

Invariantni sym. polynomy

Liouv. alg. $\mathfrak{g}M$ Ad_{g^x} $ad_{x^x} = X^x c_{F^x}$

velst. budl. EM T_{A^B} $t_{x^B}^A = X^x t_{F^B}^A$

$X^A_B = X^x t_{F^B}^A$ \mathfrak{A} A A
 \mathfrak{F} F F

1) multi. izobr. stupne \mathfrak{z}

$P(x_1, \dots, x_n) = P_{\alpha_1, \dots, \alpha_n} x_1^{\alpha_1} \dots x_n^{\alpha_n}$

$P(x_1, \dots, x_n) = P_{A_1, \dots, A_n} x_{B_1}^{A_1} \dots x_{B_n}^{A_n}$

$P_{\alpha_1, \dots, \alpha_n} = t_{\alpha_1, B_1}^{A_1} \dots t_{\alpha_n, B_n}^{A_n} P_{A_1, \dots, A_n}$

2) symetrie

$P_{\alpha_1, \dots, \alpha_n} = P_{\alpha_{\sigma(1)}, \dots, \alpha_{\sigma(n)}}$ σ permut.
 $P(x_1, \dots, x_n) = P(x_{\sigma(1)}, \dots, x_{\sigma(n)})$

3) polynomy

\mathfrak{z} -mocien $P(x) = P(x_1, \dots, x_n)$
 $P(x) = P(x_1, \dots, x_n)$

$P(x_1, \dots, x_n) = \frac{1}{\mathfrak{z}!} \frac{\partial^{\mathfrak{z}}}{\partial \tau_1 \dots \partial \tau_{\mathfrak{z}}} P(\tau_1 x_1 + \dots + \tau_{\mathfrak{z}} x_{\mathfrak{z}})$

$P(x) = \sum_{\mathfrak{z}} P_{\mathfrak{z}}(x)$

$(P \circ Q)(x) = P(x) Q(x)$

$(P \circ Q)_{\alpha_1, \dots, \alpha_{p+q}} = P_{\alpha_1, \dots, \alpha_p} Q_{\alpha_{p+1}, \dots, \alpha_{p+q}}$

4) invariant-ost vnci dca gany

$X \rightarrow \tilde{X} = T_{m_{\tau}} \cdot X \cdot T_{m_{\tau}}^{-1} \approx X + \tau [M, X]$
 $(1 + \tau M) (1 - \tau M)$ M^x

$X \rightarrow \tilde{X} = Ad_{m_{\tau}} \cdot X \approx X + \tau ad_M X$
 $\tau [M, X]$

M^z $M^A_B = M^z t_{A^B}$
 $P(\tilde{x}_1, \tilde{x}_2) = P(x_1, x_2)$

$P(\tilde{x}) = P(x)$
 $M P = 0$ $M P = 0$

$0 = \tau [P(M \cdot x_1, x_2, \dots, x_n) + P(x_1, M \cdot x_2, \dots) + \dots + P(x_1, \dots, M \cdot x_n)]$

$0 = P_{\alpha_1, \dots, \alpha_n} M^{\alpha_1} + P_{\alpha_1, \dots, \alpha_n} M^{\alpha_2} + \dots + P_{\alpha_1, \dots, \alpha_n} M^{\alpha_n}$
 $0 = M P_{\alpha_1, \dots, \alpha_n}$ $M P = 0$

Invariantni sym. polynomy

Linearna alga \mathfrak{g} M Ad_h^α $ad_X^\alpha = X^k c_{k\alpha}^\alpha$

vekt. bund. EM $T_{h, E}^A$ $t_X^A = X^k t_{F, E}^A$

$$X^A_B = X^k t_{F, E}^A \quad \begin{matrix} \exists & A & A \\ \neq & F & F \end{matrix}$$

1) multi. izob. stupnje α

$$P(x_1, \dots, x_n) = P_{\alpha_1, \dots, \alpha_n} x_1^{\alpha_1} \dots x_n^{\alpha_n}$$

$$P(X_1, \dots, X_n) = P_{A_1, \dots, A_n}^{B_1, \dots, B_n} X_{1, B_1}^{A_1} \dots X_{n, B_n}^{A_n}$$

$$P_{\alpha_1, \dots, \alpha_n} = t_{\alpha_1, B_1}^{A_1} \dots t_{\alpha_n, B_n}^{A_n} P_{A_1, \dots, A_n}^{B_1, \dots, B_n}$$

2) simetrični

$$P_{\alpha_1, \dots, \alpha_n} = P_{\alpha_1, \dots, \alpha_n}$$

$$P(X_1, \dots, X_n) = P(X_{B_1}, \dots, X_{B_n}) \quad \sigma \text{ permut.}$$

3) polynomy

k -mocina

$$P(X) = P(X_1, \dots, X_k)$$

$$P(X) = P(X_1, \dots, X_k)$$

$$P(X_1, \dots, X_k) = \frac{1}{k!} \frac{\partial^k}{\partial \tau_1 \dots \partial \tau_k} P(\tau_1 X_1 + \dots + \tau_k X_k)$$

$$P(X) = \sum_k P_k(X)$$

P_k k -mocina

$$(P \circ Q)(X) = P(X) Q(X)$$

$$(P \circ Q)_{\alpha_1, \dots, \alpha_{p+q}} = P_{(\alpha_1, \dots, \alpha_p)} Q_{(\alpha_{p+1}, \dots, \alpha_{p+q})}$$

M
 e \nearrow m_τ 4) invariant - ost n i e g g g

$$X \rightarrow \tilde{X} = T_{m_\tau} \cdot X \cdot T_{m_\tau}^{-1} \approx X + \tau [M, X]$$

$(\mathbb{1} + \tau M)$ $(\mathbb{1} - \tau M)$ M^x

$$\mathcal{X} \rightarrow \tilde{\mathcal{X}} = \text{Ad}_{m_\tau} \cdot \mathcal{X} \approx \mathcal{X} + \tau \text{ad}_M \mathcal{X}$$

