Palette of gravitoma(ch)gnetic effects

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Dr Ernst Mach

* 18 Feb 1838
in Chrlice (today part of Brno)
Moravia

+ 19 Feb 1916
Vaterstetten *)
Bavaria

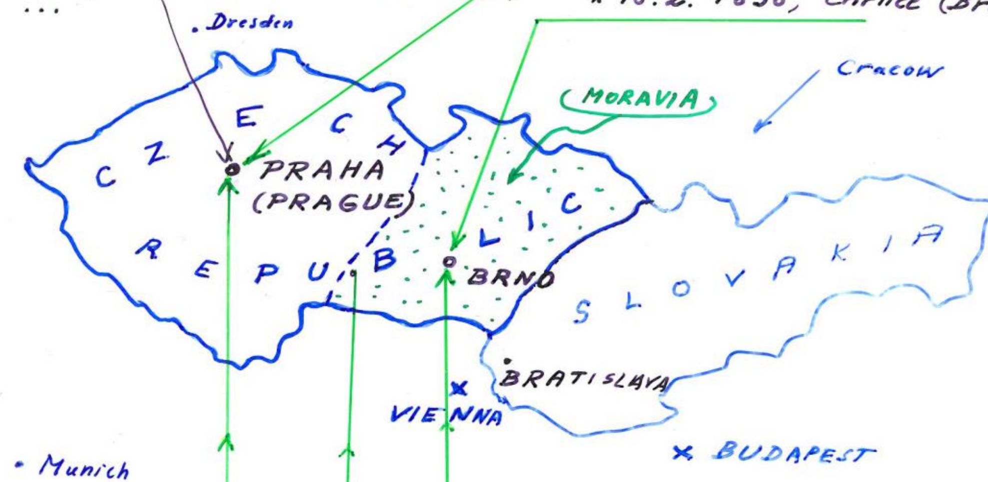
last year 170 annivers.

*(today part of Munich)

"SPIRITUS LOCI"

ERNST MACH
1867 – 1895, KU, Prague
* 18. 2. 1838, Chrlice (Brno)

Tycho Brahe
J. Kepler (1600–), Ch. Doppler (1842)
...



K. Gödel, *1906 Brno
G. Mendel, L. Janáček, ...
A. EINSTEIN, 1911–1912
(Mandl 1936, ...)

B. Bolzano
(G. Meyrink, ...)
O. Březina
F. Záviška, ...

„ The highest philosophy of the scientific investigator is to bear an incomplete conception of the world *) and to prefer it to any apparently complete but inadequate conception *"

E. Mach, Science of Mechanics. p. 560

*) Euclidean barracks in absolute space (kasárna, казарм. Kaserne)

*) „ The investigator must feel the need of... knowledge of the <u>immediate</u> connections... of the masses of the universe. There will hover before him .. an ideal insight into the principles of the whole matter, from which accelerated and inertial motion result in the same way "

E. Mach, S. of Mechanics.

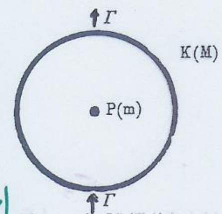
3.

Gibt es eine Gravitationswirkung, die der elektrodynamischen Induktionswirkung analog ist?

Von

Prof. Dr. Einstein-Prag.

Die in der Ueberschrift aufgeworfene Frage kann in Anlehnung an einen übersichtlichen Spezialfall in folgender Weise formuliert werden. Es werde ein System ponderabler Massen betrachtet, bestehend aus der Kugelschale K mit homogen über die Kugelfläche verteilter Masse M und dem im Mittelpunkt dieser Kugelschale angeordneten materiellen Punkt P mit der Masse m. Wirkt auf den festgehaltenen materiellen Punkt P eine Kraft, wenn ich der Schale K eine Beschleunigung Γ erteile? Die folgenden Ueber- [1] legungen werden uns dazu führen, eine solche Kraftwirkung als tatsächlich vorhanden anzusehen und uns die Grösse derselben in erster Annäherung ergeben.



force to hold P:

$$\boxed{\frac{3}{2}\frac{GmM\Gamma}{c^2R}}$$

1. Nach der Relativitäts-Theorie ist die träge Masse eines abgeschlossenen physikalischen Systems von dessen Energieinhalt in solcher Weise abhängig, dass ein Energiezuwachs des Systems um E die träge Masse um $\frac{E}{c_2}$ vergrössert, wenn c die Vakuum-Lichtgeschwin- [2] digkeit bedeutet. Bezeichnet man also mit M die träge Masse von K [3] bei Abwesenheit von P, und mit m die träge Masse von P bei Abwesenheit von K, oder mit anderen Worten mit M + m die träge Masse des aus P und K zusammen bestehenden Systems für den Fall, dass m sich in unendlicher Entfernung von K befindet, so folgt, dass die träge Masse des aus K und m bestehenden Systems, für den Fall, dass sich m im Mittelpunkt von K befindet, den Wert

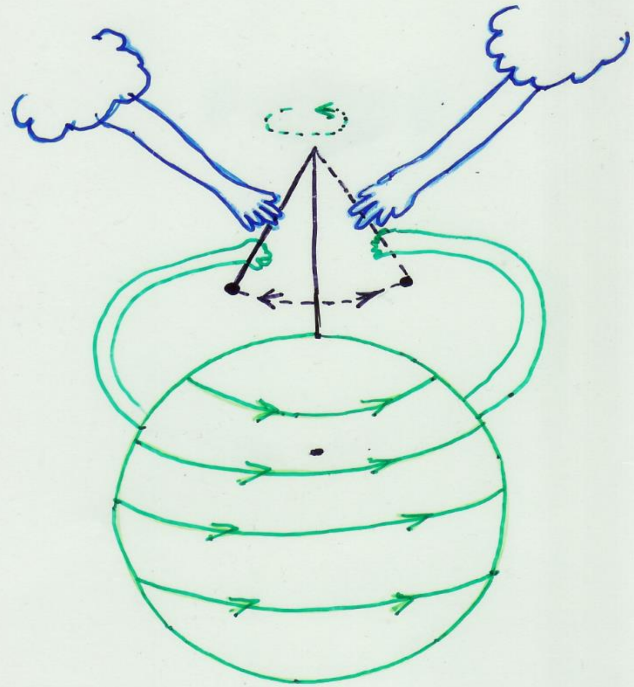
$$M + m - \frac{k\,M\,m}{R c^2} \quad \ldots . (1)$$

7. "Is There a Gravitational Effect Which Is Analogous to Electrodynamic Induction?"

[*Einstein 1912e*]

PUBLISHED July 1912

„ STARRY NIGHT "



E. M.

$$\omega\,(\text{drag}) = 2J_{\oplus}/r_{\oplus}^{3}$$

≈ 221 milliarc"/y

PHYSICS ?

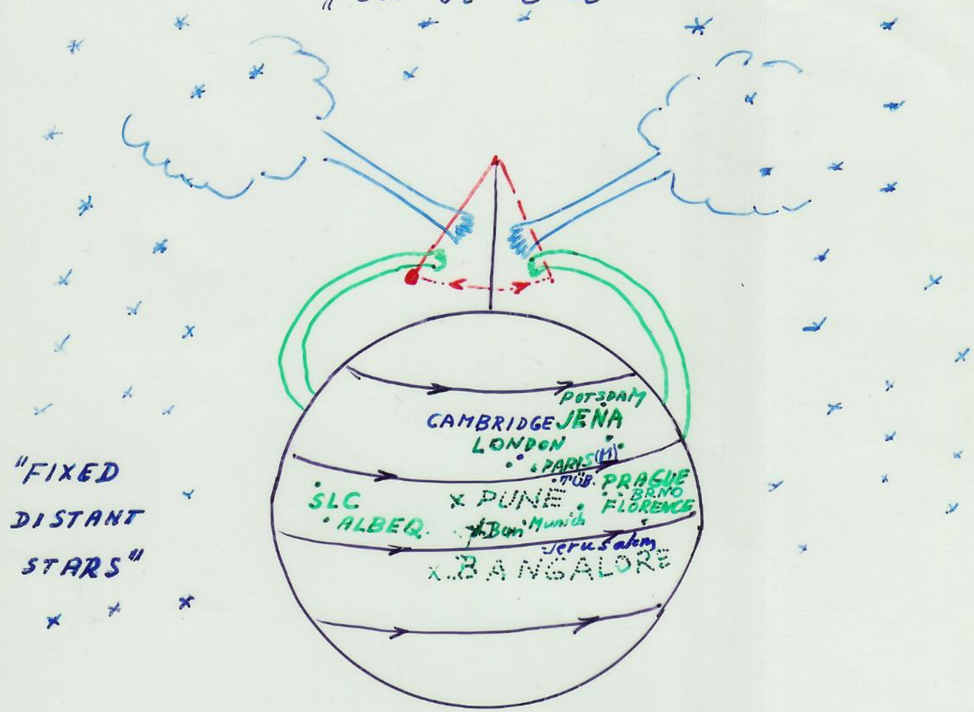
ASTROLOGY ?

Gell-Mann

America's rating jobs:

1. business ... 8. physicists :
2. politicians ... 9. astrologists 20. astronomers
: : mathematicians

„MACH"



GR : $\omega(\text{drag}) \sim 2J_{\oplus}/r_{\oplus}^{3} = 221$ milliarcsec/year

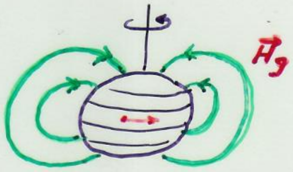
ELECTROMAGNETISM

Rotating charged sphere

⇝ el. current ⇝ magn. field $\vec{B}$

GRAVOMAGNETISM

(in GR)



Rotating massive sphere

⇝ mass-current ⇝

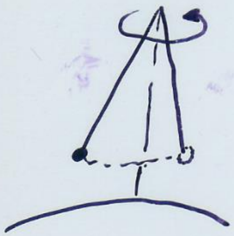
gravomagnetic field $\vec{H}_g$

ch

GR drag - gravitomagnetism - given by

$-4\,\vec{J}\times\vec{r}\,/\,r^3$

angular momentum of Earth



Frame drag at the Pole:

$$\omega = \frac{2J_\oplus}{r_\oplus^3} \ldots = 221 \text{ milliseconds of arc per year}$$

$$\left[\ \omega \sim \frac{GM_\oplus}{c^2R_\oplus}\,\Omega_\oplus = 309 \text{ milliarcsec/yr}\ \right]$$

Braginskij V.B., Polnarev A.G., Thorne K.S.
„Foucault Pendulum at the South Pole: Proposal For an Experiment to Detect the Earth's General Relativistic Gravitomagnetic Field"
Phys. Rev. Lett. 53, 863 (1984)

- Bardeen-Petterson effect - neutronstars, black holes

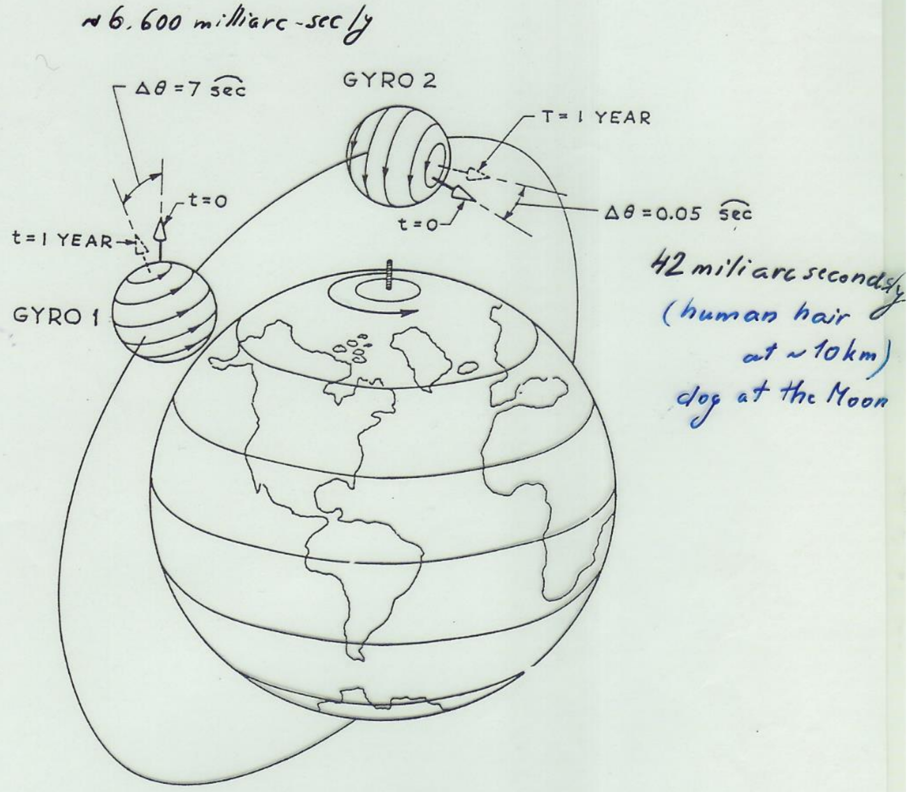


Figure 1

STANFORD GYRO EXPERIMENT *)

