the necessary concentration for developing the basic idea of the general theory of relativity which I had already conceived in 1908. In the quiet rooms of the Institute of Theoretical Physics of Prague's German University in Viničná Street, I discovered that the principle of equivalence implies the deflection of light rays near the Sun by an observable amount, without at that time knowing that a similar result had been derived from Newton's mechanics and his corpuscular theory of light. In Prague I also discovered the shift of spectral lines towards the red which is not yet completely confirmed. However, the decisive idea of the analogy between the mathematical formulation of the theory and the Gaussian theory of surfaces came to me only in 1912 after my return to Zürich, without being aware at*

*that time of the work of Riemann, Ricci, and Levi-Civita. This was first brought to my attention by my friend Grossmann when I posed to him the problem of looking for generally covariant tensors whose components depend only on derivatives of the coefficients of the quadratic fundamental invariant. It now appears that it is already possible to evaluate the achievements and limitations of the whole theory. It gives a deep knowledge of the physical nature of space, time, matter and gravity; however, it does not provide sufficient means for solving the problems of quanta and of the atomic constitution of the elementary electric units of which matter is composed.”*<sup>27</sup>)

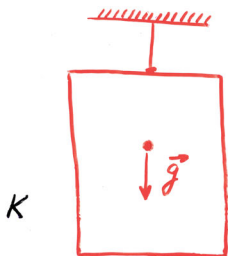
Einstein's foreword to the Czech edition of 1923 of his famous little popular book “**About the Special and General Theory of Relativity in Plain Terms**”



## The principle of equivalence

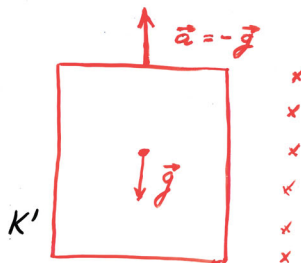
EINSTEIN :

Gravitation is universal



K

at rest in homogeneous  
gravitational field



K'

in uniformly accelerated  
motion in "empty space"  
(in inertial frame)

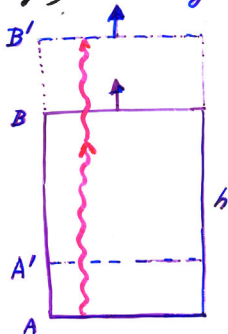
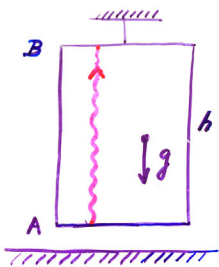


## Redshift

Red shift

(1911, Prague)

$$|\vec{a}| = |\vec{g}|$$



$$\frac{\Delta\nu}{\nu} = \frac{\Delta\Phi}{c^2}$$

$$\frac{\Delta\nu}{\nu} = \frac{v}{c} = \frac{gt}{c} \approx \frac{gh}{c^2} = \frac{\Delta\Phi}{c^2}$$



## A. Einstein: Creator & Rebel (Banesh Hoffmann)

In the paper of 1907... Einstein had already begun his attack on the problem of acceleration, and he returned to it in his Prague paper of 1911. His arguments, particularly in its 1911 form, must rank as one of the most remarkable in the history of science.

A. E. 1911 [Über den Einfluss der Schwerkraft... auf  $C^M$ ]

So lange wir uns auf rein mechanische Vorgänge aus dem Gültigkeitsbereich von Newtons Mechanik beschränken, sind wir der Gleichwertigkeit der Systeme  $K$  und  $K'$  sicher. Unsere Auffassung wird jedoch nur dann tiefere Bedeutung haben, wenn die Systeme  $K$  und  $K'$  in bezug auf alle physikalischen Vorgänge gleichwertig sind, d. h. wenn die Naturgesetze in bezug auf  $K$  mit denen in bezug auf  $K'$  vollkommen übereinstimmen. Indem wir dies annehmen, erhalten wir ein Prinzip, das, falls es wirklich zutrifft, eine große heuristische Bedeutung besitzt.



## Mark Twain

Mark Twain (Life on Mississippi):

"The nice thing about Science is that one gets such wholesale returns of conjecture from such a trifling investment of fact"





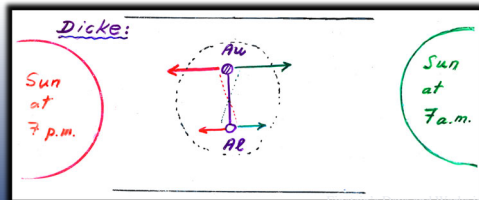
## The principle of equivalence – present day formulation

- test bodies fall with the same acceleration independently of their structure or composition (WEP)
- the outcome of any local non-gravitational experiment is independent of:
  - the velocity of the local inertial frame in which it is performed (LLI)
  - of where and when in the universe is performed (LPI)

