Invariantní sym. polynomy

Lieova alg. $\mathfrak{g}M$ $\quad \mathrm{Ad}_{g}{}^{\alpha}{}_{\varphi}$ $\quad \mathrm{ad}_{\mathcal{X}}{}^{\alpha}{}_{\varphi} = \mathcal{X}^{\kappa} c_{\kappa\varphi}{}^{\alpha}$

vekt. bundl. EM $\quad T_{g}{}^{A}{}_{B}$ $\quad t_{\mathcal{X}}{}^{A}{}_{B} = \mathcal{X}^{\kappa} t_{\kappa}{}^{A}{}_{B}$

$$X^{A}{}_{B} = \mathcal{X}^{\kappa} t_{\kappa}{}^{A}{}_{B} \qquad \begin{matrix} \alpha & A & A \\ \varphi & F & F \end{matrix}$$

1) multilin. zobr. stupně k

$$P(\mathcal{X}_1,\dots,\mathcal{X}_k) = P_{\alpha_1\dots\alpha_k}\mathcal{X}_1^{\alpha_1}\cdots\mathcal{X}_k^{\alpha_k}$$

$$P(X_1,\dots,X_k) = P^{B_1\dots B_k}_{A_1\dots A_k} X_1{}^{A_1}{}_{B_1}\cdots X_k{}^{A_k}{}_{B_k}$$

$$P_{\alpha_1\dots\alpha_k} = t_{\alpha_1}{}^{A_1}{}_{B_1}\cdots t_{\alpha_k}{}^{A_k}{}_{B_k} P^{B_1\dots B_k}_{A_1\dots A_k}$$

2) symetrie

$$P_{\alpha_1\dots\alpha_k} = P_{(\alpha_1\dots\alpha_k)}$$

$$P(X_1,\dots,X_k) = P(X_{\sigma_1},\dots,X_{\sigma_k}) \qquad \sigma \text{ permut.}$$

3) polynomy

k-mocnina $\quad P(X) = P(X,\dots,X)$

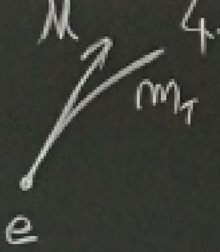
$\qquad P(\mathcal{X}) = P(\mathcal{X},\dots,\mathcal{X})$

$$P(X_1,\dots,X_k) = \frac{1}{k!}\frac{\partial^k}{\partial\tau_1\cdots\partial\tau_k}P(\tau_1X_1+\dots+\tau_kX_k)$$

$$P(X) = \sum_k P_k(X) \qquad P_k \text{ } k\text{-mocnina}$$

$$(P\circ Q)(X) = \underset{p}{P}(X)\,\underset{q}{Q}(X)$$

$$(P\circ Q)_{\alpha_1\dots\alpha_{p+q}} = P_{(\alpha_1\dots\alpha_p}Q_{\alpha_{p+1}\dots\alpha_{p+q})}$$



4) invariantnost vůči akci grupy

$$X \to \tilde{X} = \underset{(\mathbb{1}+\tau M)}{T_{m_\tau}}\cdot X\cdot \underset{(\mathbb{1}-\tau M)}{T_{m_\tau}^{-1}} \approx X + \tau[M,X] \qquad M^{\alpha}{}_{\kappa}$$

$$\mathcal{X} \to \tilde{\mathcal{X}} = \mathrm{Ad}_{m_\tau}\cdot\mathcal{X} \approx \mathcal{X} + \tau\,\mathrm{ad}_{\mathcal{M}}\mathcal{X}$$

$$\tau[\mathcal{M},\mathcal{X}]$$

$$\mathcal{M}^{\alpha} \qquad M^{A}{}_{B} = \mathcal{M}^{\alpha}t_{\alpha}{}^{A}{}_{B}$$

$$P(\tilde{X}_1,\dots,\tilde{X}_k) = P(X_1\dots X_k)$$

$$P(\tilde{X}) = P(X)$$

$$\mathbb{M}P = 0 \qquad \mathbb{M}P = 0$$

$$0 = \tau\big[P(\mathcal{M}\cdot\mathcal{X}_1,\mathcal{X}_2,\dots\mathcal{X}_k) + P(\mathcal{X}_1,\mathcal{M}\cdot\mathcal{X}_2,\dots) + \dots P(\mathcal{X}_1,\dots,\mathcal{M}\cdot\mathcal{X}_k)\big]$$

$$0 = P_{\kappa\alpha_2\dots\alpha_k}\mathcal{M}^{\kappa}{}_{\alpha_1} + P_{\alpha_1\kappa\dots\alpha_k}\mathcal{M}^{\kappa}{}_{\alpha_2} + \dots + P_{\alpha_1\alpha_2\dots\kappa}\mathcal{M}^{\kappa}{}_{\alpha_k}$$