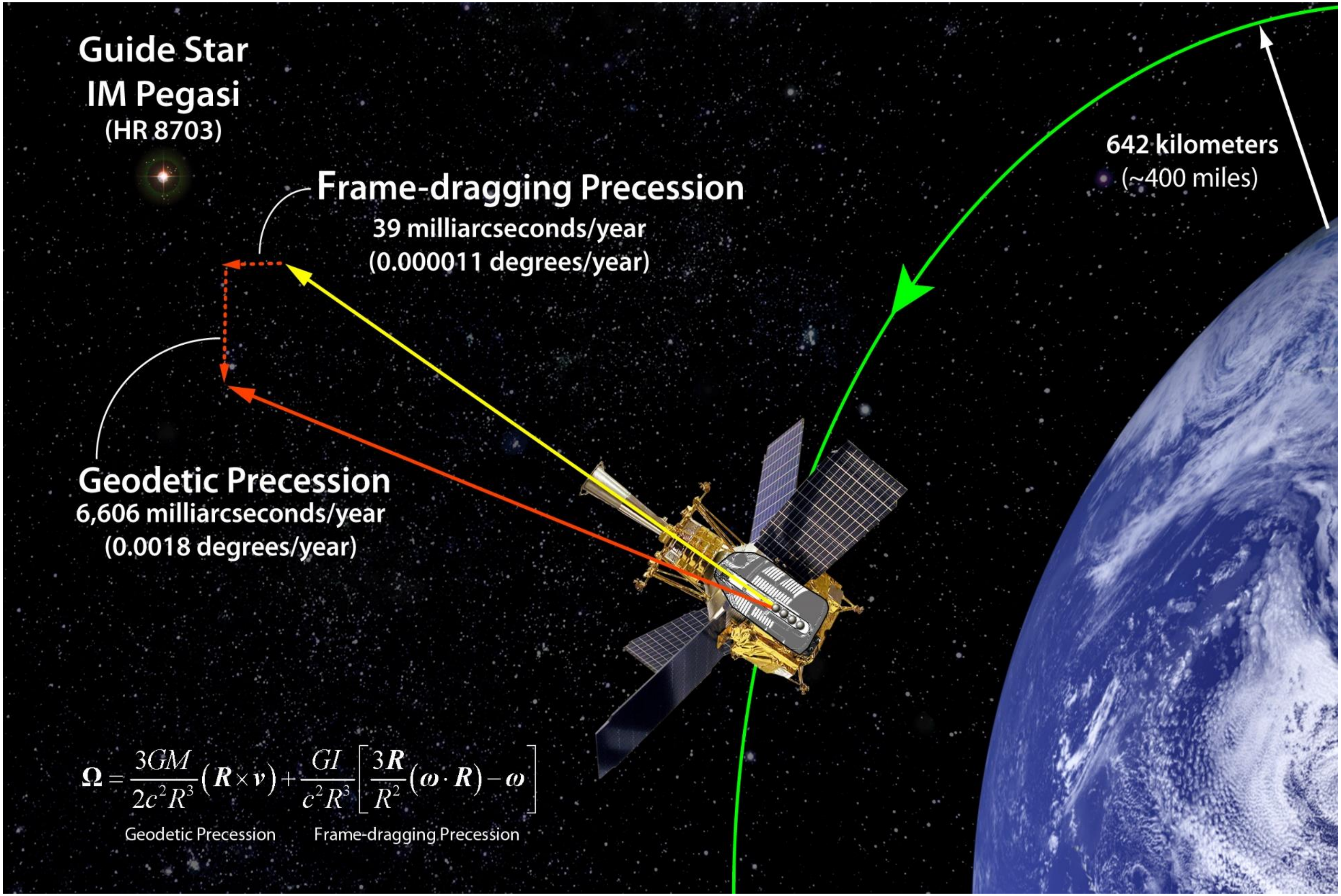
Idea: 1959 Launch: April 20, 2004

First results: April 14, 2007 (APS, Jacksonville)

Final results: End of 2007 NO!

$35 mil. ↓ $700 mil.

*) „GRAVITY PROBE B" (NASA)



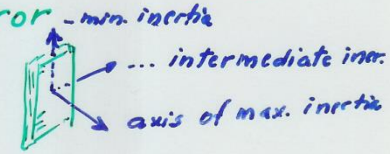
Guide Star
IM Pegasi
(HR 8703)
Frame-dragging Precession
39 milliarcseconds/year
(0.000011 degrees/year)
642 kilometers
(~400 miles)
Geodetic Precession
6,606 milliarcseconds/year
(0.0018 degrees/year)
$$\Omega = \frac{3GM}{2c^2R^3}(R \times v) + \frac{GI}{c^2R^3}\left[\frac{3R}{R^2}(\omega \cdot R) - \omega\right]$$
Geodetic Precession
Frame-dragging Precession

Mission Update - November 12, 2009

The accuracy of GP-B results has improved 17 x since APS meeting in April 2007

In past 2.5 years modeling and removing three Newtonian sources of error

1) damped polhode motion



2) misalignment torques

torques on the gyros when spacecraft's axis of symmetry not aligned with gyro's axes

3) roll-polhode resonance

All 3 effects due to "patch-effect" anomalies

„while mechanically both rotor and housing are exceedingly spherical, electrically they are not

patch charges arise from varying surface electrical potentials in 'polycrystalline materials'

Analysis up to now:

COMBINED 4-GYRO RESULT GIVES STATISTICAL UNCERTAINTY OF 14% (±5 miliarcsec) FOR THE FRAME DRAGGING

OPEN ACCESS
IOP Publishing
Classical and Quantum Gravity
Class. Quantum Grav. 32 (2015) 224001 (29pp)
doi:10.1088/0264-9381/32/22/224001

17 November

The Gravity Probe B test of general relativity

C W F Everitt[1], B Muhlfelder[1], D B DeBra[1], B W Parkinson[1], J P Turneaure[1], A S Silbergleit[1], E B Acworth[1], M Adams[1], R Adler[1], W J Bencze[1], J E Berberian[1], R J Bernier[1], K A Bower[1], R W Brumley[1], S Buchman[1], K Burns[1], B Clarke[1], J W Conklin[1], M L Eglington[1], G Green[1], G Gutt[1], D H Gwo[1], G Hanuschak[1], X He[1], M I Heifetz[1], D N Hipkins[1], T J Holmes[1], R A Kahn[1], G M Keiser[1], J A Kozaczuk[1], T Langenstein[1], J Li[1], J A Lipa[1], J M Lockhart[1], M Luo[1], I Mandel[1], F Marcelja[1], J C Mester[1], A Ndili[1], Y Ohshima[1], J Overduin[1], M Salomon[1], D I Santiago[1], P Shestople[1], V G Solomonik[1], K Stahl[1], M Taber[1], R A Van Patten[1], S Wang[1], J R Wade[1], P W Worden Jr[1], N Bartel[6], L Herman[6], D E Lebach[6], M Ratner[6], R R Ransom[6], I I Shapiro[6], H Small[6], B Stroozas[6], R Geveden[2], J H Goebel[3], J Horack[2], J Kolodziejczak[2], A J Lyons[2], J Olivier[2], P Peters[2], M Smith[3], W Till[2], L Wooten[2], W Reeve[4], M Anderson[4], N R Bennett[4], K Burns[4], H Dougherty[4], P Dulgov[4], D Frank[4], L W Huff[4], R Katz[4], J Kirschenbaum[4], G Mason[4], D Murray[4], R Parmley[4], M I Ratner[4], G Reynolds[4], P Rittmuller[4], P F Schweiger[4], S Shehata[4], K Triebes[4], J VandenBeukel[4], R Vassar[4], T Al-Saud[5], A Al-Jadaan[5], H Al-Jibreen[5], M Al-Meshari[5] and B Al-Suwaidan[5]

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[2] NASA Marshall Space Flight Center, USA
[3] NASA Ames Space Flight Center, USA
[4] Lockheed Martin, USA
[5] King Abdulaziz City Science and Technology (KACST), Saudi Arabia

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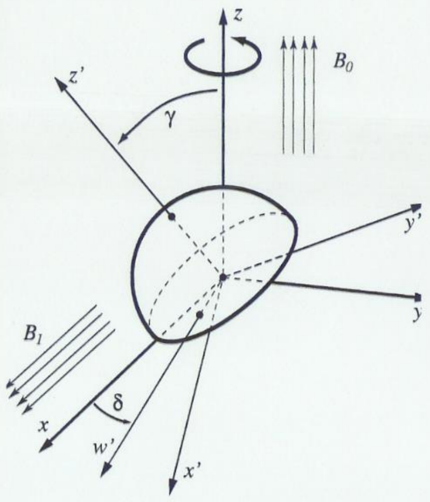
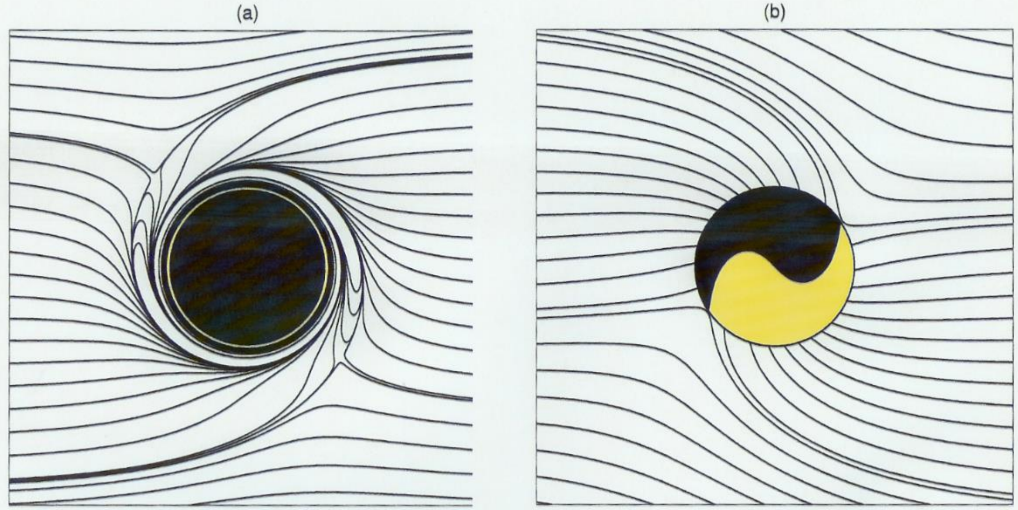
on-axisymmetric fields – asymptotically uniform non-aligned fields