$\tau [M, \mathcal{X}]$

$$M^a \quad M^A_B = M^a t^A_B$$

$$P(\tilde{X}_1, \tilde{X}_2) = P(X_1, X_2)$$

$$P(\tilde{X}) = P(X)$$

$$M P = 0 \quad M P = 0$$

$$0 = \tau [P(M \cdot X_1, X_2, \dots, X_n) + P(X_1, M \cdot X_2, \dots) + \dots + P(X_1, \dots, M \cdot X_n)]$$

$$0 = P_{\alpha_2 \dots \alpha_n} M^{\alpha_1} + P_{\alpha_1 \alpha_2 \dots \alpha_n} M^{\alpha_2} + \dots + P_{\alpha_1 \alpha_2 \dots \alpha_n} M^{\alpha_n}$$

$$0 = M P_{\alpha_1 \dots \alpha_n} \quad M P = 0$$

$$P(\tilde{X}_1, \tilde{X}_2) = P(X_1, X_2)$$

$$0 = P([M, X_1], X_2, \dots) + P(X_1, [M, X_2], \dots) + \dots$$

$$= P_{\substack{K A_2 \dots \\ L B_2 \dots}} (M_N^L X_{1K}^N - X_{1N}^L M_K^N) X_{2B_2}^{B_2} + \dots$$

$$+ P_{\substack{A_1 K \dots \\ B_1 L \dots}} X_{1A_1}^{B_1} (M_N^L X_{2K}^N - X_{2N}^L M_K^N) + \dots$$

$$0 = P_{\substack{A_1 A_2 \dots \\ L B_2 \dots}} M_{B_1}^L - P_{\substack{K A_2 \dots \\ B_1 B_2 \dots}} M_K^{B_1} + P_{\substack{A_1 A_2 \dots \\ B_1 L \dots}} M_{B_2}^L - P_{\substack{A_1 K \dots \\ B_1 B_2 \dots}} M_K^{B_2}$$

$$0 = M P_{\substack{A_1 A_2 \dots \\ B_1 B_2 \dots}} \quad M P = 0$$

Invariantní sym. polynomy

5) konstantnost polynomu

$$D_m C_{\mathbb{F}^k}^k = 0 \quad D_m t_{\mathbb{R}^A}^{\mathbb{R}^B} = 0$$

$$DP = 0 \quad DP = 0$$

$$\partial P = 0 \quad DP = \partial P + AP$$

$$A_{\mathbb{R}^k}^{\mathbb{R}^k} = \mathbb{R}^k C_{\mathbb{F}^k}^k$$

$$\text{str}_2(X_1, \dots, X_k) = \frac{1}{2!} \sum_G \text{tr}(X_{G_1} X_{G_2} \dots X_{G_k})$$

$$\text{str}_2(X) = \text{tr}(X^2)$$

$$\text{str}_2(X) \quad \text{str}_2 \propto K_{\mathbb{R}^2}$$

6) zobecnění na $\Lambda^p \otimes EM$

$$\chi_{m_1, \dots, m_p}^{\alpha} \quad X_{m_1, \dots, m_p}^{\mathbb{R}^A} \quad \mathbb{R}$$

$$\hat{P}(\chi_1, \chi_2, \dots) \equiv P(\chi_1 \wedge \chi_2 \wedge \dots)$$

$$= P_{\alpha_1 \alpha_2 \dots} \chi_1^{\alpha_1} \wedge \chi_2^{\alpha_2} \wedge \dots$$

\wedge skew-symetrický $\omega \wedge \sigma = (-1)^{pq} \sigma \wedge \omega$

$\hat{P}_2(X)$ je metrický pro \mathbb{R} i \mathbb{C}

$$\hat{P}(X)$$

4) invariantnost vůči aksi group

$$X \rightarrow \tilde{X} = T_{m_\tau} \cdot X \cdot T_{m_\tau}^{-1} \approx X + \tau [M, X]$$

$(\mathbb{1} + \tau M) \quad (\mathbb{1} - \tau M) \quad M_{\mathbb{F}^k}$

$$\chi \rightarrow \tilde{\chi} = \text{Ad}_{m_\tau} \cdot \chi \approx \chi + \tau (\text{ad}_M) \chi$$

$\tau [M, \chi]$

$$M_{\mathbb{R}^k} \quad M_{\mathbb{R}^A}^{\mathbb{R}^B} = M_{\mathbb{F}^k}^{\mathbb{R}^A} t_{\mathbb{R}^B}^{\mathbb{R}^A}$$

$$P(\tilde{\chi}_1, \tilde{\chi}_2) = P(\chi_1, \chi_2)$$

$$P(\tilde{X}) = P(X) \quad P(\tilde{\chi}) = P(\chi)$$

$$MP = 0$$

$$M^{\mathbb{R}^k} P = 0$$

$$M_{\mathbb{R}^B}^{\mathbb{R}^A} = M_{\mathbb{F}^k}^{\mathbb{R}^A} t_{\mathbb{R}^B}^{\mathbb{R}^A}$$

$$M_{\mathbb{R}^B}^{\mathbb{R}^k} = M_{\mathbb{F}^k}^{\mathbb{R}^k} C_{\mathbb{F}^k}^{\mathbb{R}^k}$$

M
 e \nearrow m_τ
 4) invariant - ost n'ici adici grupy

$$X \rightarrow \tilde{X} = T_{m_\tau} \cdot X \cdot T_{m_\tau}^{-1} \approx X + \tau [M, X]$$

$(\mathbb{1} + \tau M)$ $(\mathbb{1} - \tau M)$ M^α

$$\mathcal{X} \rightarrow \tilde{\mathcal{X}} = \text{Ad}_{m_\tau} \cdot \mathcal{X} \approx \mathcal{X} + \tau \text{ad}_M \mathcal{X}$$