Cliff Will, Living Reviews

### The present best limits on

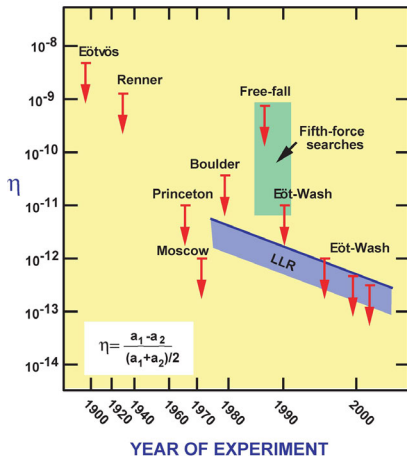
$$\eta = \frac{a_A - a_B}{\frac{1}{2}(a_A + a_B)} \quad \left\{ \begin{array}{l} (0.3 \pm 1.8) \times 10^{-13}, \text{ Eöt-Wash} \\ (-1.0 \pm 1.4) \times 10^{-13}, \text{ LLR} \end{array} \right. \quad (1)$$





## Test of weak equivalence principle

### TESTS OF THE WEAK EQUIVALENCE PRINCIPLE



Test of weak equivalence principle  
C. Will, Living reviews in general relativity (2006)



# Satellite Test of Equivalence Principle

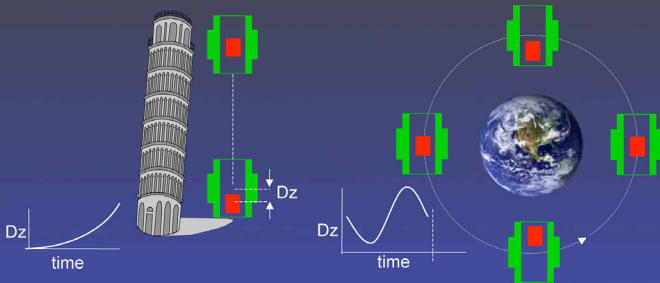


## Satellite Test of the Equivalence Principle -- STEP

Newton's Mystery

$$\left\{ \begin{array}{l} F = ma \\ F = GMm/r^2 \end{array} \right.$$

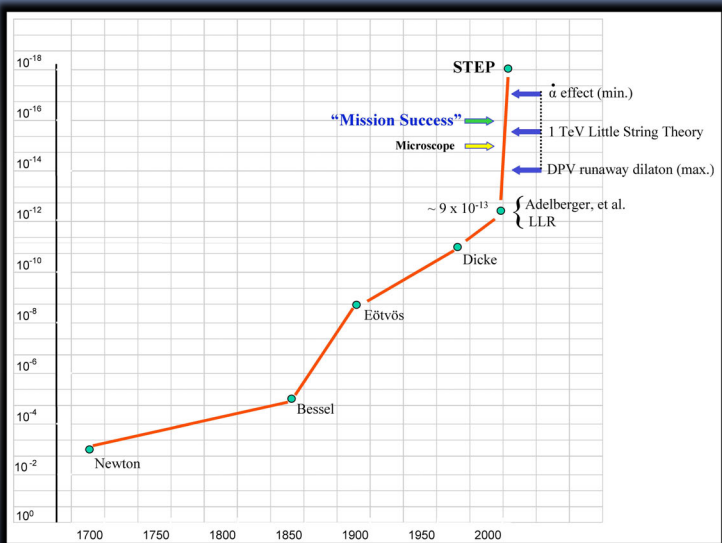
mass - the receptacle of inertia  
 mass - the source of gravitation



Orbiting drop tower experiment  $\left\{ \begin{array}{l} * \text{ More time for separation to build} \\ * \text{ Periodic signal} \end{array} \right.$



## Space > 5 Orders of Magnitude Leap (Everitt)





## Bending of light

4. Über den Einfluß  
der Schwerkraft auf die Ausbreitung des Lichtes;  
von A. Einstein.

Die Frage, ob die Ausbreitung des Lichtes durch die Schwere beinflußt wird, habe ich schon an einer vor 3 Jahren erschienenen Abhandlung zu beantworten gesucht.<sup>1)</sup> Ich komme

Prag, Juni 1911.

(Eingegangen 21. Juni 1911.)

908 A. Einstein. Einfluß der Schwerkraft usw.

Nach Gleichung (4) erleidet ein an einem Himmelskörper vorbeigehender Lichtstrahl eine Ablenkung nach der Seite sinkenden Gravitationspotentials, also nach der dem Himmelskörper zugewandten Seite von der Größe

$$\alpha = \frac{1}{c^2} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{kM}{r^2} \cos \vartheta \cdot ds = \frac{2kM}{c^2 \Delta},$$

wobei  $k$  die Gravitationskonstante,  $M$  die Masse des Himmelskörpers,  $\Delta$  den Abstand des Lichtstrahles vom Mittelpunkt des Himmelskörpers bedeutet. Ein an der Sonne vorbeigehender Lichtstrahl erleidet demnach eine Ablenkung vom Betrage  $4 \cdot 10^{-6}$

- [11] = 0,83 Bogensekunden. Um diesen Betrag erscheint die Winkeldistanz des Sternes vom Sonnenmittelpunkt durch die Krümmung des Strahles vergrößert. Da die Fixsterne der der Sonne zugewandten Himmelspartien bei totalen Sonnenfinsternissen sichtbar werden, ist diese Konsequenz der Theorie mit der Erfahrung vergleichbar. Beim Planeten Jupiter erreicht die zu erwartende Verschiebung etwa  $\frac{1}{100}$  des angegebenen Betrages. Es wäre dringend zu wünschen, daß sich Astronomen der hier aufgerollten Frage annähmen, auch wenn die im vorigen gegebenen Überlegungen ungenügend fundiert oder gar abenteuerlich erscheinen sollten. Denn abgesehen von jeder Theorie muß man sich fragen, ob mit den heutigen Mitteln ein Einfluß der Gravitationsfelder auf die Ausbreitung des Lichtes sich konstatieren läßt.