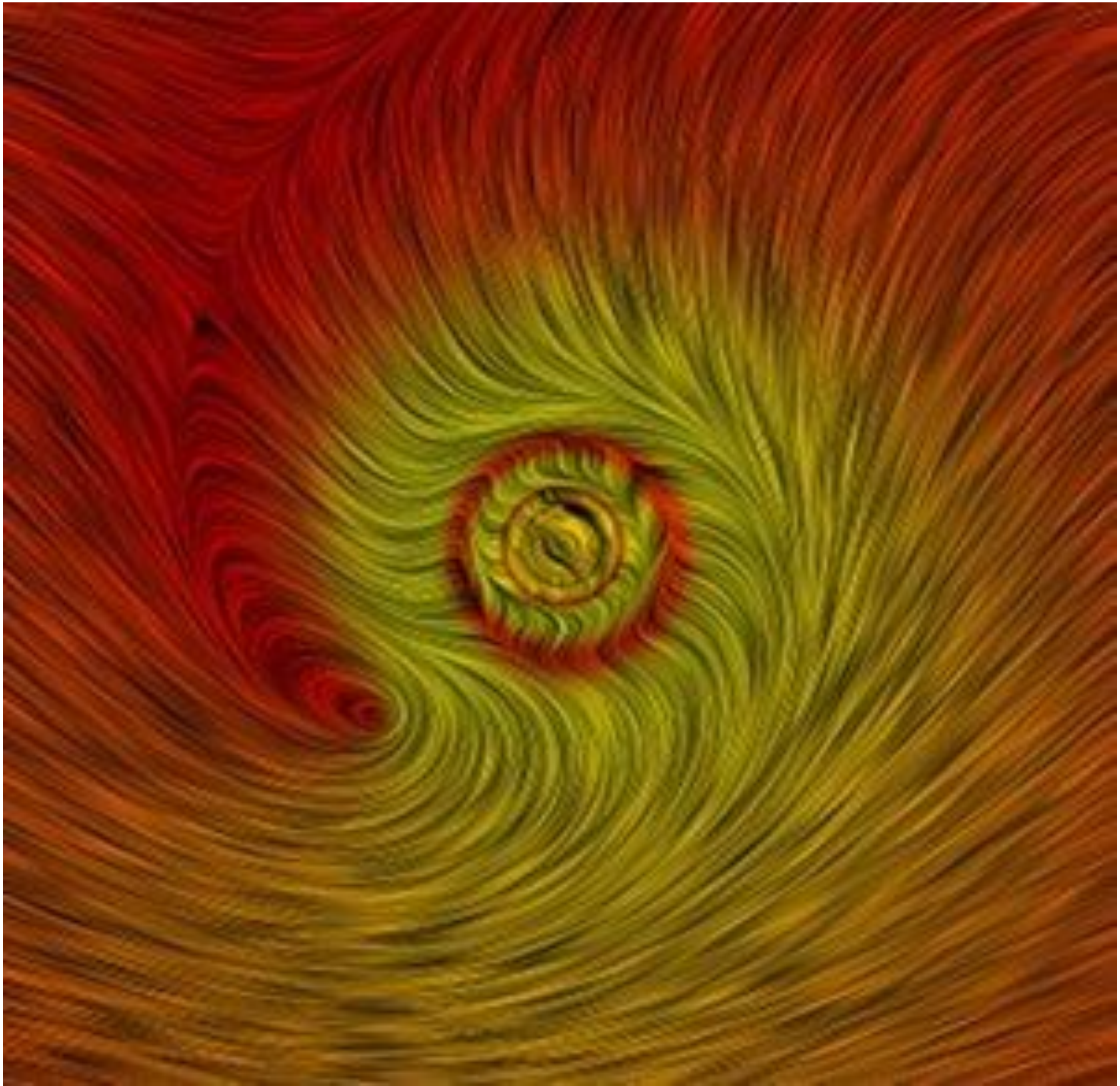


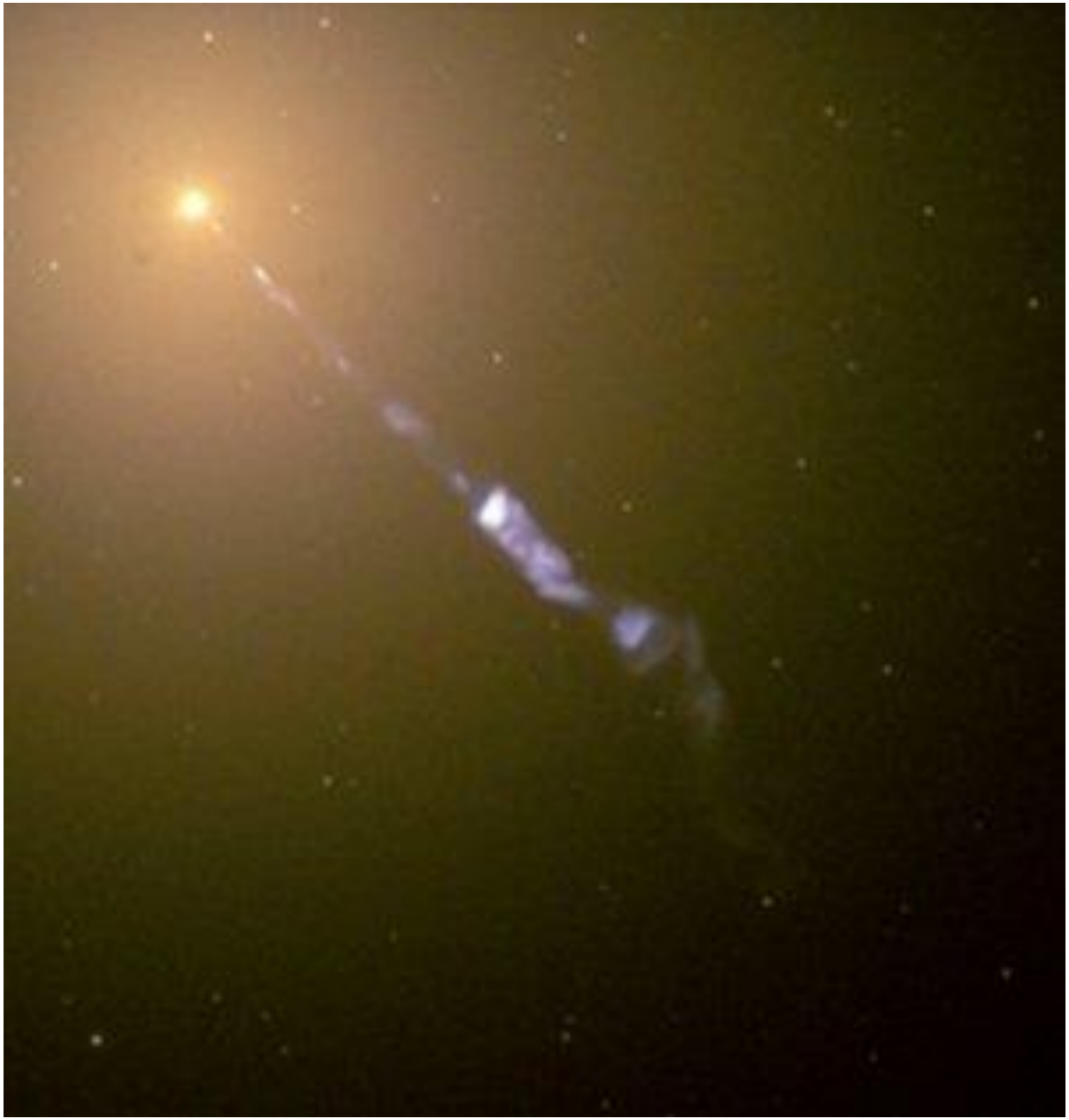


ıes of the magnetic field which is asymptotically uniform and perpendicular to the rotation axis. The equatorial plane is shown as ɘwed from top, i.e. along the rotation axis, (a) in the frame of zero angular momentum observers orbiting at constant radius; (b) in ɘ frame of freely falling observers. In the panel (b), two regions of ingoing/outgoing lines are distinguished by different levels of ading of the horizon. The hole rotates counter-clockwise ($a = M$).

$= 0$: $\Phi = B_0 \pi r_+^2 (1 - \frac{a^4}{r_+^4})$, $r_+ = M + \sqrt{M^2 - a^2}$

$\neq 0$: $\forall |a| \leq 1 \; \exists \delta_{\rm max} \Rightarrow \Phi_{\rm max}$ $\left(a = M \Rightarrow \delta_{\rm max} \sim -63°,\ \phi \sim 2.25 B_1 \pi r_+^2\right)$





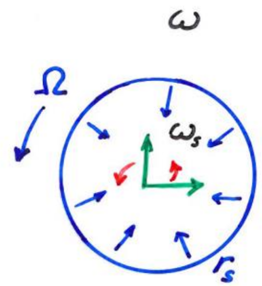
Instantaneous Inertial Frames & Retarded Electromagnetic Fields in Relativistic Collapse with Rotation

Katz, Lynden-Bell, Bi., CQG

Extending & generalizing

Lindblom & Brill (Phys. Rev. D 1974)

(see also H. Pfister, Ch. Klein, etc.)



$$\Omega = \frac{d\varphi_{shell}}{dt} \quad \text{small (neglect } r_s^2\Omega^2)$$

A collapsing spherical shell (of dust) in slow (rigid) rotation produces a slightly perturbed Schwarzschild spacetime outside the shell

$r \geq r_s$ (in $\{t, r, \theta, \varphi\}$ coordinates):

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{1}{1 - \frac{2M}{r}} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, (d\varphi - \omega \, dt)^2 \qquad (1)$$

perturbation: frame-dragging potential

PERTURB FE's ($\ell = 1$; odd parity):

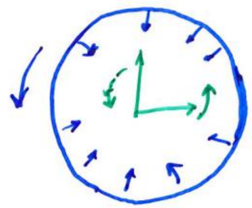
$$\boxed{\omega(r) = \frac{2J}{r^3}}$$

J: shell's (small) fixed total angul. momentum

Insight into Inside

$$\equiv \omega[r_s(t)] = \frac{2J}{[r_s(t)]^3}$$

$$(\sim) \quad d\bar{\varphi} = d\varphi - \omega_s\, dt$$



Local inertial frames (LIF's) inside ($\bar{\varphi} = \text{const}$) all rotate rigidly with the same angular velocity w.r.t. to observers at rest relative to infinity ($\varphi = \text{const}$)... „static obs.":

$$\frac{d\bar{\varphi}}{dt} = 0 \overset{(\sim)}{\Longrightarrow} \boxed{\frac{d\varphi}{dt} = \omega_s} \qquad \omega_s(t)$$

As measured in LIF's own proper time the rate of rotation is

$$\frac{d\varphi}{d\bar{t}} = \bar{\omega}_s = \omega_s \left.\frac{dt}{d\bar{t}}\right|_s$$

Static observers inside experience Euler acceleration (their Coriolis and centrifugal $\sim \bar{\omega}_s^2$) and the congruence of their world lines twists

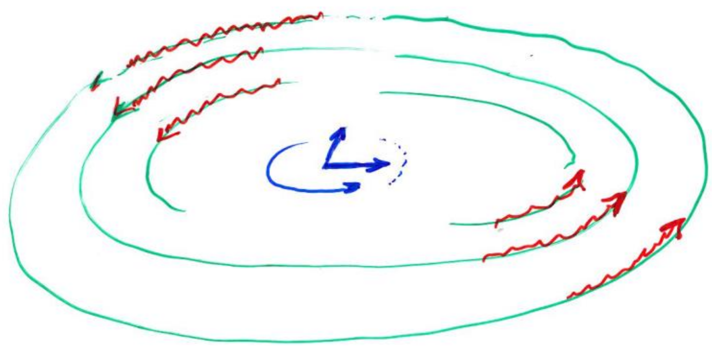
Gravitational waves and dragging effects

2 papers in CQG 2008 (@ J.K., DLB) - 2010

yrs. 2000 <t: Bini, de Felice, Herrera, Valiente, Tucker
gyroscopes (spinning particles)
immersed directly in a gravit. wave

We wish to tackle a more fundamental question:

Whether energy and angular momentum in purely vacuum spacetimes can cause the local inertial frames to rotate



If yes, is this effect „instantaneous" as in case of rotating matter?