$\tau [M, \mathcal{X}]$

$$M^\alpha \quad M_{\underline{B}}^{\underline{A}} = M^\alpha t_{\alpha \underline{B}}^{\underline{A}}$$

$$P(\tilde{X}_1, \tilde{X}_2) = P(X_1, X_2)$$

$$P(\tilde{X}) = P(X) \quad P(\tilde{\mathcal{X}}) = P(\mathcal{X})$$

$$M \parallel P = 0$$

$$M \parallel P = 0$$

$$M_{\underline{B}}^{\underline{A}} = M^k t_{\underline{F} \underline{B}}^{\underline{A}}$$

$$M_{\underline{B}}^{\alpha} = M^F c_{\underline{F} \underline{B}}^{\alpha}$$

Invariantni sym. polynomy

5) konstantnost polynoma

$$D_m C_{\mathbb{F}}^{\mathbb{R}} = 0 \quad D_m t_{\mathbb{R}}^{\mathbb{D}} = 0$$

$$DP = 0 \quad DP = 0$$

$$\partial P = 0 \quad DP = \partial P + \mathbb{A}P$$

$$A_{\mathbb{F}}^{\mathbb{R}} = \mathbb{R}^{\mathbb{R}} C_{\mathbb{F}}^{\mathbb{R}}$$

$$\text{str}_2(X_1, \dots, X_k) = \frac{1}{k!} \sum_G \text{tr}(X_{G_1} X_{G_2} \dots X_{G_k})$$

$$\text{str}_2(X) = \text{tr}(X^2)$$

$$\text{str}_2(X) \quad \text{str}_2 \propto K_{\mathbb{R}\mathbb{B}}$$

6) zobecnění na $\Lambda^p \otimes EM$

$$X_{m_1 \dots m_p}^\alpha$$

$$X_{m_1 \dots m_p}^{\mathbb{A}} \quad \mathbb{B}$$

$$\hat{P}(X_1, X_2, \dots) \equiv P(X_1 \uparrow, X_2 \uparrow, \dots)$$

$$= P_{\alpha_1 \alpha_2 \dots} X_1^{\alpha_1} \wedge X_2^{\alpha_2} \wedge \dots$$

\wedge skew-symetrický $\omega \wedge \sigma = (-1)^{pq} \sigma \wedge \omega$
 $\begin{matrix} p & q \end{matrix}$

$\hat{P}_\alpha(X)$ je metrický pro \mathbb{K} sudé

$$\hat{P}(X)$$

Chern-Weilova věta

D na gM resp. EM

kovnost \tilde{F}_{min} F_{min}^A

inv. sym. pol P P

1) $\hat{P}(F)$ resp. $\hat{P}(F)$ jsou uzavřené

$$d\hat{P}(F) = 0 \quad d\hat{P}(F) = 0$$

$$2) \quad D \quad F \quad F \quad \tilde{D} \quad \tilde{F} \quad \tilde{F}$$

$$\hat{P}(\tilde{F}) - \hat{P}(F) = dTP(\tilde{D}, D)$$

$$\hat{P}(\tilde{F}) - \hat{P}(F) = dTP(\tilde{D}, D)$$

TP forma transgrese

$$d\hat{P}(F) = D \wedge \hat{P}(F) = 0$$

hom. poly. st. $= D \wedge P(F, F, \dots)$

$$= \underbrace{(D \wedge P)}_0(F, F, \dots) + \underbrace{\hat{P}(D \wedge F, F, \dots)}_0 + \hat{P}(F, \underbrace{D \wedge F}_0, \dots)$$

$$D \wedge (P_{\alpha_1, \alpha_2} \tilde{F}_{k_1, l_1}^{\alpha_1} \wedge \tilde{F}_{k_2, l_2}^{\alpha_2} \wedge \dots)$$

$$\tilde{D}, D \quad \Delta = \tilde{D} - D$$

$$D_\tau = \tau \tilde{D} + (1-\tau)D = D + \tau\Delta = \tilde{D} - (1-\tau)\Delta$$

$$F_\tau = F + \tau D \wedge \Delta + \tau^2 \Delta^{\wedge 2} = \tau(F + D \wedge \Delta + \Delta^{\wedge 2}) + (1-\tau)F - \tau(1-\tau)\Delta^{\wedge 2}$$

$$= \tau \tilde{F} + (1-\tau)F - \tau(1-\tau)\Delta^{\wedge 2}$$

$$\frac{\partial}{\partial \tau} D_\tau = \Delta \quad \frac{\partial}{\partial \tau} F_\tau = D \wedge \Delta + 2\tau \Delta^{\wedge 2} = D \wedge \Delta + [\tau \Delta, \Delta] = D_\tau \wedge \Delta$$

$$\frac{\partial}{\partial \tau} \hat{P}(F_\tau) = k \hat{P}\left(\frac{\partial}{\partial \tau} F_\tau, F_\tau, \dots\right) = k \hat{P}(D_\tau \wedge \Delta, F_\tau, \dots) = k D_\tau \wedge \hat{P}(\Delta, F_\tau, \dots) = k d\hat{P}(\Delta, F_\tau, \dots)$$

hom. poly. st.

$$[\Delta, \Delta] \quad \Delta_m^A \wedge \Delta_n^B \quad [\Delta_m, \Delta_n]^A \quad \Delta^{\wedge 2}$$

$$[\Delta_m, \Delta_n] = \Delta_m \wedge \Delta_n - \Delta_n \wedge \Delta_m = \Delta_m \wedge \Delta_n$$

$$[A_m, B_n] = A_m \cdot B_n - B_n \cdot A_m \neq A_m \wedge B_n$$

$$A_m \wedge B_n = A_m \cdot B_n - A_n \cdot B_m$$

$$[A_m \wedge B_n] = A_m \wedge B_n - B_n \wedge A_m =$$

$$-A_m \cdot B_n - A_n \cdot B_m - B_n \cdot A_m + B_m \cdot A_n$$

$$[\Delta_m, \Delta_n] = \Delta_m \wedge \Delta_n - \Delta_n \wedge \Delta_m = \Delta_m \wedge \Delta_n$$

$$[\Delta_m \wedge \Delta_n] = 2[\Delta_m, \Delta_n] = 2\Delta_m \wedge \Delta_n$$

$$P'(\Delta, F) = \sum_k k P_k(\Delta, F, F, \dots)$$

$$= \sum_k (P_k(\Delta, F, F, \dots) + P_k(F, \Delta, F, \dots) + P_k(F, F, \Delta, \dots))$$