$$0 = \mathbb{M}P_{\alpha_1\dots\alpha_k} \qquad \mathbb{M}P = 0$$

Invariantní sym. polynomy

Lieova alg. $\mathfrak{g}M$ $\quad \mathrm{Ad}_{h}{}^{\underline{\alpha}}{}_{\underline{\kappa}}$ $\quad \mathrm{ad}_{x}{}^{\underline{\alpha}}{}_{\underline{\beta}} = x^{\kappa} c_{\kappa\underline{\beta}}{}^{\underline{\alpha}}$

vekt. bundl. $\mathbb{E}M$ $\quad T_{h}{}^{\underline{A}}{}_{\underline{B}}$ $\quad t_{x}{}^{\underline{A}}{}_{\underline{B}} = x^{\kappa} t_{\kappa}{}^{\underline{A}}{}_{\underline{B}}$

$$X^{\underline{A}}{}_{\underline{B}} = x^{\kappa} t_{\kappa}{}^{\underline{A}}{}_{\underline{B}} \qquad \begin{matrix} \mathfrak{K} & \mathcal{A} & \mathsf{A} \\ \mathfrak{F} & \mathcal{F} & \mathsf{F} \end{matrix}$$

1) multilin. zobr. stupně k

$$P(x_1,\dots,x_k) = P_{\underline{\alpha}_1\dots\underline{\alpha}_k}\, x_1^{\underline{\alpha}_1}\cdots x_k^{\underline{\alpha}_k}$$

$$P(X_1,\dots,X_k) = P^{\underline{B}_1\dots\underline{B}_k}_{\underline{A}_1\dots\underline{A}_k}\, X_1{}^{\underline{A}_1}{}_{\underline{B}_1}\cdots X_k{}^{\underline{A}_k}{}_{\underline{B}_k}$$

$$P_{\underline{\alpha}_1\dots\underline{\alpha}_k} = t_{\underline{\alpha}_1}{}^{\underline{A}_1}{}_{\underline{B}_1}\cdots t_{\underline{\alpha}_k}{}^{\underline{A}_k}{}_{\underline{B}_k}\, P^{\underline{B}_1\dots\underline{B}_k}_{\underline{A}_1\dots\underline{A}_k}$$

2) symetrické

$$P_{\underline{\alpha}_1\dots\underline{\alpha}_k} = P_{(\underline{\alpha}_1\dots\underline{\alpha}_k)}$$

$$P(X_1,\dots,X_k) = P(X_{\sigma_1},\dots,X_{\sigma_k}) \qquad \sigma \text{ permut.}$$

3) polynomy

k-liniowa $\quad P(X) = P(X, \dots, X)$

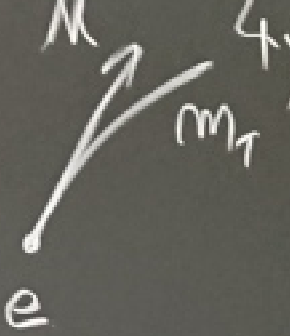
$$\mathcal{P}(\mathcal{X}) = \mathcal{P}(\mathcal{X}, \dots, \mathcal{X})$$

$$P(X_1, \dots, X_k) = \frac{1}{k!} \frac{\partial^k}{\partial \tau_1 \dots \partial \tau_k} P(\tau_1 X_1 + \dots + \tau_k X_k)$$

$$P(X) = \sum_k P_k(X) \qquad P_k \text{ k-liniowe}$$

$$(P \circ Q)(X) = \underset{p}{P}(X)\, \underset{q}{Q}(X)$$

$$(P \circ Q)_{\alpha_1 \dots \alpha_{p+q}} = P_{(\alpha_1 \dots \alpha_p} Q_{\alpha_{p+1} \dots \alpha_{p+q})}$$



4) invariantnost vůči akci grupy

$$X \to \tilde{X} = \underset{(\mathbb{1}+\tau M)}{T_{m_\tau}} \cdot X \cdot \underset{(\mathbb{1}-\tau M)}{T_{m_\tau}^{-1}} \approx X + \tau[M, X]$$

$$M^\alpha{}_\beta$$

$$\mathcal{X} \to \tilde{\mathcal{X}} = Ad_{m_\tau} \cdot \mathcal{X} \approx \mathcal{X} + \tau\, ad_{\mathcal{M}}\, \mathcal{X}$$