- <u>Corvino & Schoen</u> (2006) „analytic gluing technique"

Kerr → WAVES → Kerr metric

Rotating gravitational waves in the symmetry reduced GR

pure waves, no string, only $\partial/\partial z$ symmetry, not cylindrical $(\partial/\partial\varphi)$ symmetry

$$ds^2 = e^{-2\psi} g_{ab}\, dx^a dx^b - e^{2\psi} \underbrace{dz}_{3}{}^2$$

$\psi(x^c)$, $g_{ab}(x^c)$ $\quad a, b, c \ldots \underset{t}{0}, \underset{\rho}{1}, \underset{\varphi}{2}$

$$R_{ab} = 0 \Rightarrow \boxed{\mathcal{R}_{ab} = 2\partial_a\psi\,\partial_b\psi}$$

$$R_{33} = 0 \Rightarrow \boxed{g^{ab}\nabla_a\nabla_b\psi = 0}$$

$\mathcal{R}_{ab}$ … Ricci of 3 space $g_{ab}\,dx^a dx^b$

In 3 dimensions $\mathcal{R}_{abcd}$ determined by $\mathcal{R}_{ab}$ (so here by ψ):

$$\mathcal{R}_{abcd} = 2\Big[\big(\mathcal{R}_{a[c} - \tfrac{1}{4} g_{a[c}\mathcal{R}\big) g_{d]b} - \big(\mathcal{R}_{b[c} - \tfrac{1}{4} g_{b[c}\mathcal{R}\big) g_{d]a}\Big]$$

1 Killing only $\to$ „formidable task"

$\to$ assume ψ and derivatives small

develop approximation procedure

$\psi = \varepsilon \Psi(x^c) \Rightarrow \mathcal{R}_{ab} \sim O(\varepsilon^2)$, $\mathcal{R}_{abcd} \sim O(\varepsilon^2)$

$\Rightarrow$ may write

$$g_{ab} = \eta_{ab} + \varepsilon^2 \gamma_{ab}(x^c), \quad g^{ab} = \eta^{ab} - \varepsilon^2 \gamma^{ab}$$

So we can construct a genuinely rotating („φ-dependent") solution of the wave eq. in flat space and still satisfy Field Eqs. in terms $\sim O(\varepsilon^2)$ by solving $\boxed{\mathcal{R}_{ab} = 2\,\partial_a\psi\,\partial_b\psi}$

$\Rightarrow g_{ab} = g_{ab}(t, \rho, \varphi)$.

However, we are primarily interested in the rotation of inertial frames („gyros") at the axis (which is regular) – φ-dependent terms in ω do not affect the rotation there

$\Rightarrow$ concetrate on the axially symm. terms in the Fourier expansion of ω as function of φ

$\Rightarrow$ solve equations for g_{ab} averaged over φ

$$\Rightarrow \mathcal{R}_{ab} = 2 \underbrace{\langle \partial_a \psi \, \partial_b \psi \rangle}_{\equiv S_{ab}} = 2 \int_0^{2\pi} \partial_a \psi \, \partial_b \psi \, d\varphi$$

So source(*) axisymmetric, hence also g_{ab}
but $\partial/\partial\varphi$ not hypersurface orthogonal

$$\boxed{d\sigma^2 = e^{2\gamma}(dt^2 - d\rho^2) - W^2(d\varphi - \omega\, dt)^2}$$

(*) no problem at the axis (like matter cyl.)

dragging

$\gamma(t,\rho)$, $\quad W(t,\rho) = \rho + \varepsilon^2 w(t,\rho)$

ε will be „absorbed"

Left h. sides

EFEs:

$$\mathcal{R}_{00} = -\ddot{\gamma} + \gamma'' + \frac{1}{\rho}\gamma' - \frac{1}{\rho}\ddot{W}$$

$$\mathcal{R}_{11} = \ddot{\gamma} - \gamma'' + \frac{1}{\rho}\gamma' - \frac{1}{\rho}W''$$

$$\mathcal{R}_{01} = \frac{1}{\rho}\dot{\gamma} - \frac{1}{\rho}\dot{W}'$$

$$\mathcal{R}_{22} = \rho(\ddot{W} - W'')$$

$$\boxed{\mathcal{R}_{02} = \frac{1}{2\rho}(\rho^3\omega')'}$$

constraint eq.

$$\mathcal{R}_{12} = \frac{1}{2}\rho^2\dot{\omega}'$$

Inertial frame rotation induced by rotating gravitational waves

Two equations considered explicitly:

WE: $$\ddot{\psi} - \psi'' - \frac{1}{\rho}\psi' - \frac{1}{\rho^2}\partial_\varphi^2 \psi = 0 \qquad (1)$$

$\mathcal{R}_{02} = 2S_{02}$: $$\frac{1}{2\rho}\left(\rho^3 \langle\omega\rangle'\right)' = 2\langle \dot{\psi}\,\partial_\varphi \psi\rangle \overset{\text{def}}{\equiv} j \qquad (2)$$

~ angular mom. density

Elem. Solutions of (1) expressed in Bessel functions $J_m(\underline{\omega}\rho)$

not $\langle\omega\rangle$

$$\psi = A e^{i(m\varphi - \underline{\omega} t)} J_m(\underline{\omega}\rho) \qquad \text{(real part)}$$

$m \neq 0$ ... rotating wave: $m\varphi - \omega t = \text{const}$

$\rightarrow$ $\boxed{\varphi = \frac{\omega}{m}t + \text{const}}$

Inspired by exact Bonnor-Weber-Wheeler pulse take superposition ("ω" integrates out)

$$\boxed{\psi = B\int_0^\infty (a\omega)^m e^{-a\underline{\omega}} e^{i(m\varphi - \underline{\omega} t)} J_m(\omega\rho)\, a\,d\omega + \text{c.c.}}$$

$a > 0$ constant – effective duration of the pulse

15*

Weber–Wheeler–Bonnor pulse

$$ds^2 = e^{-2\psi}\left[e^{2\gamma}(dt^2 - d\rho^2) - \rho^2 d\varphi^2\right] - e^{2\psi} dz^2$$

$$\psi = \psi(t,\rho)\,, \quad \gamma = \gamma(t,\rho)$$

EFEs: $\gamma' = \rho\left(\psi'^2 + \dot\psi^2\right)\,, \quad \dot\gamma = 2\rho\,\dot\psi\,\psi'$

and WE: $\psi'' + \frac{1}{\rho}\psi' - \ddot\psi = 0$

$$\tilde\rho = \frac{\rho}{a}\,, \quad \tilde t = \frac{t}{a}\,, \quad b = \frac{\sqrt{2}\,c}{a}$$

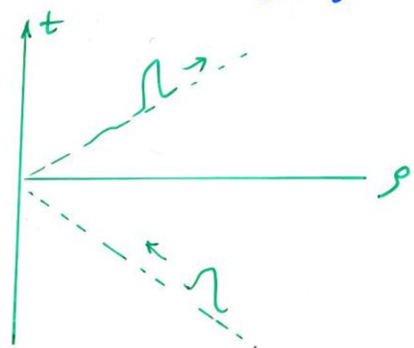
measure of the "amplitude"

the width of pulse

c, a constants

$$\psi = b\left\{\frac{1+\tilde\rho^2-\tilde t^2+\left[(1+\tilde\rho^2-\tilde t^2)^2+4\tilde t^2\right]^{1/2}}{(1+\tilde\rho^2-\tilde t^2)^2+4\tilde t^2}\right\}^{1/2}$$

$$\gamma = \frac{b^2}{4}\left\{1 - 2\tilde\rho^2\frac{(1+\tilde\rho^2-\tilde t^2)^2-4\tilde t^2}{\left[(1+\tilde\rho^2-\tilde t^2)^2+4\tilde t^2\right]^2} - \frac{1-\tilde\rho^2+\tilde t^2}{\left[(1+\tilde\rho^2-\tilde t^2)^2+4\tilde t^2\right]^{1/2}}\right.$$



Bateman Manuscript Project of Erdélyi et al formula (8.6.5.)

$$\psi = B a^{m+1} e^{im\varphi} 2^m \frac{\Gamma(m+\frac{1}{2})}{\sqrt{\pi}} \rho^m (\alpha^2+\rho^2)^{-m-\frac{1}{2}} + c.c.$$

$$\alpha = \alpha(t) = a + it$$

Define $\tilde{\rho} = \frac{\rho}{a}$, $\tilde{t} = \frac{t}{a}$

use $2^m \frac{\Gamma(m+\frac{1}{2})}{\sqrt{\pi}} = (2m-1)!!$

in real terms

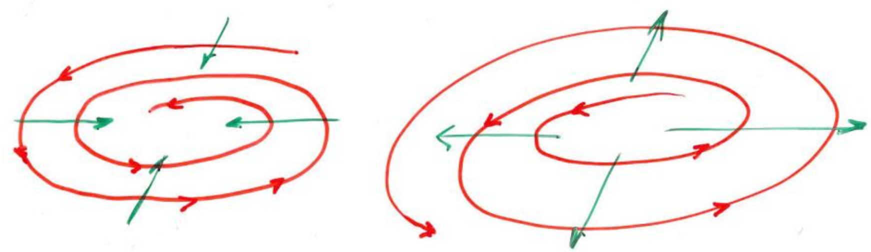
$$\psi(\tilde{t}, \tilde{\rho}, \varphi) =$$

$$= 2B(2m-1)!! \frac{\tilde{\rho}^m \cos\big[\overbrace{m\varphi - (m+\tfrac{1}{2})\chi}^{\boxed{\text{phase}}}\big]}{\left[(1+\tilde{\rho}^2-\tilde{t}^2)^2+4\tilde{t}^2\right]^{\frac{1}{2}(m+\frac{1}{2})}}$$

where $\chi = \arctan \frac{2\tilde{t}}{1+\tilde{\rho}^2-\tilde{t}^2}$

$\tilde{\rho}^2$

$\tilde{t} \to -\tilde{t}$ sym.



Integrating the Einstein eq. for $\mathcal{R}_{02}$, ω

$$\langle \omega \rangle = \frac{1}{2} \int_{\tilde{\rho}^2}^{\infty} \frac{1}{\rho_2^4} \Big[\int_0^{\tilde{\rho}^2} \underset{\uparrow}{j} \, d\rho_1^2 \Big] d\rho_2^2$$

$-2\langle \dot{\psi} \partial_y \psi \rangle$

⋮ integrating by parts ...

$$\boxed{\begin{aligned} \langle \omega \rangle = {} & \frac{2B^2}{a} m \left[(2m-1)!! \right]^2 \times \\ & \times \Big[\frac{m}{u} I_{m-1} + 2(2m+1) \frac{\tilde{t}^2}{u} I_m + (m-1) H_{m-1} \\ & \qquad + 2(2m+1) \tilde{t}^2 H_m \Big] \end{aligned}}$$

$$\boxed{\begin{aligned} & I_m(u) \equiv \int_0^u u^m Q^{-m-3/2} \, du \\ & H_m(u) \equiv \int_u^{\infty} u^{m-1} Q^{-m-3/2} \, du \\ & u \overset{\text{df}}{\equiv} \tilde{\rho}^2, \qquad Q = (1 + u - \tilde{t}^2)^2 + 4\tilde{t}^2 \end{aligned}}$$

Evaluation of $I_m(u)$, $H_m(u)$

... detailed Appendix in CQG

Rotation of inertial frames at small and great distances

On axis

$$\langle\omega\rangle_0 = \frac{B^2}{a}\frac{(2m)!}{2^{2m-1}}\frac{1+m(1+\tilde{t}^2)}{(1+\tilde{t}^2)^2}\Bigg| + \ldots O(\tilde{\rho}^{2m})$$

Greatest at $\tilde{t} = 0$

No time lag between the wave arriving closest to the axis – most of the energy never gets nearer than $\tilde{\rho} \simeq 0.4$