$$\frac{\partial}{\partial \tau} \hat{P}(F_\tau) = d\hat{P}'(\Delta, F_\tau)$$

$$\hat{P}(\tilde{F}) - \hat{P}(F) = d \int_0^1 \hat{P}'(\Delta, F_\tau) d\tau$$

$$TP(\tilde{D}, D)$$

$$TP(\tilde{D}, D) = \int_0^1 P'(\Delta, F_\tau) d\tau$$

Chern-Weilova věta

D má g M řady EM

kovnost \underline{F}^{α} \underline{F}^A
 \underline{m} \underline{B}

inv. sym. pol. P P

\Downarrow

1) $\hat{P}(\hat{F})$ řady $\hat{P}(F)$ jsou uzavřené

$$d\hat{P}(\hat{F}) = 0 \quad d\hat{P}(F) = 0$$

2) $D \quad \hat{F} \quad F \quad \tilde{D} \quad \tilde{F} \quad \tilde{F}$

$$\hat{P}(\tilde{F}) - \hat{P}(\hat{F}) = dTP(\tilde{D}, D)$$

$$\hat{P}(\tilde{F}) - \hat{P}(F) = dTP(\tilde{D}, D)$$

TP forma transgrese

$$d\hat{P}(\hat{F}) = D \wedge \hat{P}(\hat{F}) = 0$$

from poly at $z = D \wedge P(\hat{F}, \hat{F}, \dots)$

$$= \underbrace{(D \wedge P)}_0(\hat{F}, \hat{F}, \dots) + \hat{P}(\underbrace{D \wedge \hat{F}}_0, \hat{F}, \dots) + \hat{P}(\hat{F}, \underbrace{D \wedge \hat{F}}_0, \dots) + \dots$$

$$D_m \wedge \left(P_{\alpha_1 \alpha_2} \hat{F}_{k_1 l_1}^{\alpha_1} \wedge \hat{F}_{k_2 l_2}^{\alpha_2} \wedge \dots \right)$$

$$[\Delta, \Delta] \quad \Delta_m^A \quad [\Delta_k, \Delta_n]^A \quad \Delta^{12}$$

$$[\Delta_m, \Delta_n] = \Delta_m \cdot \Delta_n - \Delta_n \cdot \Delta_m = \Delta_m \wedge \Delta_n$$

$$[A_m, B_n] = A_m \cdot B_n - B_n \cdot A_m \neq A_m \wedge B_n$$

$$A_m \wedge B_n = A_m \cdot B_n - A_n \cdot B_m$$

$$[A_m, B_n] = A_m \wedge B_n - B_n \wedge A_m =$$

$$= A_m \cdot B_n - A_n \cdot B_m - B_n \cdot A_m + B_m \cdot A_n$$

$$[\Delta_m, \Delta_n] = \Delta_m \cdot \Delta_n - \Delta_n \cdot \Delta_m = \Delta_m \wedge \Delta_n$$

$$[\Delta_m, \Delta_n] = 2[\Delta_m, \Delta_n] = 2\Delta_m \wedge \Delta_n$$

$$\begin{aligned}
 P'(\Delta, x) &= \sum_{\mathcal{R}} \mathcal{R} P_{\mathcal{R}}(\Delta, x_1, x_1, \dots) \\
 &= \sum_{\mathcal{R}} (P_{\mathcal{R}}(\Delta, x_1, x_1, \dots) + P_{\mathcal{R}}(x_1, \Delta, x_1, \dots) + P_{\mathcal{R}}(x_1, x_1, \Delta, \dots))
 \end{aligned}$$

$$\frac{\partial}{\partial \tau} \hat{P}(F_{\tau}) = d \hat{P}'(\Delta, F_{\tau})$$

$$\hat{P}(\tilde{F}) - \hat{P}(F) = \underbrace{d \int_0^1 \hat{P}'(\Delta, F_{\tau}) d\tau}_{TP(\tilde{D}, D)}$$

$$TP(\tilde{D}, D) = \int_0^1 P'(\Delta, F_{\tau}) d\tau$$

Charakteristické třídy

Primový polynom

$$\mu = [\hat{P}(F)] \in H(M)$$

- kohomol. gr.
- mezivěsí na D
- P homogenní at $2k \rightarrow 2k$ -formy
- P nehomogenní μ_{2k} homog. kom.

$$\int_M \underbrace{\mu \wedge \nu \wedge \dots}_{\text{stupně d}} = \int_M \hat{P}(F) \wedge \hat{Q}(F) \wedge \dots$$

$$\int_M (\hat{P}(F) + dx) \wedge \hat{Q}(F) \wedge \dots = \int_M \dots + \int_M d(\alpha \wedge \hat{Q}(F) \wedge \dots)$$

Chernovy charakteristické třídy

EM komplex vekt. b. a herm. str.

$$\hat{C}(F) = \det \left(\mathbb{1} + \frac{i\alpha}{2\pi} F \right)$$

$$C = [\hat{C}(F)] \quad \text{nehomogen}$$

$$\hat{C}(F) = 1 + \alpha \hat{C}_1(F) + \alpha^2 \hat{C}_2(F) + \dots$$

$$C_2 = [\hat{C}_2(F)]$$

$$\det(\mathbb{1} + \lambda F) = \sum_{A_1, B_1}^N \sum_{A_2, B_2}^N \dots \sum_{A_N, B_N}^N (\mathbb{1} + \lambda F)_{B_1}^{A_1} \dots (\mathbb{1} + \lambda F)_{B_N}^{A_N}$$

$$= \sum_{k=0}^N \lambda^k \sum_{A_1, B_1, C_1, \dots, A_k, B_k, C_k}^N \binom{N}{k} F_{B_1}^{A_1} \wedge \dots \wedge F_{B_k}^{A_k}$$

$$= \sum_{k=0}^N \lambda^k F_{[A_1} \wedge \dots \wedge F_{B_k]}$$

$$= 1 + \lambda F^A_A$$

$$+ \frac{1}{2} \lambda^2 (F^A_A \wedge F^B_B - F^A_{B_1} F^B_{A_1})$$

$$+ \frac{1}{3!} \lambda^3 (F^A_{A_1} F^B_{B_1} F^C_{C_1} + F^D_{B_1} F^B_{C_1} F^C_{A_1} + F^A_{C_1} F^B_{A_1} F^C_{B_1} - F^A_{A_1} F^B_{C_1} F^C_{B_1} - F^A_{B_1} F^B_{A_1} F^C_{C_1} - F^A_{C_1} F^B_{B_1} F^C_{A_1})$$

+ ...

$$= 1 + \lambda \operatorname{tr} F + \frac{1}{2} \lambda^2 ((\operatorname{tr} F)^2 - \operatorname{tr} F^2)$$

$$+ \frac{1}{3!} \lambda^3 ((\operatorname{tr} F)^3 + 2 \operatorname{tr} F^3 - 3 \operatorname{tr} F \wedge (\operatorname{tr} F^2))$$

+ ...