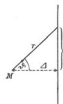


Fig. 3.

Prag, Juni 1911.

(Eingegangen 21. Juni 1911.)



**Max Pechstein:**  
**Bildnis des Erwin Finlay Freundlich, 1919**





## Bending of light

Schwarzschild Metric

(Communicated Jan. 13, 1916)

$$(S) \quad ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 \underbrace{(d\theta^2 + \sin^2\theta d\varphi^2)}_{= d\Omega^2}$$

Indep. J. Droste (student of H.A. Lorentz), May 1916

Impact on experimental relativityEddington, Robertson, ... Will PPN

parametrized post-Newtonian

→ dimensionless parameters ↔ experiment

The simplest generalization of (S) (no dropping) address pref. frames

$$ds^2 = -\left[1 - \frac{2M}{r} + 2\beta \frac{M^2}{r^2}\right] dt^2 + \left(1 + 2\gamma \frac{M}{r}\right) dr^2$$

(in general 10 PPN parameters)

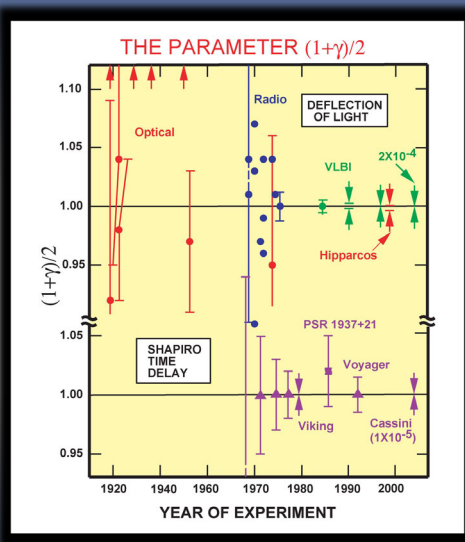
GR:  $\beta = \gamma = 1$  Advance of the pericentre

$$\Delta\psi = \frac{2}{3}(2\gamma - \beta) \frac{6\pi M}{a(1-e^2)}$$

Total deflection of waves

$$\Delta\Phi = \frac{2(1+\gamma)M}{r_0}$$



The parameter  $\gamma$ 

A 2004 analysis of  $\approx 2$  million VLBI observations of 541 radio sources at 87 VLBI sites:

$$\gamma - 1 = (-1.7 \pm 4.5) \times 10^{-4}$$

Scalar-tensor theories must have

$$\omega > 40000$$

to be compatible

$$\gamma = \frac{1 + \omega}{2 + \omega}$$





*Physics Today, March 2009*

feature  
article

# Testing relativity from the 1919 eclipse— a question of bias

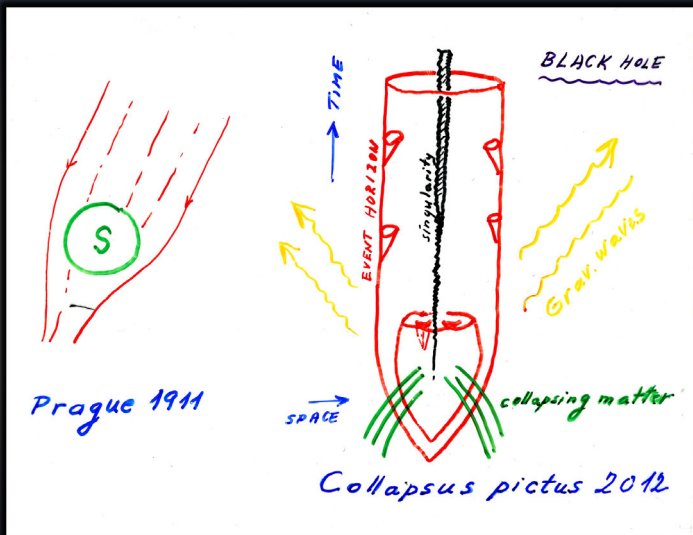
Daniel Kennefick

When interpreting experimental results, context is everything. The researchers who took and analyzed the most important eclipse data had good reasons for judging the experiment a victory for Albert Einstein.

Daniel Kennefick is an assistant professor of physics at the University of Arkansas at Fayetteville.