$\Rightarrow$ like with the shell rotating and collapsing – non-local effect given by the constraint equation instantaneous

Far from the axis

$$\langle\omega\rangle \simeq \frac{B^2}{a}\frac{m(m!)(2m-1)!!}{2^{m-1}}\frac{1}{\tilde{\rho}^2}, \quad \tilde{\rho} \gg 1$$

Due to the rotation of IF rod at the origin points towards

$$\phi(t) = \phi_0 + \int_{-\infty}^{t} \langle\omega\rangle_0 \, dt$$

$$= \phi_0 + B^2 \frac{(2m+1)!}{2^{2m}} \left[\arctan \tilde{t} + \frac{\pi}{2} + \frac{\tilde{t}}{(2m+1)(1+\tilde{t}^2)} \right]$$

can choose $= 0$ at $t = 0$

$$\langle\omega\rangle \simeq \langle\omega\rangle_0 \left[1 - \frac{(2m-1)!!}{(m+1)!} \frac{1}{(1+\tilde{t}^2)^{m+1}} \times \right.$$

$$\left. \times \frac{(2m-5)+(2m+7)\tilde{t}^2}{(m+1)+m\tilde{t}^2} \left(\frac{\tilde{\rho}^2}{1+\tilde{t}^2} \right)^m \right]$$

very small at $\tilde{\rho} \ll 1$, m high

like shell

Metric at $\tilde{\rho}^2 \ll (1+\tilde{t}^2)$

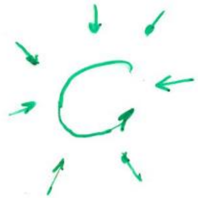
$$ds^2 \cong e^{2\psi} \left[dt^2 - d\rho^2 - \rho^2 (d\varphi - \langle\omega\rangle_0 dt)^2 - e^{-2\psi} dz^2 \right.$$

$\psi \sim \tilde{\rho}^m$ very small

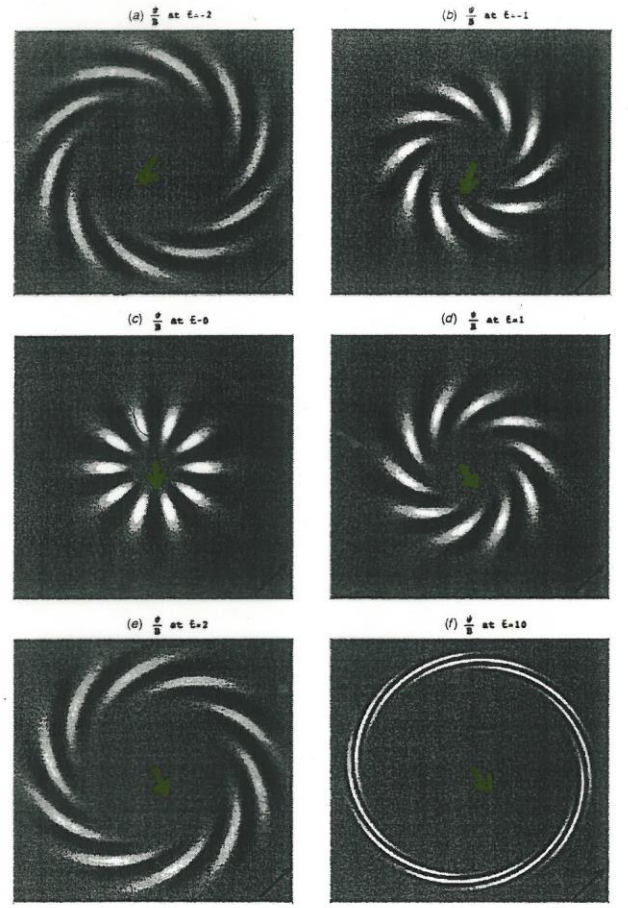
in rotating axes $\tilde{\varphi} = \varphi - \phi(t)$: $ds^2 = e^{2\psi} \underbrace{(dt^2 - d\rho^2 - \rho^2 d\tilde{\varphi}^2)}_{\text{flat in the reduced space}} - e^{-2\psi} dz^2$

18**

ψ at different times



anticlockwise rotating waves



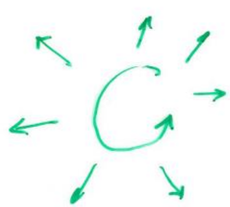


Figure 5. This shows the $m = 10$ wave which always rotates anticlockwise. As it comes inwards (*a*) at $\bar{t} = -2$ it is in the form of a leading spiral with the outer parts of the arms ahead of the central parts. By $\bar{t} = -1$ (*b*) the spiral has started to open. By $\bar{t} = 0$ (*c*) the central parts have caught up and the spiral has changed to a cartwheel structure but rotation keeps it beyond $\bar{\rho} \simeq 0.4$. By $\bar{t} = 1$ (*d*) the spiral has become trailing as befits a wave that now feeds angular momentum outwards. By $\bar{t} = 2$ (*e*) the spiral becomes tighter and the flat central cylinder becomes larger. We show $\bar{t} = 10$ (*f*) at a small scale but note the beautiful tight wrapping of the narrow arms. Also note the opposite spirality of the conjugate pairs $\bar{t} = \pm 2$ and $\bar{t} = \pm 1$. Figures encompass a radius $\bar{\rho} \simeq 7$ ($\bar{\rho} \simeq 17$ for $\bar{t} = 10$). The height of ψ/B reduced by a factor of 10^{-4} is between 0 and 1. Lighting falls at 45° from the left. The view is along the z axis from above at a distance of $10^{-4}\psi/B = 40$.

orientation of rod at the origin:

$$\phi(t) = \varphi_0 + \int_{-\infty}^{t} \langle \omega \rangle_0 \, dt = \ldots \text{explicitly known}$$

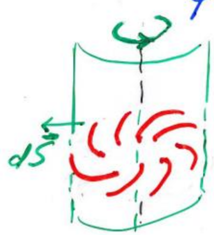
Angular momentum transport by gravitational torques

analogy with angular momentum transport in spiral galaxies

classical gravitational stress tensor is

$$\sigma_{k\ell} = \frac{1}{\kappa}\left(2\,\partial_k\psi\,\partial_\ell\psi - \delta_{k\ell}\sum_m |\partial_m\psi|^2\right)$$

ψ – is now classical gravitational potential



Gravitational couple transferring angular momentum outwards a cylinder is

$$C_{grav} = \int \varepsilon_{3k\ell}\, \underbrace{X^k}_{\text{radius vector}}\, \overbrace{\sigma^{\ell m}}^{\text{stress tensor}}\, \underbrace{dS_m}_{\text{outward pointing surface element}}$$

$$\Rightarrow \quad C_{grav} = \frac{2}{\kappa}\int \partial_\varrho\psi\,\partial_\varphi\psi\,\varrho\,d\varphi\,dz$$

to carry angular momentum outwards must be a positive correlation between $\partial_\varrho\psi$ and $\partial_\varphi\psi$ averaged over the cylinder $\Rightarrow$

trailing sense of spirality to countours of $\psi = \text{const.}$

outer parts of spiral galaxy trail inner parts in the sense of rotat.

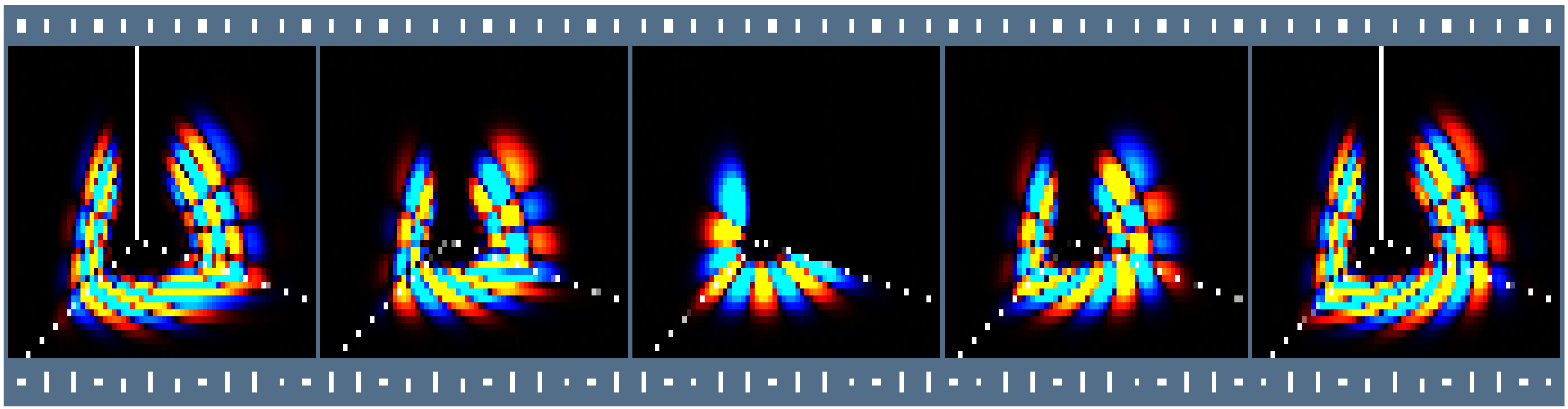
Similarly with gravitational waves:

for angular momentum transport outward, the spiral formed by countours of ψ must trail the inner parts but oppositely when they form a leading spiral with the outside further advanced than the inside and the angular momentum is transported inward (upper part of Fig.)

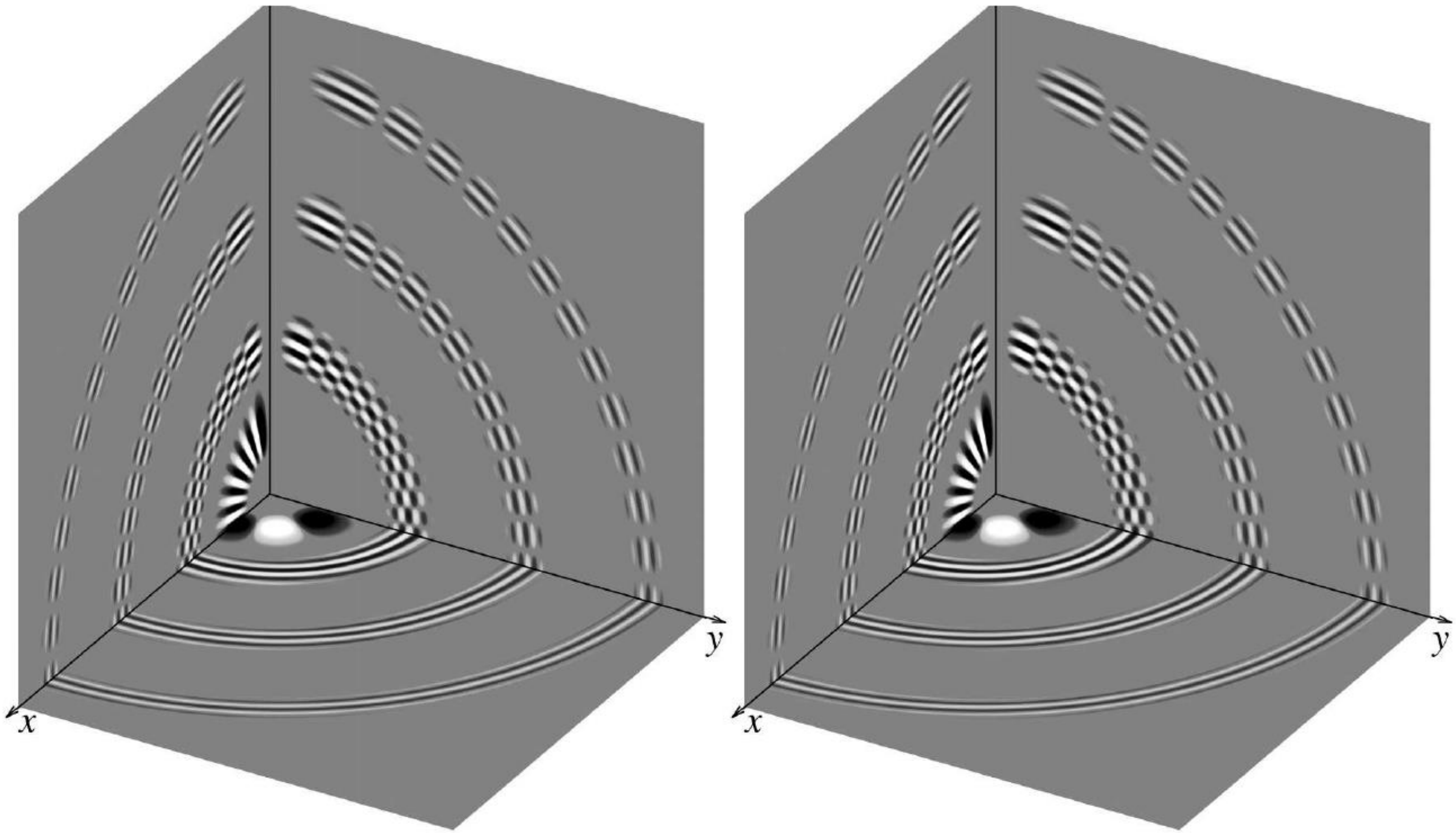
At $t = 0$ no angular momentum transport
$\Rightarrow$ countours of ψ form a rotating cartwheel

with no spirality (Figure)

The waves themselves never reach much further than $\tilde{\rho} = 1$ (flat space around axis). But
the 2nd order effect of the angular momentum causes the rotation of the inertial frame within the waves



Rotating scalar waves



Snapshots of impact (left, $t = -6, -4, -2, 0$) and departure (right, $t = 0, 2, 4, 6$) of scalar spherical version of Weber-Wheeler-Bonnor pulse with $l = 27, m = 5$.