Charakteristické třídy

P iho sym. polynom

$$\mu = [\hat{P}(F)] \in H(M)$$

- kochanol. gr.
- mezávisti me D
- P homog. st $k \rightarrow 2k$ -formy
- P nehomog. μ_k homog. kom.

$$\int_M \underbrace{\mu \wedge \nu \wedge \dots}_{\text{stupně } d} = \int_M \hat{P}(F) \wedge \hat{Q}(F) \wedge \dots$$

$$\int_M (\hat{P}(F) + d\alpha) \wedge \hat{Q}(F) \wedge \dots = \int_M \dots + \int_M d(\alpha \wedge \hat{Q}(F) \wedge \dots)$$

$$P(x) = P(x, \dots, x)$$

$$P(x) = P(x_1, \dots, x_n) = P_{x_1 \dots x_n} x^{\alpha_1} \dots x^{\alpha_n}$$

$$x^{\beta} \rightarrow x^{\alpha} \quad \mathbb{E}_{\alpha}^{\beta}$$

Chernovy charakteristické třídy

EM komplex věst. b. a herm. str.

$$\hat{C}(F) = \det \left(\underline{1} + \frac{i\alpha}{2\pi} F \right)$$

$C = [\hat{C}(F)]$ *nehomogení*

$$\hat{C}(F) = 1 + \alpha \hat{C}_1(F) + \alpha^2 \hat{C}_2(F) + \dots$$

$$C_2 = [\hat{C}_2(F)]$$

$$\det(\underline{1} + \lambda F) = \sum_{A_1, \dots, A_N}^{[N]} \binom{N}{A_1, \dots, A_N} (\underline{1} + \lambda F)_{B_1}^{A_1} \wedge \dots \wedge (\underline{1} + \lambda F)_{B_N}^{A_N}$$

$$= \sum_{s=0}^N \lambda^s \sum_{A_1, \dots, A_s} \binom{N}{A_1, \dots, A_s} F_{B_1}^{A_1} \wedge \dots \wedge F_{B_s}^{A_s}$$

$$= \sum_{s=0}^N \lambda^s F_{[A_1} \wedge \dots \wedge F_{A_s]}$$

$$= 1 + \lambda F^A_A$$

$$+ \frac{1}{2} \lambda^2 (F^A_A \wedge F^B_B - F^A_B \wedge F^B_A)$$

$$+ \frac{1}{3!} \lambda^3 (F^A_A \wedge F^B_B \wedge F^C_C + F^A_B \wedge F^B_C \wedge F^C_A + F^A_C \wedge F^B_A \wedge F^C_B$$

$$- F^A_A \wedge F^B_C \wedge F^C_B - F^A_B \wedge F^B_A \wedge F^C_C - F^A_C \wedge F^B_B \wedge F^C_A)$$

+ ...

$$= 1 + \lambda \operatorname{tr} F + \frac{1}{2} \lambda^2 ((\operatorname{tr} F)^{\wedge 2} - \operatorname{tr} F^{\wedge 2})$$

$$+ \frac{1}{3!} \lambda^3 ((\operatorname{tr} F)^{\wedge 3} + 2 \operatorname{tr} F^{\wedge 3} - 3 (\operatorname{tr} F) \wedge (\operatorname{tr} F^{\wedge 2}))$$

+ ...

Chernova charakteristika

$$\hat{Ch}(F) = \text{tr} \exp\left(\frac{i\lambda}{2\pi} F\right)$$

$$= \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} \text{tr} F^{A_k} = \sum_k \hat{Ch}_k(F)$$

$$ch = [\hat{Ch}(F)] \quad ch_k = [\hat{Ch}_k(F)]$$

$$\hat{Ch}_k(F) = \frac{\lambda^k}{k!} \text{tr}(F^{A_k}) = \frac{\lambda^k}{k!} \text{str}_k(F, \dots, F)$$

$$\text{str}_k(A_1, \dots, A_k) = \sum_{\mu \in S_k} \text{tr}(A_{\sigma_1} \dots A_{\sigma_k})$$

$$\text{str}_{k \times 1 \dots \times k} = t_{\begin{pmatrix} A_1 \\ \vdots \\ A_k \end{pmatrix}} t_{\begin{pmatrix} A_1 \\ \vdots \\ A_k \end{pmatrix}} \dots t_{\begin{pmatrix} A_k \\ \vdots \\ A_1 \end{pmatrix}}$$

$$\log \det(1 + \lambda F) = \text{tr} \log(1 + \lambda F)$$

$$\text{tr}(\lambda F - \frac{1}{2} \lambda^2 F^{A_2} + \frac{1}{3} \lambda^3 F^{A_3} - \dots)$$

$$= \sum_{k=1}^{\infty} (-1)^{k-1} \frac{\lambda^k}{k} \text{tr} F^{A_k}$$

$$= \sum_{k=1}^{\infty} (-1)^{k-1} (k-1)! \hat{Ch}_k(F)$$

$$= \hat{Ch}_1 - \hat{Ch}_2 + 2 \hat{Ch}_3 - \dots$$

$$\Rightarrow \log\left(1 + \sum_{k=1}^{\infty} \hat{C}_k\right) =$$

$$= (\hat{C}_1 + \hat{C}_2 + \hat{C}_3 + \dots) - \frac{1}{2} (\hat{C}_1 + \hat{C}_2 + \dots)^2 + \frac{1}{3} (\hat{C}_1 + \dots)^3 + \dots$$

$$= \hat{C}_1 + (\hat{C}_2 - \frac{1}{2} \hat{C}_1^2) + (\hat{C}_3 - \hat{C}_2 \hat{C}_1 + \frac{1}{3} \hat{C}_1^3) + \dots$$

$$\hat{Ch}_1 = \hat{C}_1 \quad \hat{Ch}_2 = -\hat{C}_2 + \frac{1}{2} \hat{C}_1^2$$