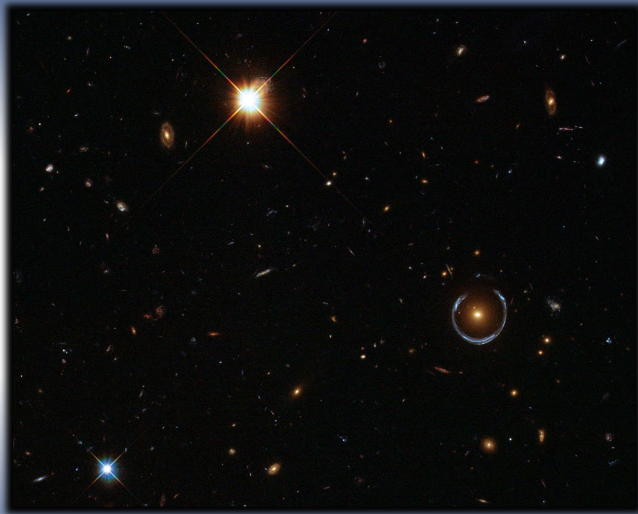


## Gravitational collapse





## Einstein's ring



ESA / Hubble & NASA



## Collision of 2 galaxy clusters (1E0657-558)

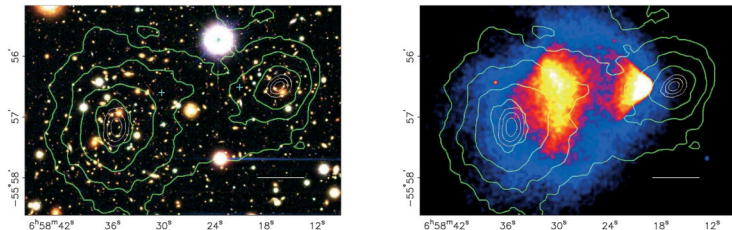


Fig. 1.—*Left panel:* Color image from the Magellan images of the merging cluster 1E 0657–558, with the white bar indicating 200 kpc at the distance of the cluster. *Right panel:* 500 ks *Chandra* image of the cluster. Shown in green contours are the weak-lensing  $\kappa$  reconstructions, with the outer contour levels at  $\kappa = 0.16$  and increasing in steps of 0.07. The white contours show the errors on the positions of the  $\kappa$  peaks and correspond to 68.3%, 95.5%, and 99.7% confidence levels. The blue plus signs show the locations of the centers used to measure the masses of the plasma clouds in Table 2.

Cowley et al (2006)



## On the history of gravitational lensing

- Einstein's Scratch Notebook 1912
- Notes by Eddington 1920,  
Chwolson Astron. Nacht. 1924
- Einstein, Science, December 4 1936  
“Lens-like Action of a Star by the Deviation of Light in the Graviational Field” – interaction with a Czech amateur scientist Rudi Mandl  
J. Renn, T. Sauer, J. Stachel: On the Origin of Gravitational Lensing:  
A Postscript to Einstein's 1936 Science Paper, Science 1997  
  
Detailed account in  
J. Renn, T. Sauer: “Eclipses of the Stars – Mandl, Einstein, and the Early History of Gravitational Lensing” in “Revising the Foundations of Relativistic Physics – J. Stachel's Festschrift” Ed. A. Ashtekar *et al*, Kluwer (2003)



## František Link

ASTROPHYSIQUE. — *Sur les conséquences photométriques de la déviation d'Einstein.* Note de M. F. LINK, présentée par M. Charles Fabry.

Dès le début de la théorie de la relativité on a cherché de vérifier la déviation des rayons lumineux passant normalement au champ de gravitation d'un corps céleste. La déviation  $\omega$  est

constant même très près de l'étoile occultée et à peine supérieur à sa valeur normale. Lorsque la distance apparente s'approche de la valeur

$$\rho_{\infty} \doteq \sqrt{Kk\alpha_1}.$$

L'intensité commence à augmenter pour atteindre à cette distance la valeur infinie, si l'étoile occultée était rigoureusement ponctuelle. Ce cas

$\rho_{\infty} = \sqrt{Kk\alpha_1}$  corresponds to angular size of Einstein ring  $\Theta_E$

**Link F.: Comptus Rendus 202 (16 Mar 1936), 917-919**



## Sur les conséquences photométriques de la déviation d'Einstein

### SUR LES CONSÉQUENCES PHOTOMÉTRIQUES DE LA DÉVIATION D'EINSTEIN.

Par M. F. LINK.

SOMMAIRE. — La déviation d'Einstein des rayons lumineux dans le champ de gravitation a pour conséquence une modification

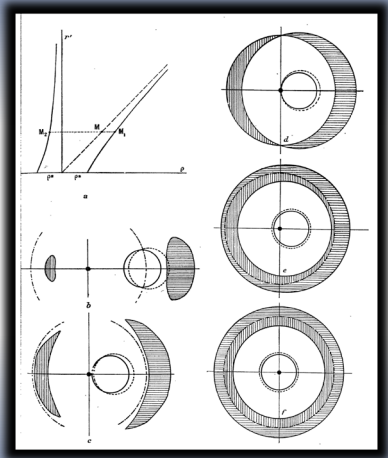
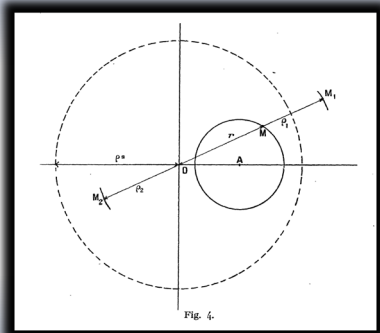
il le frapperait en  $dS$ . Le rapport  $\frac{dS'}{dS}$  donne la modification de l'éclairement en N. On l'obtient d'une façon analogue que nous avons traité l'affaiblissement par la réfraction dans les éclipses de Lune [5]:

$$(5) \quad \frac{i}{j} = \frac{dS'}{dS} = s = \left[ 1 - \frac{\omega}{\alpha_1 + \alpha_2} \frac{a}{R'_0} \right] \left[ 1 - \frac{d\omega}{dR'_0} \frac{a}{\alpha_1 + \alpha_2} \right].$$