$$\tau[\mathcal{M}, \mathcal{X}]$$

$$\mathcal{M}^\alpha \qquad M^A{}_B = \mathcal{M}^\alpha t_\alpha{}^A{}_B$$

$$P(\tilde{X}_1, \ldots, \tilde{X}_r) = P(X_1 \ldots X_r)$$

$$P(\tilde{X}) = P(X)$$

$$M P = 0 \qquad \mathbb{M} P = 0$$

$$0 = \tau\left[P(M\cdot\mathcal{X}_1, \mathcal{X}_2, \ldots \mathcal{X}_r) + P(\mathcal{X}_1, M\cdot\mathcal{X}_2, \ldots) + \ldots P(\mathcal{X}_1, \ldots, M\cdot\mathcal{X}_r)\right]$$

$$0 = P_{\mu\alpha_2\ldots\alpha_r} M^\mu{}_{\alpha_1} + P_{\alpha_1\mu\ldots\alpha_r} M^\mu{}_{\alpha_2} + \ldots + P_{\alpha_1\alpha_2\ldots\mu} M^\mu{}_{\alpha_r}$$

$$0 = \mathbb{M} P_{\alpha_1\ldots\alpha_r} \qquad \mathbb{M} P = 0$$

$$P(\tilde{X}_1, \ldots, \tilde{X}_k) = P(X_1 \ldots X_k)$$

$$0 = P([M, X_1], X_2, \ldots) + P(X_1, [M, X_2], \ldots) + \ldots$$

$$= P^{\underline{K}\,\underline{A}_2 \ldots}_{\underline{L}\,\underline{B}_2 \ldots} \left(M^{\underline{L}}_{\underline{N}} X_{1\,\underline{K}}^{\underline{N}} - X_{1\,\underline{N}}^{\underline{L}} M^{\underline{N}}_{\underline{K}} \right) X_{2\,\underline{A}_2}^{\underline{B}_2} \ldots +$$

$$+ P^{\underline{A}_1 \underline{K} \ldots}_{\underline{B}_1 \underline{L} \ldots} X_{1\,\underline{A}_1}^{\underline{B}_1} \left(M^{\underline{L}}_{\underline{N}} X_{2\,\underline{K}}^{\underline{N}} - X_{2\,\underline{N}}^{\underline{L}} M^{\underline{N}}_{\underline{K}} \right) \ldots + \ldots$$

$$0 = P^{\underline{A}_1 \underline{A}_2 \ldots}_{\underline{L}\,\underline{B}_2 \ldots} M^{\underline{L}}_{\underline{B}_1} - P^{\underline{K}\,\underline{A}_2 \ldots}_{\underline{B}_1 \underline{B}_2 \ldots} M^{\underline{A}_1}_{\underline{K}} + P^{\underline{A}_1 \underline{A}_2}_{\underline{B}_1 \underline{L}} M^{\underline{L}}_{\underline{B}_2} - P^{\underline{A}_1 \underline{K} \ldots}_{\underline{B}_1 \underline{B}_2 \ldots} M^{\underline{A}_2}_{\underline{K}}$$

$$0 = \mathbb{M}\, P^{\underline{A}_1 \underline{A}_2 \ldots}_{\underline{B}_1 \underline{B}_2 \ldots} \qquad \mathbb{M} P = 0$$

Invariantní sym. polynomy

5) Konstantnost polynomu

$$D_{\underline{m}} c_{\kappa\beta}{}^{\alpha} = 0 \qquad D_{\underline{m}} t_{\alpha}{}^{A}{}_{B} = 0$$

$$DP = 0 \qquad DP = 0$$

$$\partial P = 0 \qquad DP = \partial P + \mathbb{A} P$$

$$A^{\alpha}{}_{\beta} = \mathcal{A}^{\kappa} c_{\kappa\beta}{}^{\alpha}$$

$$\mathrm{str}_k(X_1,\dots,X_k) = \frac{1}{k!}\sum_{\sigma} \mathrm{tr}(X_{\sigma_1} X_{\sigma_2} \cdots X_{\sigma_k})$$

$$\mathrm{str}_k(X) = \mathrm{tr}(X^k)$$

$$\mathrm{str}_k(\mathcal{X}) \qquad \mathrm{str}_2 \propto k_{\alpha\beta}$$

6) zobecnění na $\Lambda^p \otimes EM$

$$\mathcal{X}^{\alpha}_{m_1\dots m_p} \qquad X_{m_1\dots m_p}{}^{A}{}_{B}$$

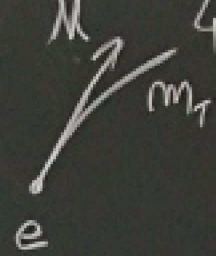
$$\hat{P}(\mathcal{X}_1, \mathcal{X}_2, \dots) \equiv P(\mathcal{X}_1 \overset{\wedge}{,} \mathcal{X}_2 \overset{\wedge}{,} \dots)$$

$$= P_{\alpha_1\alpha_2\dots}\, \mathcal{X}_1^{\alpha_1} \wedge \mathcal{X}_2^{\alpha_2} \wedge \dots$$