$$\hat{Ch}_3 = \frac{1}{2} \hat{C}_3 - \frac{1}{2} \hat{C}_2 \hat{C}_1 + \frac{1}{6} \hat{C}_1^3$$

$$\hat{C}(F) = 1 + \lambda F^A$$

$$+ \frac{1}{2} \lambda^2 (F^A_A \wedge F^B_B - F^A_B \wedge F^B_A)$$

$$+ \frac{1}{3!} \lambda^3 (F^A_A \wedge F^B_B \wedge F^C_C + F^A_B \wedge F^B_C \wedge F^C_A + F^A_C \wedge F^B_A \wedge F^C_B - F^A_A \wedge F^B_B \wedge F^C_C - F^A_B \wedge F^B_A \wedge F^C_C - F^A_C \wedge F^B_B \wedge F^C_A)$$

$$- 1 + \lambda \text{tr} F + \frac{1}{2} \lambda^2 ((\text{tr} F)^2 - \text{tr} F^{A_2})$$

$$+ \frac{1}{3!} \lambda^3 ((\text{tr} F)^3 + 2 \text{tr} F^{A_3} - 3(\text{tr} F) \wedge (\text{tr} F^{A_2}))$$

$$\hat{C}_1 = \hat{Ch}_1$$

$$\hat{C}_2 = -\hat{Ch}_2 + \frac{1}{2} \hat{Ch}_1^2$$

$$\hat{C}_3 = 2 \hat{Ch}_3 - \hat{Ch}_2 \wedge \hat{Ch}_1 + \frac{1}{6} \hat{Ch}_1^3$$

Chernova karakteristika

$$\hat{\text{Ch}}(F) = \text{tr} \exp\left(\frac{i\alpha}{2\pi} F\right)$$
$$= \sum_{q=0}^{\infty} \frac{\lambda^q}{q!} \text{tr} F^{Aq} = \sum_q \text{Ch}_q(F)$$

$$\text{ch} = [\hat{\text{Ch}}(F)] \quad \text{ch}_q = [\hat{\text{Ch}}_q(F)]$$

$$\hat{\text{Ch}}_q(F) = \frac{\lambda^q}{q!} \text{tr}(F^{Aq}) = \frac{\lambda^q}{q!} \text{str}_q(F, \dots, F)$$

$$\text{str}_q(A_1, \dots, A_q) = \sum_{\sigma \in S_q} \text{tr}(A_{\sigma_1} \cdots A_{\sigma_q})$$

$$\text{str}_{q \times 1 \cdots \times q} = t_{\alpha_1}^{A_1} t_{\alpha_2}^{A_2} \cdots t_{\alpha_q}^{A_q}$$

$$\begin{aligned}
 \hat{C}(F) &= 1 + \lambda F^A_A \\
 &+ \frac{1}{2} \lambda^2 (F^A_A \wedge F^B_B - F^A_B \wedge F^B_A) \\
 &+ \frac{1}{3!} \lambda^3 (F^A_A \wedge F^B_B \wedge F^C_C + F^A_B \wedge F^B_C \wedge F^C_A + F^A_C \wedge F^B_A \wedge F^C_B \\
 &\quad - F^A_A \wedge F^B_C \wedge F^C_B - F^A_B \wedge F^B_A \wedge F^C_C - F^A_C \wedge F^B_A \wedge F^C_A) \\
 &+ \dots \\
 &= 1 + \lambda \operatorname{tr} F + \frac{1}{2} \lambda^2 ((\operatorname{tr} F)^{\wedge 2} - \operatorname{tr} F^{\wedge 2}) \\
 &+ \frac{1}{3!} \lambda^3 ((\operatorname{tr} F)^{\wedge 3} + 2 \operatorname{tr} F^{\wedge 3} - 3 (\operatorname{tr} F) \wedge (\operatorname{tr} F^{\wedge 2}))
 \end{aligned}$$

$$\begin{aligned}
 \hat{C}_1 &= \hat{C} h_1 \\
 \hat{C}_2 &= -\hat{C} h_2 + \frac{1}{2} \hat{C} h_1^{\wedge 2} \\
 \hat{C}_3 &= 2 \hat{C} h_3 - \hat{C} h_2 \wedge \hat{C} h_1 + \frac{1}{6} \hat{C} h_1^{\wedge 3}
 \end{aligned}$$

$$\hat{\log} \det(\mathbb{1} + \lambda F) = \text{tr} \hat{\log}(\mathbb{1} + \lambda F)$$

$$\text{tr} \left(\lambda F - \frac{1}{2} \lambda^2 F^{\wedge 2} + \frac{1}{3} \lambda^3 F^{\wedge 3} - \dots \right)$$

$$= \sum_{k=1}^{\infty} (-1)^{k-1} \frac{\lambda^k}{k} \text{tr} F^{\wedge k}$$

$$= \sum_{k=1}^{\infty} (-1)^{k-1} (k-1)! \hat{\mathcal{C}}_k(F)$$

$$= \hat{\mathcal{C}}_1 - \hat{\mathcal{C}}_2 + 2 \hat{\mathcal{C}}_3 - \dots$$

$$\Rightarrow \hat{\log} \left(1 + \sum_{k=1}^{\infty} \hat{\mathcal{C}}_k \right) =$$

$$= (\hat{\mathcal{C}}_1 + \hat{\mathcal{C}}_2 + \hat{\mathcal{C}}_3 + \dots) - \frac{1}{2} (\hat{\mathcal{C}}_1 + \hat{\mathcal{C}}_2 + \dots)^{\wedge 2} + \frac{1}{3} (\hat{\mathcal{C}}_1 + \dots)^{\wedge 3} - \dots$$

$$= \hat{\mathcal{C}}_1 + \left(\hat{\mathcal{C}}_2 - \frac{1}{2} \hat{\mathcal{C}}_1^{\wedge 2} \right) + \left(\hat{\mathcal{C}}_3 - \hat{\mathcal{C}}_2^{\wedge 1} \hat{\mathcal{C}}_1 + \frac{1}{3} \hat{\mathcal{C}}_1^{\wedge 3} \right) + \dots$$

$$\hat{\mathcal{C}}_1 = \hat{\mathcal{C}}_1 \quad \hat{\mathcal{C}}_2 = -\hat{\mathcal{C}}_2 + \frac{1}{2} \hat{\mathcal{C}}_1^{\wedge 2}$$

$$\hat{\mathcal{C}}_3 = \frac{1}{2} \hat{\mathcal{C}}_3 - \frac{1}{2} \hat{\mathcal{C}}_2^{\wedge 1} \hat{\mathcal{C}}_1 + \frac{1}{6} \hat{\mathcal{C}}_1^{\wedge 3}$$