Link, F.: Bulletin Astronomique **10** (1937), 73-90



# František Link







## Prague works on gravitation 1912

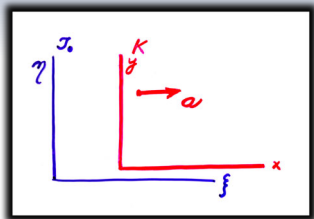
- **“The speed of light and the statics of the Gravitational Field”**  
(received 25 Feb)
- On the Theory of the Static Gravitational Field and “Note added in Proof”  
(reviewed 23 March)
- Is There a Gravitational Effect which is Analogous to Electrodynamical Induction (published July)
- **Relativity and Gravitation.** Reply to a Comment by M. Abraham  
(received 4 July)



## Prague works on gravitation 1912

$$\mathcal{I}(\tau, \xi, \eta, \zeta)$$

$$\mathcal{K}(t, x, y, z)$$



assume  $\xi = \lambda(x) + \alpha(x)t^2 + \mathcal{O}(t^3)$   
 $\tau = \beta(x) + \gamma(x)t + \delta(x)t^2 + \mathcal{O}(t^3)$

$$\eta = y, \quad \zeta = z \quad \text{At } t = 0, \xi = 0, x = 0$$

$$ds_{\mathcal{I}}^2 = -c_*^2 d\tau^2 + d\xi^2 + d\eta^2 + d\zeta^2, \quad c_* = 1$$

$$ds_{\mathcal{K}}^2 = -c^2(x) dt^2 + dx^2 + dy^2 + dz^2$$

from

$$ds_{\mathcal{I}}^2 = 0 \Leftrightarrow ds_{\mathcal{K}}^2 = 0 \Rightarrow \xi = x + \frac{1}{2}act^2, \quad \tau = ct, \quad c = c_0 + ax$$



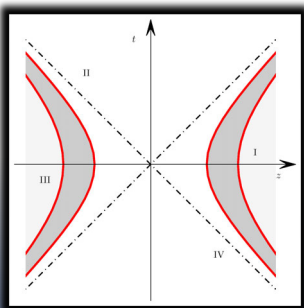
## Today

Today transformation to the rigid uniformly accelerated (“Rindler”) frame

$$\xi = \frac{1}{a} (\cosh at - 1) + x \cosh at \quad (2)$$

$$\tau = \frac{1}{a} \sinh at + x \sinh at \quad (3)$$

for small  $t$  (i.e. neglecting  $\mathcal{O}(t^3)$ )  $\Rightarrow$  Einstein's Prague transformation



boost – rotation symmetric spacetimes

- radiative
- plausible Newtonian limit
- asymptotically flat
- C-metric
- etc.



## Equations for static gravitational field

Equation for static gravitational field

$$\Delta c = 0 \quad (\text{in vacuum}) \quad \Delta c = kc\rho \quad (\text{in matter})$$

contradictions with conservation of energy and momentum ( $\int \vec{f} dV \neq 0$ ,  
where  $\vec{f} = -\rho \text{ grad } c$ )

⇒ modification of the field equation

$$\Delta c = k \left[ c\rho + \frac{1}{2k} \frac{\text{grad}^2 c}{c} \right]$$

⇒ **nonlinear field equation**

“local view” on the equivalence principle

$$\xi = x + \frac{1}{2}c \frac{dc}{dx} t^2, \quad c(x) \text{ arbitrary}$$

**see D. Giulini's talk this afternoon**





# Gravity Probe B

146 C. W. F. EVERITT, W. M. FAIRBANK, AND W. O. HAMILTON

Figure 1

**STANFORD GYRO EXPERIMENT**

Idea: 1959      Launch: April 20, 2007

First results: April 14, 2007 (APS, Jacksonville)

Final results: End of 2007 **NO!**

\*) **GRAVITY PROBE B** (NASA)

\$35 mil.

↓

\$700 mil.

**Frame-dragging Effect**  
0.041 arcseconds/year  
(0.000011 degrees/year)

**Geodetic Effect**  
4.6 arcseconds/year  
(0.0018 degrees/year)

Guide Star HR 8799 (HR 8799)

4.6 arcseconds/year (100 miles)

- Basic formula: **Leonard Schiff**

$$\dot{\vec{I}} = \frac{3GM}{2c^2 R^3} (\vec{R} \times \vec{v}) + \frac{GI}{c^2 R^3} \left[ \frac{3\vec{R}}{R^2} (\vec{\omega} \cdot \vec{R}) - \vec{\omega} \right]$$

- Oblateness correction: **\* Dan Wilkins (Phys**



# Effects of rotating gravitational waves

Jiří Bičák, Joseph Katz, Tomáš Ledvinka, and Donald Lynden-Bell

Rev. D **85**(12), 124003 (2012)

$$\psi_{lm} = \frac{B_l 2^l l! N_l^m P_l^m(\cos\theta) \tilde{r}^l}{[((1 + \tilde{r}^2 - \tilde{t}^2)^2 + 4\tilde{t}^2)^{(l+1)/2}] \cos[m\varphi - \lambda(t, r)]}$$