Rotating scalar waves

In linear analysis one can use simple prescribtion

$$\psi_{lm}(t,r,\theta,\phi) \sim \mathrm{Re}\, \frac{\left(\frac{r}{a}\right)^{l}}{\left[\frac{(a+it)^2+r^2}{a^2}\right]^{l+1}} Y_{lm}(\theta,\varphi)e^{-i\omega t}$$

In nonlinear terms, we may need an explicit form

$$\psi_{lm}(t,r,\theta,\phi) \sim \frac{\left(\frac{r}{a}\right)^{l}}{\left[\frac{(a^2+r^2-t^2)^2+4a^2t^2}{a^4}\right]^{\frac{l+1}{2}}} P_l^m(\cos\theta)\cos(m\phi-\lambda(t,r))$$

$$\lambda(t,r) = (l+1)\arctan\frac{2at}{a^2+r^2-t^2}$$

Expansion of (odd parity) symmetric second-rank covariant tensor into tensor harmonics

$$h^{(i)}_{\mu\nu} = \sum_{lm} \frac{\sqrt{2l(l+1)}}{r} \left[-h^{(i)}_{0lm}(t,r) c_{0lm\mu\nu} + i h^{(i)}_{1lm}(t,r) c_{lm\mu\nu} + \frac{i\sqrt{(l-1)(l+2)}}{2r} h^{(i)}_{2lm}(t,r) d_{lm\mu\nu} \right],$$

$$c_{0lm} = \frac{r}{\sqrt{2l(l+1)}} \begin{pmatrix} 0 & 0 & \frac{1}{\sin\theta}\partial_\varphi Y_{lm} & -\sin\theta\partial_\theta Y_{lm} \\ 0 & 0 & 0 & 0 \\ \frac{1}{\sin\theta}\partial_\varphi Y_{lm} & 0 & 0 & 0 \\ -\sin\theta\partial_\theta Y_{lm} & 0 & 0 & 0 \end{pmatrix},$$

$$c_{lm} = \frac{ir}{\sqrt{2l(l+1)}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sin\theta}\partial_\varphi Y_{lm} & -\sin\theta\partial_\theta Y_{lm} \\ 0 & \frac{1}{\sin\theta}\partial_\varphi Y_{lm} & 0 & 0 \\ 0 & -\sin\theta\partial_\theta Y_{lm} & 0 & 0 \end{pmatrix},$$

Second-order odd parity dipole perturbations

Now we have solved the first order Einstein equations

$$G^{(1)}_{\mu\nu}[h^{(1)}] = 0$$

In general the second-order metric perturbations $h^{(2)}$ can be obtained by solving

$$G^{(1)}_{\mu\nu}[h^{(2)}] = -G^{(2)}_{\mu\nu}[h^{(1)}, h^{(1)}],$$

right-hand side is the source term in the form of an effective energy-momentum tensor

$$G^{(1)}_{\mu\nu}[h^{(2)}] = -\frac{1}{2}\Big[h^{(2);\alpha}_{\mu\nu;\alpha} + h^{(2);\alpha}_{\mu\alpha;\nu} + h^{(2);\alpha}_{\nu\alpha;\mu} - h^{(2)\alpha}{}_{\alpha\,;\mu\nu} - \bar{g}_{\mu\nu}\left(h^{(2)}{}_{\beta\alpha}{}^{;\alpha\beta} - h^{(2)\beta;\alpha}{}_{\beta\;\;;\alpha}\right)\Big].$$

Rotation $\phi' = \phi - \omega_0 t \to ds^2 = \ldots + r^2 \sin^2\theta(d\phi - \omega_0 dt)^2$ is most easily identified in

$$g^{(2)}_{t\varphi} = -\omega_0\, r^2 \sin^2\theta$$

since $t\phi$-component is associated only with the following tensor harmonic component

$$c_{0lm\,t\varphi} = \frac{r}{\sqrt{2l(l+1)}}\left(-\sin\theta\partial_\theta Y_{lm}\right)$$

the dragging of inertial frames near the origin is given by $l = 1, m = 0$ perturbation

SECOND-ORDER ODD PARITY DIPOLE PERTURBATIONS

- Projection of the second-order perturbation equation

$$R^{(1)}_{\mu\nu}[h^{(2)}] = -R^{(2)}_{\mu\nu}[h^{(1)}, h^{(1)}],$$

into tensor harmonics $c_{0\,lm}$ and d_{lm} with $l = 1, m = 0$ which are gauge-only perturbations

$$\frac{1}{2}\left[h_0^{(2)\prime\prime} - \frac{2}{r^2} h_0^{(2)}\right] = -\sqrt{\frac{3}{4\pi}} \int_\Omega R^{(2)}_{t\varphi}[h^{(1)}, h^{(1)}] \; d\Omega$$

- Chosen gauge corresponds to rigidly rotating central inertial frame
- Variation of constants then provides solution, namely near $r = 0$ the dragging angular velocity of the central inertial frame

$$\omega_0 = \frac{1}{\sqrt{12\pi}} \int_0^\infty \int_\Omega R^{(2)}_{t\varphi}[h^{(1)}, h^{(1)}]\, d\Omega \, \frac{dr}{r}.$$

Second order Ricci

- Real-valued metric perturbations written as follows

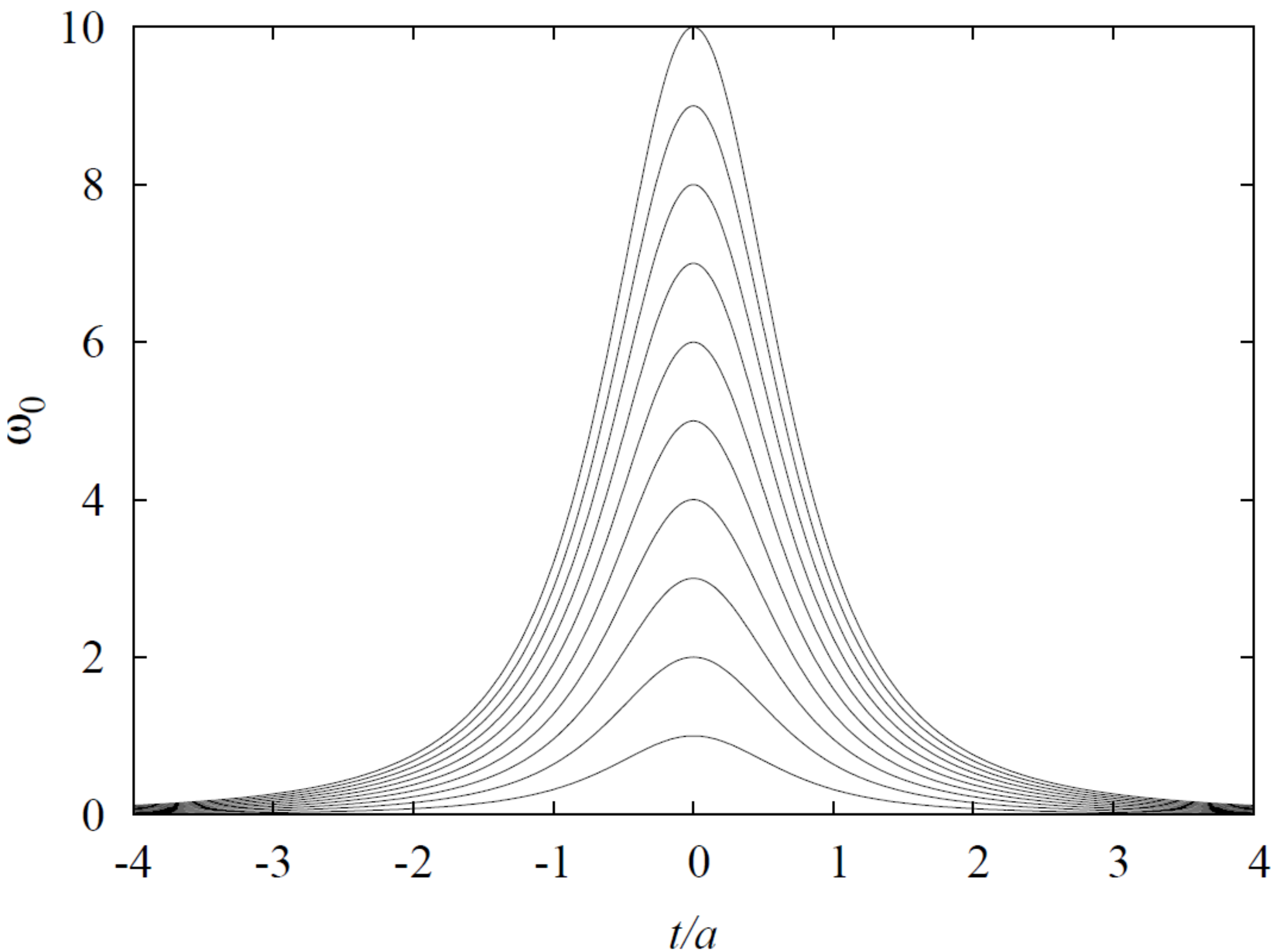
$$h_{\theta t} = \frac{1}{\sin\theta}\frac{\partial\chi}{\partial r\partial\varphi}, h_{\varphi t} = -\sin\theta\frac{\partial\chi}{\partial r\partial\theta}, h_{\theta r} = \frac{1}{\sin\theta}\frac{\partial\chi}{\partial t\partial\varphi}, h_{\varphi r} = -\sin\theta\frac{\partial\chi}{\partial t\partial\theta},$$

$$\chi = \tilde{B}_l N_l^m\, \kappa(t,r) P_l^m(\cos\theta)\cos(m\varphi - \lambda(t,r))\ , \quad \kappa = \frac{\tilde{r}^{l+2}}{[(1+\tilde{r}^2-\tilde{t}^2)^2+4\tilde{t}^2]^{(l+1)/2}}\ .$$