Pontrjaginovy charakt. třídy

AM reálný vektor. bundl s metrikou

kov. gr. $O(A, H)$

$$F = -F^T$$

$$\text{tr} F = 0 \quad Q = F \wedge F^T = -F^{\wedge 2}$$

$$\hat{P}_A(F) = \det^{\frac{1}{2}} \left(\mathbb{1} + \frac{Q}{(2\pi)^2} \right) =$$

$$= \det^{\frac{1}{2}} \left(\mathbb{1} - \frac{F}{2\pi} \right) = \det^{\frac{1}{2}} \left(\mathbb{1} + \frac{F}{2\pi} \right)$$

$$\mu = [\hat{P}_A(F)] \quad \text{tr} F^{\wedge 2k+1} = 0$$

$$\hat{P}_A(F) = \sum_{k=0}^{\infty} \hat{P}_{2k}(F) \quad \hat{P}_{2k+1}(F) = 0$$

Eulerova charakt. třída

$\mathbb{T}M$ tečnou bundle

metr. ∇_g $R_{ab}{}^m{}_n$

$\dim M = 2m$

$$\hat{P}_F \left(\frac{R}{2\pi} \right) \quad e = \left[\hat{P}_F \left(\frac{R}{2\pi} \right) \right]$$

$$\text{Pf}(X) = \frac{1}{2^m m!} \varepsilon_{a_1 b_1 \dots a_m b_m} X^{a_1 b_1} \dots X^{a_m b_m}$$

$$= X^{12} X^{34} \dots X^{2m-1, 2m} = \left\{ \begin{matrix} 1 \\ 2 \\ \dots \\ m \end{matrix} \right\}$$

$$= (\det X)^{\frac{1}{2}}$$

$$\hat{P}_F(R)_{a_1 b_1 \dots a_m b_m} = \frac{1}{2^m m!} \varepsilon_{k_1 l_1 \dots k_m l_m} R_{a_1 b_1}{}^{k_1 l_1} \wedge \dots \wedge R_{a_m b_m}{}^{k_m l_m}$$

$$= \frac{1}{2^m m!} \frac{1}{(2m)!} \varepsilon^{m_1 m_2 \dots m_{2m}} \varepsilon_{k_1 l_1 \dots k_m l_m} R_{m_1 m_2}{}^{k_1 l_1} \wedge \dots \wedge R_{m_{2m-1} m_{2m}}{}^{k_m l_m} \varepsilon_{a_1 b_1 \dots a_m b_m}$$

$$= \frac{1}{2^m m!} R_{k_1 l_1}{}^{k_2 l_2} \wedge \dots \wedge R_{k_m l_m}{}^{k_{m+1} l_{m+1}} \varepsilon_{a_1 b_1 \dots a_m b_m}$$

$$= \frac{(2m)!}{2^m m!} R_{[k_1 l_1} \dots R_{k_m l_m]} \varepsilon_{a_1 b_1 \dots a_m b_m}$$

Gauss-Bonnetova věta

$$\int_M e = \chi(M) = \text{index}_{\text{de Rham}}(M)$$

$$\begin{matrix} d & \delta & \Delta \\ d & + & \delta \end{matrix}$$

Pontrjaginovy charakt. třídy

AM reálný věst. bundl s metrikou

kal. gr. $O(A, H)$

$$F = -F^T$$

$$\text{tr} F = 0 \quad Q = F \wedge F^T = -F^{\wedge 2}$$

$$\begin{aligned} \hat{P}_A(F) &= \det^{\frac{1}{2}} \left(\mathbb{1} + \frac{Q}{(2\pi)^2} \right) = \\ &= \det^{\frac{1}{2}} \left(\mathbb{1} - \frac{F}{2\pi} \right) = \det^{\frac{1}{2}} \left(\mathbb{1} + \frac{F}{2\pi} \right) \end{aligned}$$

$$\nu = [\hat{P}_A(F)] \quad \text{tr} F^{\wedge 2k+1} = 0$$

$$\hat{P}_A(F) = \sum_{k=0}^{\infty} \hat{P}_{2k}(F) \quad \hat{P}_{2k+1}(F) = 0$$

Eulerova charakt. třída

$\mathbb{T}M$ tečný bundle

$$g_{ab} \quad \nabla_a \quad R_{ab}^{\quad c}$$

$$\dim M = 2m$$

$$\hat{P}f\left(\frac{R}{2\pi}\right) \quad e = \left[\hat{P}f\left(\frac{R}{2\pi}\right) \right]$$

$$Pf(X) = \frac{1}{2^m m!} \varepsilon_{a_1 b_1 \dots a_m b_m} X^{a_1 b_1} \dots X^{a_m b_m}$$

$$= X^{12} X^{34} \dots X^{2m-1, 2m} = \left\{ \begin{matrix} 1 \\ 2 \\ \dots \\ m \end{matrix} \right\} \left[\begin{matrix} 0 & \xi_1 & & \\ -\xi_1 & 0 & & \\ & & 0 & \xi_2 \\ & & -\xi_2 & 0 \\ & & & \dots \end{matrix} \right]$$
$$= (\det X)^{\frac{1}{2}}$$

$$\hat{P}_f(R)_{a_1 b_1 \dots a_m b_m} = \frac{1}{2_m^m} \varepsilon_{k_1 l_1 \dots k_m l_m} R_{a_1 b_1}^{k_1 l_1} \wedge \dots \wedge R_{a_m b_m}^{k_m l_m}$$

$$= \frac{1}{2_m^m} \frac{1}{(2m)!} \varepsilon_{m_1 m_2 \dots m_m} \varepsilon_{k_1 l_1 \dots k_m l_m} R_{m_1 m_2}^{k_1 l_1} \wedge \dots \wedge R_{m_{m-1} m_m}^{k_{m-1} l_{m-1}} \varepsilon_{a_1 b_1 \dots a_m b_m}$$

$$= \frac{1}{2_m^m} R_{k_1 l_1}^{k_1 l_1} \wedge \dots \wedge R_{k_m l_m}^{k_m l_m} \varepsilon_{a_1 b_1 \dots a_m b_m}$$

$$= \frac{(2m)!}{2_m^m} R_{[k_1 l_1} \dots R_{k_m l_m]} \varepsilon_{a_1 b_1 \dots a_m b_m}$$