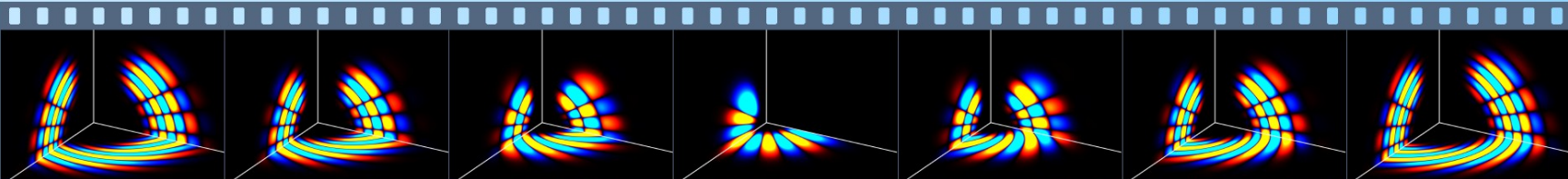
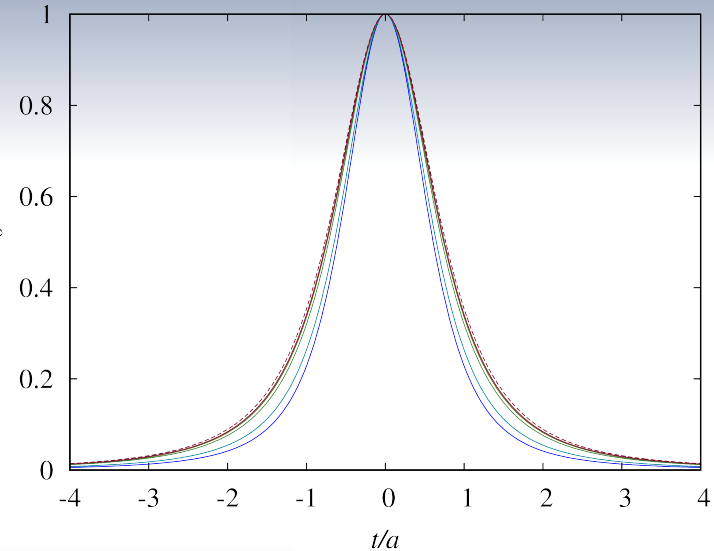
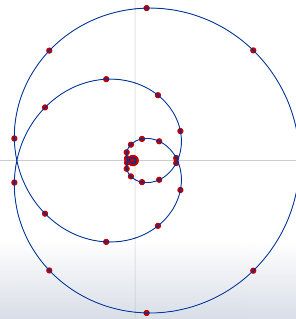
$$\lambda(t, r) = (l + 1) \arctan \frac{2\tilde{t}}{1 + \tilde{r}^2 - \tilde{t}^2} \quad \tilde{r} = \frac{r}{a}, \quad \tilde{t} = \frac{t}{a}$$

$$h_{\mu\nu}^{(i)} = \sum_{lm} \left[ -\frac{\sqrt{2l(l+1)}}{r} h_{0lm}^{(i)}(t, r) c_{0lm\mu\nu} + i \frac{\sqrt{2l(l+1)}}{r} h_{1lm}^{(i)}(t, r) c_{lm\mu\nu} \right] \hat{\omega}^{\mu\nu}$$

$$G_{\mu\nu}^{(1)}[h^{(2)}] = -G_{\mu\nu}^{(2)}[h^{(1)}, h^{(1)}]$$

$$g_{t\varphi}^{(2)} = -\sqrt{\frac{3}{4\pi}} h_0^{(2)}(t, r) \sin^2\theta = -\omega_0 r^2 \sin^2\theta$$

$$\omega_0 = \frac{1}{4\pi} \int_0^\infty \int R_{t\varphi}^{(2)}[h^{(1)}, h^{(1)}] d\Omega \frac{dr}{r}$$





## Relativität und Gravitation. Erwiderung...

### 12. *Relativität und Gravitation.*

*Erwiderung  
auf eine Bemerkung von M. Abraham;  
von A. Einstein.*

In einer in diesen Annalen erscheinenden Notiz hat M. Abraham auf einige von mir geäußerte kritische Bedenken zu seinen Untersuchungen über Gravitation geantwortet, sowie

Allerdings scheint der Weg zu diesem Ziele ein recht schwieriger zu sein. Man sieht schon aus dem bisher behandelten, höchst speziellen Falle der Gravitation ruhender Massen, daß die Raum-Zeit-Koordinaten ihre einfache physikalische Deutung einbüßen werden, und es ist noch nicht abzusehen, welche Form die allgemeinen raumzeitlichen Transformationsgleichungen haben könnten. Ich möchte alle Fachgenossen bitten, sich an diesem wichtigen Problem zu versuchen.

ist beispielsweise die Energiedichte im statischen Schwerfeld  $\frac{c^2}{\gamma} \text{grad}^2 c$ , nach meiner Theorie  $\frac{1}{2k} \frac{\text{grad}^2 c}{c}$ . Das Eingehen von  $c$  ist in beiden Theorien verschieden.