- Computer algebra & pencil and paper

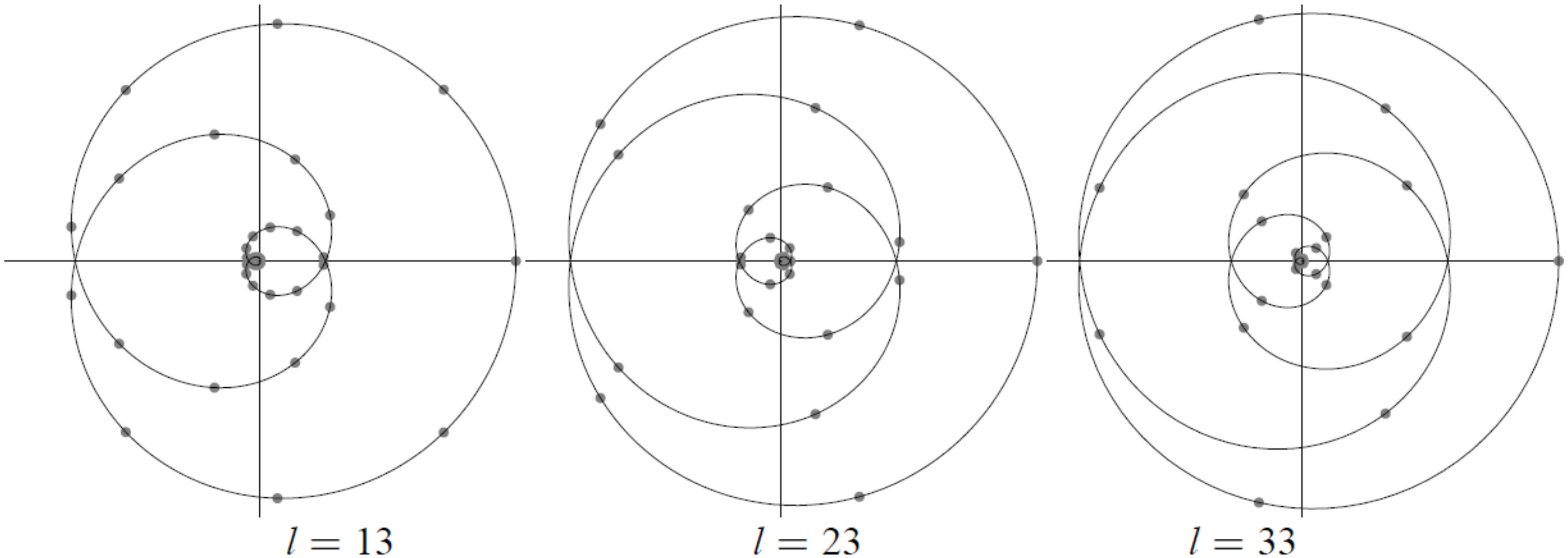
$$\begin{aligned}
R^{(2)}_{t\varphi} = -\frac{1}{2}\frac{1}{r^2}\Big(& 2\,\chi_{,\theta rr\theta}\chi_{,\varphi t} - \chi_{,r\varphi r}\chi_{,\theta t\theta} + 2\,\chi_{,tr\varphi\theta}\chi_{,r\theta} - \chi_{,rr\varphi\theta}\chi_{,t\theta} - \chi_{,\theta tr\theta}\chi_{,\varphi r} \\
&+ \chi_{,rr\theta}\chi_{,t\varphi\theta} + \chi_{,\theta r\theta}\chi_{,r\varphi t} - \chi_{,t\theta}\chi_{,tt\varphi\theta} - \chi_{,tt\theta}\chi_{,t\varphi\theta} - \chi_{,\varphi t}\chi_{,\theta tt\theta}\Big) + \frac{\chi_{,\theta r\theta}\chi_{,\varphi t}}{r^3} \\
&+ \frac{\left(\chi_{,r\theta}\chi_{,r\varphi t} - \chi_{,tr\theta}\chi_{,\varphi r} - \chi_{,r\varphi r}\chi_{,t\theta} + \chi_{,tt\theta}\chi_{,\varphi t}\right)\cos\theta}{2r^2\sin\theta} + \frac{\chi_{,r\theta}\chi_{,\varphi t}\cos\theta}{r^3\sin\theta} \\
&- \frac{\chi_{,r\varphi\varphi t}\chi_{,\varphi r} + \chi_{,\varphi\varphi r}\chi_{,r\varphi t} - \chi_{,t\varphi t}\chi_{,\varphi\varphi t}}{2r^2\sin^2\theta} + \frac{\chi_{,\varphi t}}{2r^3\sin^2\theta}\left(2\chi_{,\varphi\varphi r} + 2r\chi_{,\varphi\varphi tt} - r\chi_{,\varphi\varphi rr}\right)\ .
\end{aligned}$$

Central inertial frame dragging



Angular velocity of the central inertial frame $\omega_0(l, m; t)$ for $l = 10$ and $m = 1, 2, \ldots 10$. The vertical axis is scaled in units of $\omega_0(10, 1; 0)$.

Star's trajectories



When appropriately scaled and rotated, the trajectories of all stars are the same, i.e. the curve is an image of a straight line in the complex plane $z = 1 + is$, $s \in (-\infty, \infty)$ mapped by the function $f(z) = z^{-l-2}$ (positions at time $t/a = 0, \pm 0.05, \pm 0.1, \ldots$ are shown as circles).

$$\frac{\delta\varphi}{\Delta\varphi} + i\frac{\delta\theta}{\Delta\theta} = \frac{i^l\ e^{im\varphi}}{\left(1 + i\frac{T}{a}\right)^{l+2}}\ .\quad \text{where}$$

$$\Delta\theta = B_l N_l^m 2(l-1)!\frac{mP_l^m(\cos\theta)}{\sin\theta}, \qquad \Delta\varphi \qquad = -B_l N_l^m 2(l-1)! P_l'^m(\cos\theta),$$

By Mach's principle we mean:

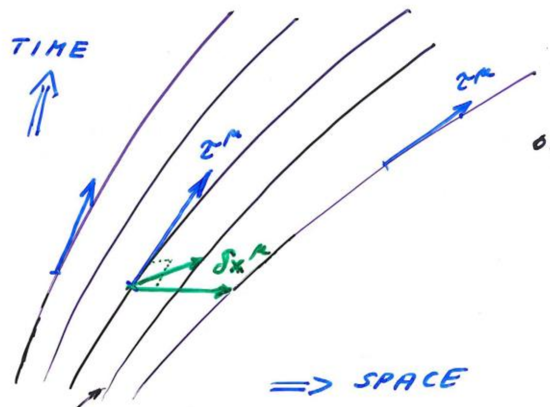
„ all motions, velocities, rotations and accelerations are relative; local inertial frames are determined through the distributions of energy and momentum in the Universe by some weighted averages of the apparent motions "

H. Bondi, Cosmology 1952

We show how such averages are to be taken for perturbed FLRW universes and demonstrate „ Mach" for the spherical universes

Cosmological perturbation theory, instantaneous „MACHIAN" gauges & local inertial frames

- General spacetime (universe)



filled with cosmological observers' (frames) COFs

congruence of COs:

$$x^\mu = x^\mu(y^i; p)$$

$\mu = 0, 1, 2, 3$

y^i = const, $i, j, l, \dots$ 1, 2, 3

τ^μ ... unit, timelike 4-velocity of CO

$$\delta x^\mu_{(i)\perp} = P^\mu_\nu \frac{\partial x^\nu}{\partial y^i}$$ .... connecting vectors spacelike, $\perp \tau^\mu$

are „Lie propagated"

projection $\perp \tau^\mu$

if 3 indep. unit: $m^\mu_{(i)}$

$\{\tau^\mu, m^\mu\}$ COF

Decomposition of $\tau_{\mu;\nu}$

$$\tau_{\mu;\nu} = \tau_\nu \alpha_\mu + \omega_{\mu\nu} + \sigma_{\mu\nu} + \frac{1}{3}\theta P_{\mu\nu}$$

acceleration, vorticity (rotation), shear, expansion e.g. $\theta = \tau^\nu{}_{;\nu}$

$P_\mu^{\ \lambda}\, \delta x^\mu_{(i)\perp ;\nu}\, \tau^\nu = \tau^\lambda_{\ ;\nu}\, \delta x^\nu_\perp$

From the propagation laws („Lie transport")

of $\delta x^\mu_{(i)\perp} = \underbrace{\delta l_{(i)}}_{\text{scalars „distance"}}\, m^\mu_{(i)}$ along the congruence:

unit vectors

1)

$$\boxed{\frac{\frac{d}{d\tau}\left(\delta l_{(i)}\right)}{\delta l_{(i)}} = \left(\sigma_{\mu\nu} + \frac{1}{3}\theta P_{\mu\nu}\right) m^\mu_{(i)}\, m^\nu_{(i)}}$$

generalized Hubble law

(viz $\frac{\dot{l}}{l} = H$ Hubble's const

in FRW $\theta = \frac{3\dot{a}}{a}$)

2)

$$\boxed{\underbrace{P_\mu^{\ \lambda}\, m^\mu_{(i);\nu}\, \tau^\nu}_{= \frac{D_F m^\mu_{(i)}}{d\tau}} = \left[\omega_\nu^{\ \lambda} + \sigma_\nu^{\ \lambda} + (\sigma\!:\!m.m.)\, P_{..}\right] m^\nu_{(i)}}$$

Fermi-Walker time derivative

= 0 for 'gyro'

rotation of COF w.r.t. local inertial axes („gyroscopes")

$\Rightarrow$ Rotation of LIFs

(Local Inertial Frames)

w.r.t. COFs, or vice versa

from now on:

$$\boxed{\text{Linearly perturbed FRW models}}$$

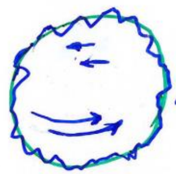
$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \underline{h_{\mu\nu}} \qquad \tilde{h}_{\mu\nu} \quad \text{if } dt = a(\eta)\,d\eta$$

"conformal time"

$$d\bar{s}^2 = dt^2 - a^2(t)\, f_{kl}(x^m)\, dx^k dx^l$$

e.g.

$$d\bar{s}^2 = dt^2 - a^2(t)\left[d\chi^2 + \Sigma_k^2 (d\theta^2 + \sin^2\theta\, d\varphi^2)\right]$$



perturbed $k=+1$

$$\Sigma_k = \begin{cases} \sin\chi & k=+1 \quad S^3 \\ \chi & k=0 \quad E^3 \\ \sinh\chi & k=-1 \quad H^3 \end{cases}$$

Now find:

vorticity $\quad \omega_{kl} = \delta\omega_{kl} = \frac{1}{2}\left(h_{0k,l} - h_{0l,k}\right)$

shear $\quad \sigma_{kl} = \delta\sigma_{kl} = \frac{1}{2}\dot{h}_{kl} - \frac{1}{6}\dot{h}^m_m \bar{g}_{kl} - \frac{\dot{a}}{a} h_{kl}$

$$\left(\omega_{0\alpha} = \delta\omega_{0\alpha} = \sigma_{0\alpha} = \delta\sigma_{0\alpha} = 0\right)$$

Expansion

$$\boxed{\theta = \bar{\theta} + \delta\theta = \frac{3\dot{a}}{a} + \frac{1}{2}\left(\dot{h}^m_m - \frac{3\dot{a}}{a} h^0_0 - \nabla_n h^m_0\right)}$$

background

The accelerations of COFs w.r.t. LIFs (i.e. "– accel." of LIFs w.r.t. COFs – mutually at rest ...)

$$\boxed{\alpha_\mu = \tau_{\mu;\nu}\,\tau^\nu} \leftarrow \text{general, covariant form}$$

In perturbed FRW

$$\boxed{\alpha^l = \bar{g}^{lm}\left(-\tfrac{1}{2}h_{00,m} + \dot{h}_{0m}\right)}$$

in Newtonian limit $\nabla\Phi$

Summary: in perturbed FRW universes, to determine rotation and acceleration of Local Inertial Frames we need to know $\boxed{h_{00,l} \quad h_{0l,m} \quad \dot{h}_{0l}}$

TASKS: – can these be determined instanteneously from matter variables $\delta T_{\mu\nu}$, possibly $\delta\dot{T}_{\mu\nu}$? (δT_{00}, δT_{0i}, δT_{mi})