Gauss-Bonnet in a nete

$$\int_M e = \chi(M) = \text{index}_{\text{deRham}}(M)$$

$d \delta \Delta$

$d-\delta$

$$m=1 \quad \text{dim} \mathcal{M} = 2$$

$$2\pi \epsilon_{ab} = \frac{1}{2} R_{kl}{}^{kl} \quad \epsilon_{ab} = K \epsilon_{ab}$$

$$\frac{1}{2\pi} \int_{\mathcal{M}} K \epsilon = \chi(\mathcal{M}) = b_0 - b_1 + b_2$$

$\stackrel{\text{Algebra}}{=} 1 - 0 + 1 = 2$

$$m=2 \quad \text{dim} \mathcal{M} = 4$$

$$(2\pi)^2 \epsilon_{abcd} = \frac{1}{8} (R_{kl}{}^{kl} \wedge R_{mn}{}^{mn}) \epsilon_{abcd}$$

$$= \frac{1}{8} (R_{kl}{}^{kl} R_{mn}{}^{mn} + R_{mn}{}^{kl} R_{kl}{}^{mn} - 4 R_{km}{}^{kl} R_{ln}{}^{mn})$$

$$= \frac{1}{8} (R^2 - 4 \text{Ric}^2 + \mathcal{R}^2) \epsilon_{abcd}$$

$$R_{klmn} R^{klmn} \quad \text{Ric}_{kl} \text{Ric}^{kl} \quad \mathcal{R} \mathcal{R}$$

$$\frac{1}{8} \frac{1}{(2\pi)^2} \int_{\mathcal{M}} (R^2 - 4 \text{Ric}^2 + \mathcal{R}^2) \epsilon =$$

$$= \chi(\mathcal{M}) = b_0 - b_1 + b_2 - b_3 + b_4$$

Lovelock gravitace

$$S_{\text{Lov}} = \sum_k \int_{\mathcal{M}} \alpha_k \mathcal{L}_k g^{\frac{1}{2}}$$

$$k < m = \text{dim} \mathcal{M} / 2$$

ϵ_{abcd}	dim 4	$m=2$	\mathcal{L}_0	\mathcal{L}_1	
			\wedge	\mathcal{H}	
		$m=3$	\mathcal{L}_0	\mathcal{L}_1	\mathcal{L}_2

$$\hat{\text{Pf}}(R)_{a_1 b_1 \dots a_m b_m} = \frac{1}{2^m m!} \epsilon_{k_1 l_1 \dots k_m l_m} R_{a_1 b_1}{}^{k_1 l_1} \wedge \dots \wedge R_{a_m b_m}{}^{k_m l_m}$$

$$= \frac{1}{2^m m!} \frac{1}{(2\pi)^m} \epsilon^{m_1 m_2 \dots m_m} \epsilon_{a_1 b_1 \dots a_m b_m} R_{m_1 m_2}{}^{k_1 l_1} \wedge \dots \wedge R_{m_{m-1} m_m}{}^{k_{m-1} l_{m-1}} \epsilon_{a_1 b_1 \dots a_m b_m}$$

$$= \frac{1}{2^m m!} R_{k_1 l_1}{}^{k_1 l_1} \wedge \dots \wedge R_{k_m l_m}{}^{k_m l_m} \epsilon_{a_1 b_1 \dots a_m b_m}$$

$$= \underbrace{\frac{(2m)!}{2^{2m} m!} R_{[k_1 l_1} \dots R_{k_m l_m]}}_{\mathcal{L}_m} \epsilon_{a_1 b_1 \dots a_m b_m}$$

$$m=1 \quad \dim M = 2$$

$$2\pi c_{ab} = \frac{1}{2} R_{kr}{}^{kr} \quad \varepsilon_{ab} = K \varepsilon_{ab}$$

$$\frac{1}{2\pi} \int_M K \varepsilon = \chi(M) = b_0 - b_1 + b_2$$

$$\stackrel{\text{Algebra}}{=} 1 - 0 + 1 = 2$$

$$m=2 \quad \dim M = 4$$

$$\begin{aligned} (2\pi)^2 \epsilon_{abcd} &= \frac{1}{8} (R_{kl}{}^{kl} \wedge R_{mn}{}^{mn}) \epsilon_{abcd} \\ &= \frac{1}{8} (R_{kl}{}^{kl} R_{mn}{}^{mn} + R_{mn}{}^{kl} R_{kl}{}^{mn} - 4 R_{km}{}^{kl} R_{ln}{}^{mn}) \epsilon_{abcd} \\ &= \frac{1}{8} (R^2 - 4 \text{Ric}^2 + R^2) \epsilon_{abcd} \end{aligned}$$

$$R_{klmn} R^{klmn} \quad \text{Ric}_{kl} \text{Ric}^{kl} \quad RR$$

$$\frac{1}{8} \frac{1}{(2\pi)^2} \int_M (R^2 - 4 \text{Ric}^2 + R^2) \epsilon =$$

$$= \chi(M) = b_0 - b_1 + b_2 - b_3 + b_4$$

Lovelock gravitace

$$S_{\text{Lov}} = \sum_k \int_M \alpha_k \mathcal{L}_k g^{\frac{1}{2}}$$

$$k < m = \text{dim} - 1/2$$

$$\underbrace{\frac{(2m)!}{2^{2m} m!} R_{[k_1 l_1} \dots R_{k_m l_m]}}_{\mathcal{L}_m}$$

dim = 4	m = 2	\mathcal{L}_0	\mathcal{L}_1	
		\wedge	\mathcal{R}	
	m = 3	\mathcal{L}_0	\mathcal{L}_1	\mathcal{L}_2