1) M. Abraham, Physik. Zeitschr. 13. Nr. 19. p. 2. 1912.

(Eingegangen 4. Juli 1912.)





## Prague, summer 1912

Prague, summer 1912

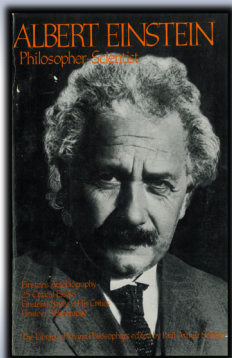
- (i) local significance of equivalence principle
  - (ii) eqs. of motion of mass points (variational principle); eqs. of elmag. field when gravity is present
  - (iii) nonlinear field eq. for gravity (energy density of gr. field itself as source)
  - (iv) all equations must be form invariant with respect to a larger group than Lorentz group
  - (v) "spacetime coordinates lose their simple physical meaning" ||
- (but) gravitation described wholly by one function - (variable) velocity of light and static

- 1 local significance of equivalence principle
- 2 eqs. of motion for mass points (variational principle), eqs. of elmag. field when gravity is present
- 3 nonlinear field eq. for gravity (energy density of gr. field itself as source)
- 4 all equations must be form invariant with respect to a larger group than Lorentz group
- 5 spacetime coordinates lose their simple physical meaning

(but) gravitation described wholly by one function — (variable) speed of light — and static (stationary)



## Philosopher – Scientist



### AUTOBIOGRAPHICAL NOTES

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This happened in 1908. Why were another seven years required for the construction of the general theory of relativity? The main reason lies in the fact that it is not so easy to free oneself from the idea that co-ordinates must have an immediate metrical meaning. The transformation took place in approximately the following fashion.

This happened in 1908. Why were another seven years required for the construction of the general theory of relativity? The main reason lies in the fact that it is not so easy to free oneself from the idea that co-ordinates must have an immediate metrical meaning. The transformation took place in approximately the following fashion.



## Zurich, winter 1912

Zürich - winter 1912

(with H. Grossmann)

gravitation described by 10 functions

 $g_{\mu\nu}$ 

(i) eq. of geodesics

$$\delta \int \sqrt{-g_{\mu\nu} dx^\mu dx^\nu} = 0$$

(ii) Maxwell eqs. in gr. field  
invariant under  $x^\mu \rightarrow x'^\mu(x^\nu)$ (iii) The source of gravity  $T^{\mu\nu}$   
 $T^{\mu\nu};_{\nu} = 0$ 

(iv) Field equations

$$\Gamma_{\mu\nu} = \kappa T_{\mu\nu}$$

$$R_{\mu\nu} \rightarrow \Delta\Phi = 4\pi G\rho$$

→ error with "covariance"

cooperation with M. Grossmann  
gravitation described by 10 functions

 $g_{\mu\nu}$ 

1 eq. of geodesics

$$\delta \int \sqrt{-g_{\mu\nu} dx^\mu dx^\nu} = 0$$

2 Maxwell eqs. in gr. field invariant  
under  $x^\mu \Rightarrow x'^\mu(x^\nu)$ 3 the source of gravity  $T^{\mu\nu}$ 

$$T^{\mu\nu};_{\nu} = 0$$

4 field equations

$$\Gamma_{\mu\nu} = \kappa T_{\mu\nu}$$



## Einstein Field Equations

25. 11. 1915, Berlin

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$$

Einstein's field equations

“Belated Decision in the Hilbert-Einstein Priority Dispute”, L. Corry,  
J. Renn, ... Science **278**, 14 Nov 1997



20

Sobota 15. května 2004 LIDOVÉ NOVINY

### Zajímavosti týdne

# Einstein očima Johanny z Čech

V těchto dnech by Princetonská univerzita měla začít plnit to, co před měsícem sblíbla: odstartovat sérii článků o vztahu mezi Albertem Einsteinem a Johannou Fantovou. Jedním z podkladů jsou i nedávno objevené zápisky, které si fyzikova přítelkyně vedla v posledních dvou letech jeho života.

Marie Homolová

**T**y zápisky se natly šťastnou náhodou. Princetonská univerzita se rozhodla připravit seriál o slavných párech, které se vyskytovaly v její historii. A protože se vědělo, že Einstein, který za amerického pobytu v Princetonu působil, a Fantová, která tam pracovala v knihovně, měli k sobě blízko, otevřeli autoři seriálu letos v únoru její dokumentační. Kromě jiného tu lze svému překvapení našli i deník, obsahující na 62 stránkách více než dvě stovky zá-

„Chtěla jsem k tomu, jak Einsteina chápe, přihat nový pohled.“ napsala Fantová v úvodu. „Nevzdalyho jako žijící legendu ani jako věhlasného vědce, ale jako člověka.“

#### Z Prahy do Princetonu

Johanna Fantová se narodila roku 1901 v severních Čechách jako Johanna Bobaschová. Její muž Otto byl systémem manželé Fantových, kteří před první světovou válkou vedli v pražském domě U bílého jednorožce slavný salon, kde se scházeli pražský kulturní svět. Mezi hosty se objevoval i Franz Kafka, Max Brod a... Albert Einstein, který tehdy v letech 1911 a 1912 přednášel v Praze na německé universitě fyziku.