– how uniquely in different types of universes ? (S^3, E^3, H^3, ... richer topologies) conformal time $dt = a(\eta) d\eta$

$a(\eta)$ ... expansion factor $\mathcal{H} = \frac{a'}{a} = Ha$ $\quad ' \equiv \frac{d}{d\eta}$

Einstein Field Equations for perturbations

$$\tilde{\underset{T}{h}}{}^l_k = \tilde{h}^l_k - \delta^l_k \tilde{h}^n_n \, .$$

general gauge
no harmonics
no decompositions

et

$$\mathcal{T}_k = \nabla_l \tilde{\underset{T}{h}}{}^l_k \, , \quad \mathcal{K} = \frac{3}{2} \dot{a} \tilde{h}_{00} + \frac{1}{2} a \dot{\tilde{h}}{}^n_n - \nabla_l \tilde{h}^l_0$$

foll ows Eintein's perturbation equations, separating δ^l_k, the traceless pa from the trace $\delta\tilde{G}^n_n$ which we combine with $\delta\tilde{G}^0_0$ for a reason to be seen b , defining $\nabla^2 = f^{kl}\nabla_{kl}$, we have the following dimensionless equations

$$\begin{aligned}
a^2\kappa\delta\tilde{T}^0_0 &= a^2\delta\tilde{G}^0_0 = \nabla^2\tilde{h}^n_n + k\tilde{h}^n_n - 2\mathcal{H}\mathcal{K} - \nabla_k\mathcal{T}^k \, , \\
a^2\kappa\delta\tilde{T}^0_k &= a^2\delta\tilde{G}^0_k = \nabla^2\tilde{h}_{k0} + k\tilde{h}_{k0} + \nabla_{kl}\tilde{h}^l_0 \\
&\quad + \nabla_k\mathcal{K} - (\mathcal{T}_k)' \, , \\
a^2\kappa\left(\delta\tilde{T}^0_0 - \delta\tilde{T}^n_n\right) &= a^2\left(\delta\tilde{G}^0_0 - \delta\tilde{G}^n_n\right) = \nabla^2\tilde{h}_{00} \\
&\quad + 3a\left(\frac{1}{a}\mathcal{H}\right)'\tilde{h}_{00} + \frac{2}{a}\left(a\mathcal{K}\right)' \, ,
\end{aligned}$$

and finally

$$\begin{aligned}
a^2\kappa\left(\delta\tilde{T}^l_k - \delta^l_k\delta\tilde{T}^n_n\right) &= a^2\delta^l_k = -\nabla^2\tilde{\underset{T}{h}}{}^l_k + k\tilde{\underset{T}{h}}{}^l_k + \frac{1}{2a^2}\left[a^2\left(\tilde{\underset{T}{h}}{}^l_k\right)'\right]' \\
&\quad + f^{lm}\left(\nabla_{(m}\mathcal{T}_{k)} - f_{mk}\nabla_n\mathcal{T}^n\right) \\
&\quad - \frac{1}{a^2}f^{lm}\left[a^2\left(\nabla_{(m}\tilde{h}_{k)0} - f_{mk}\nabla_n\tilde{h}^n_0\right)\right]' \\
&\quad + f^{lm}\left(\nabla_{mk} - f_{mk}\nabla^2\right)\left(\tilde{h}_{00} - \tilde{h}^n_n\right) \, .
\end{aligned}$$

ne case of perfect fluid perturbations with local coordinate velocity compo

$$\tilde{V}^k = \frac{dx^k(\eta)}{d\eta}$$

nave

$$a^2\kappa\delta\tilde{T}^0_0 = a^2\kappa\delta\rho \, , \quad a^2\kappa\delta\tilde{T}^0_k = 2\left(k + \mathcal{H}^2 - \mathcal{H}'\right)\left(-\tilde{V}_k + \tilde{h}_{k0}\right) \, ,$$

Instanteneous („Machian") gauges in cosmology

Mach 1, Mach 2, 2*, Mach 3, 3*

differences in the time slicing of a perturbed universe, i.e. in the choice of time coordinate (possible changes by $\xi^0(x)$) i.e. in the choice of "snapshot"

The choice of spatial coordinates on given slices the same in all Machian gauges

$$(*) \quad \boxed{\mathcal{T}_k \equiv \nabla_\ell \underset{T}{\tilde{h}}{}^\ell{}_k = 0} \qquad \underset{T}{\tilde{h}}{}^\ell{}_k = \tilde{h}^\ell{}_k - \frac{1}{3}\delta^\ell_k \tilde{h}^m{}_m$$

$k, \ell, \ldots$ 1, 2, 3 spatial only

motivated by non-linear GR, numerical relativity „minimal-distortion shift vector" (Smarr....)

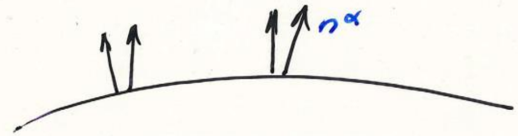
In Schwarzschild (for slicings ⊥ geodesics from ∞)

$$ds^2 = (1 - 2M/r)\, d\tau^2 - 2\sqrt{\frac{2M}{r}}\, dr\, d\tau - dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Mach 1

(*) + constant mean $\overbrace{\text{curvature}}^{\text{extrinsic}}$ slicing
(GR, QG, ...)

$\Longleftrightarrow$ uniform Hubble expansion (Bardeen)



$t = \text{const}$

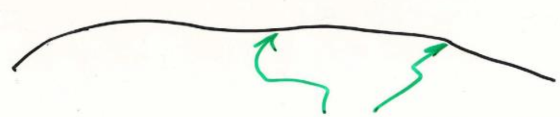
$\delta n^\alpha{}_{;\alpha} = 0 \Rightarrow$

$$\boxed{\mathcal{K} = \frac{3}{2}\dot{a}\tilde{h}_{00} + \frac{1}{2}a\dot{\tilde{h}}^m{}_m - \mathcal{P} = 0}$$

$$\underbrace{\mathcal{P} \equiv \nabla_\ell \tilde{h}^\ell_0} \qquad (= f(t))$$

Mach 2

(*) + constant mean intrinsic (scalar) curvature slicing



$t = \text{const}$

$$\mathcal{R} = \bar{\mathcal{R}} + \delta\mathcal{R} \qquad \bar{\mathcal{R}} = -\frac{6}{a^2}\underbrace{K}_{\pm 1}$$

<0 for k=+1 (signature!)

$$\boxed{0 = \delta\mathcal{R} = -\frac{2}{3a^2}\left(\nabla^2\tilde{h}^m_m + 3K\tilde{h}^m_m\right)} + \underbrace{\frac{1}{a^2}\nabla_m\mathcal{T}^m}_{=0}$$

But if the velocities of "heavenly bodies" are given, solutions are unique even in $K=+1$ (S^3) case:

$\delta T^{\mu}{}_{\nu}$ ... for perfect fluid

$\delta T_0{}^0 = \delta\rho$ $\qquad$ $\delta T_i{}^0 = \underbrace{(\bar\rho + \bar p)}_{\text{background}} (h_{i0} + V_i)$

velocity of matter

$$U^\mu = \bar U^\mu + \delta U^\mu = (1 - \tfrac{1}{2}h_{00}, V^i)$$

$$V^i = \frac{dx^i}{dt}$$

$$\nabla^2 \tilde h_{i0} + 2K \tilde h_{i0} = 2a^2\varkappa(\bar\rho + \bar p)(\tilde h_{i0} + \tilde V_i)$$

no Killing's can be added as for soltn's of homogeneous eq.

$$\nabla^2 \tilde h_{i0} + 2K \tilde h_{i0} = 0 \quad !$$

Motions in a closed universe do provide a complete determination of h_{00}, ∇h_{00}, h_{0m}, $\dot h_{0m}$

$\Rightarrow$ LIFs (Mach again)

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Cosmological perturbation theory, instantaneous gauges, and local inertial frames

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Linear perturbations of Friedmann-Robertson-Walker universes with any curvature and cosmological constant are studied in a general gauge without decomposition into harmonics. Desirable gauges are selected as those which embody best Mach's principle: in these gauges local inertial frames can be determined instantaneously via the perturbed Einstein field equations from the distributions of energy and momentum in the universe. The inertial frames are identified by their "accelerations and rotations" with respect to the cosmological frames associated with the "Machian gauges." In closed spherical universes, integral gauge conditions are imposed to eliminate motions generated by the conformal Killing vectors. The meaning of Traschen's integral-constraint vectors is thus elucidated. For all three types of Friedmann-Robertson-Walker universes the Machian gauges admit much less residual freedom than the synchronous or generalized harmonic gauge. Mach's principle is best exhibited in the Machian gauges in closed spherical universes. Independent of any Machian motivation, the general perturbation equations and discussion of gauges are useful for cosmological perturbation theory.

REMBRANDT FAUST

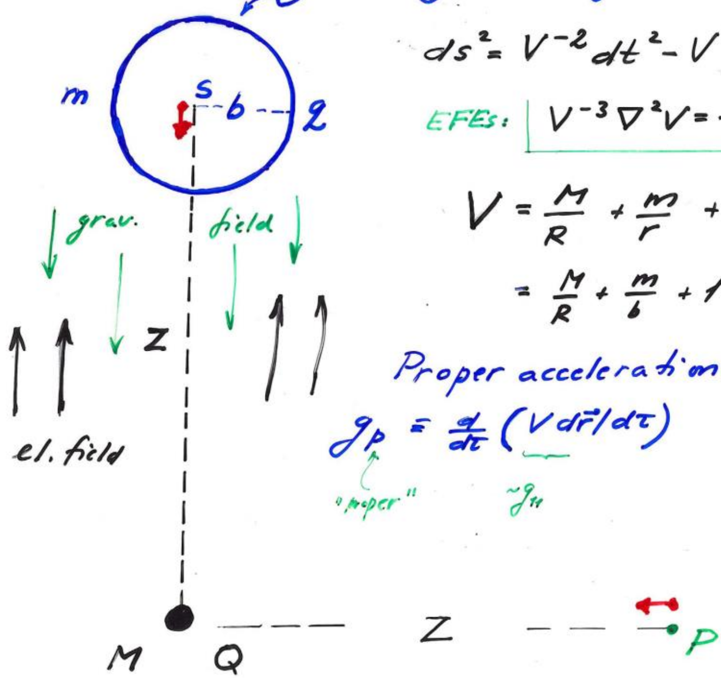
On 'Accelerated' Inertial Frames in Gravity and Electromagnetism

D. Lynden-Bell, J. Bi., J. Katz (Ann. of Phys. 2000)

- uniformly accelerated charged insulating spherical shell ... induction of field inside ...

In gravity – „linear" dragging (recall Einstein in Prague)
In static situation using „conformastats" spacetimes („electrically counteropposed dust")

has a significant grav. potential inside



$$ds^2 = V^{-2}dt^2 - V^2(dx^2 + dy^2 + dz^2)$$

EFEs: $\boxed{V^{-3}\nabla^2 V = -\frac{1}{2}\kappa\rho}$

$$V = \frac{M}{R} + \frac{m}{r} + 1 \qquad r \geq b \qquad {}^{*)}$$

$$= \frac{M}{R} + \frac{m}{b} + 1 \qquad r \leq b$$

Proper accelerations at S ↔ P

$$g_p = \frac{d}{dt}\left(\underbrace{V}_{\sim g_{11}} d\vec{r}/d\tau\right)$$

„proper"

*) R, r coord. distance from M, m

$$\boxed{\frac{(g_p)_{in}}{(g_p)_{out}} = \left(\frac{V_{in}}{V_{out}}\right)^{-2} = \left(\frac{1+\frac{M}{Z}}{1+\frac{M}{Z}+\frac{m}{b}}\right)^{2}}$$

The reduction of acceleration inside can be very large – if $\frac{m}{b}$ is large – and this can be since b can be as small as we like

$b = 0 \rightsquigarrow$ extreme RN black hole $(r = r_{schw} - m)$

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