Johanna, které tehdy bylo deset a běhala na zecia jiné adrese, se s Einsteinem sešla až v roce 1929 v Berlíně, kde jí nabídl, aby mu uspořádala jeho rozepsání, ale chaotickou knihovnu. Ačkoli je dělná dvaadvacet let věku, velmi se otrláhlá, a nakonec jí převalil ke své nejmilejší zá- bavi, židě na plachetnici. Když v roce 1939 odešla do Spojených států, znovu se k Einsteimovi, který tam už nábídl, let žil v emigraci, přiblížila.

Einstein už měl za sebou dvě manželství. V roce 1902 se oženil se srbkou studentkou fyziky Milevou, měli spolu dva syny, ale časem se rozvedli. Druhá manželka, sestřenice Elsa, ho doprovázela i do Ameriky, kam emigroval



Albert Einstein a Johanna Fantová spolu v Americe rádi jachtali na Lake Carnegie

Reportáž

týdně spolu dlouze telefonovali, ona mu vřihala vlny, on jí psal básně, obhovili i příjemné chvíle na plá-

„Unavují z dlouhého ticha chci ti tímto jasně ukázat, jak silně myšlenky na tebe buďou vždy sdílet v jedné

že všichni jsme malé Země, ale každý si myslí, že je jejich středem...“ A tak dále.

mera, který vedl americký program vývoje atomové bomby. Když se Oppenheimer, kterého si Einstein velmi považoval, ocitl před McCarthyovým Výborem pro vyšetřování neamerické činnosti, Fantová si zaznamenal, že se to u nich stalo častým příležitým konverzací. „Politická perzekuce jeho přítele byla pro něj horším vystřízlivěním“, zapsala si.

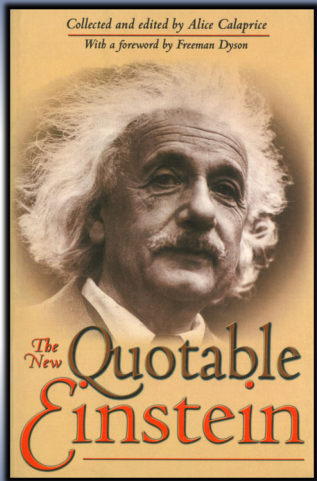
Soucítil s Oppenheimerovým zklamáním, ale o sobě říkal, že se „narodil se sloni kůži“ a nikdo mu nemůže ublížit. „Každá kritika po mně stoe jako voda.“ Přesto si nikdy nepřestal vyčítat, že je svým způsobem odpovědný za tragédie způsobené atomovou bombou. Nejen proto, že k bombě dal jistý podnět svými vědeckými závěry, ale především za druhé světové války napsal prezidentu Rooseveltovi dopis, ve kterém ho varoval, že Němci jsou blízko k sestrojení a je důležité předběhnout je. Ten dopis byl jedním z impulzů, které způsobily, že se americký projekt opravdu rozjel na plné obrátky. Odpověděl na to, co následovalo, „ho velmi stresovala“, zapsala si Fantová.

Zápisy dosvědčují, jak se trápil vývojem světa i tím, že už ho nemůže ovlivnit. Sověťům vrátil mírovou cenu, nechtěl nést nálepku boževníka. A zároveň odmítl varovat před preventivní válkou s Čínou, protože věděl, že by jeho slova padla do prázdna. Žlobil se. Je mezin Einstein „ma-

## An article about Einstein in Lidové noviny



## From: Day-by-Day Summary of Johanna Fantová's Journal



**April 13, 1954** Expresses annoyance at Oppenheimer for letting the McCarthy and Atomic Energy Commission affairs bother him so much. Already told the press that he has great respect for Oppenheimer, both as a human being and as a scientist.

**October 24, 1954** He calculated like crazy again today but accomplished nothing.

The last but one (AE died in the night, April 18, 1955): **April 10, 1955** He tried all day to compose a radio message on behalf of Israel and did not succeed in finishing it. He claims he is totally stupid – that he has always thought so, and that only once in a while was he able to accomplish something.



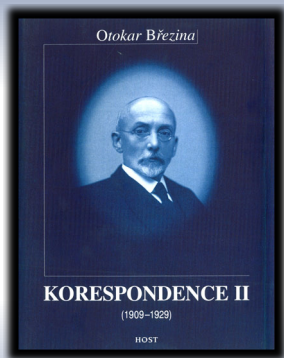
## Einstein's course, J. Florian Publisher



Einstein's course (Eddington,...),  
Stará Říše, September 1926



## Otokar Březina



There will always be minds who, by the united force of knowledge and dreams, science and poetry, will strive for a unified picture of the universal processes, an image that in equal measure corresponds both to the eternal longing of the human mind for harmony and beauty and to the thirst of the heart for justice.

Otokar Březina (in the letter to F. Novotný,  
February 7, 1920)





## Einstein's Days and Works in Prague: Relativity Then and Now

Overture: Biography & Culture

1<sup>st</sup> Movement: Lectures & Papers

2<sup>nd</sup> Movement: Principle of Equivalence  
Light bending

3<sup>th</sup> Movement: Dragging of inertial frames

4<sup>th</sup> Movement: Uniform acceleration & static fields

Finale: Basic features of a theory of gravity

Coda