$\wedge$ skew-symetrický $\underset{p}{\omega} \wedge \underset{q}{\sigma} = (-1)^{pq}\, \sigma \wedge \omega$

$\hat{P}_k(\mathcal{X})$ je netriviální pro k sudé

$\hat{P}(\mathcal{X})$



4) invariantnost vůči akci grupy

$$X \to \tilde{X} = T_{m_\tau} \cdot X \cdot T^{-1}_{m_\tau} \approx X + \tau[M, X]$$

$$(\mathbb{1} + \tau M) \quad (\mathbb{1} - \tau M) \qquad M^{\alpha}{}_{\beta}$$

$$\mathcal{X} \to \tilde{\mathcal{X}} = \mathrm{Ad}_{m_\tau} \cdot \mathcal{X} \approx \mathcal{X} + \tau\, \mathrm{ad}_{\mathcal{M}}\, \mathcal{X}$$

$$\tau[\mathcal{M}, \mathcal{X}]$$

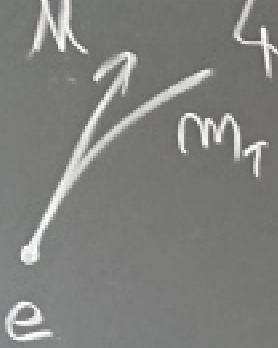
$$\mathcal{M}^{\alpha} \qquad M^{A}{}_{B} = \mathcal{M}^{\alpha} t_{\alpha}{}^{A}{}_{B}$$

$$P(\tilde{X}_1, \dots, \tilde{X}_k) = P(X_1 \dots X_k)$$

$$P(\tilde{X}) = P(X) \qquad P(\tilde{\mathcal{X}}) = P(\mathcal{X})$$

$$\mathbb{M} P = 0 \qquad \mathbb{M} P = 0$$

$$M^{A}{}_{B} = \mathcal{M}^{\kappa} t_{\kappa}{}^{A}{}_{B} \qquad M^{\alpha}{}_{\beta} = \mathcal{M}^{\kappa} c_{\kappa\beta}{}^{\alpha}$$



4) invariantnost vůči akci grupy

$$X \to \tilde{X} = T_{m_\tau} \cdot X \cdot T_{m_\tau}^{-1} \approx X + \tau [M, X]$$

$$(\mathbb{1} + \tau M) \quad (\mathbb{1} - \tau M) \qquad\qquad M^{\alpha}{}_{\beta}$$

$$\mathcal{X} \to \tilde{\mathcal{X}} = \mathrm{Ad}_{m_\tau} \cdot \mathcal{X} \approx \mathcal{X} + \tau\, \mathrm{ad}_{\mathcal{M}}\, \mathcal{X}$$

$$\tau [\mathcal{M}, \mathcal{X}]$$

$$\mathcal{M}^{\alpha} \qquad M^{A}{}_{B} = \mathcal{M}^{\alpha} t_{\alpha}{}^{A}{}_{B}$$

$$P(\tilde{X}_1, \ldots, \tilde{X}_k) = P(X_1 \ldots X_k)$$

$$P(\tilde{X}) = P(X) \qquad P(\tilde{\mathcal{X}}) = P(\mathcal{X})$$

$$\mathbb{M} P = 0 \qquad \boldsymbol{\mathcal{M}} P = 0$$

$$M^{A}{}_{B} = \mathcal{M}^{\kappa} t_{\kappa}{}^{A}{}_{B} \qquad M^{\alpha}{}_{\beta} = \mathcal{M}^{\kappa} c_{\kappa\beta}{}^{\alpha}$$

Invariantní sym. polynomy

5) konstantnost polynomu

$D_{\underline{m}} c_{\mu\beta}{}^{\alpha} = 0 \qquad D_{\underline{m}} t_{\underline{\alpha}}{}^{\underline{A}}{}_{\underline{B}} = 0$

$\mathcal{D} P = 0 \qquad \mathcal{D} P = 0$

$\partial P = 0 \qquad \mathcal{D} P = \partial P + \mathbb{A} P$

$A^{\alpha}{}_{\beta} = R^{\mu} c_{\mu\beta}{}^{\alpha}$

$\mathrm{str}_k(X_1, \ldots, X_k) = \frac{1}{k!} \sum_{\sigma} \mathrm{tr}(X_{\sigma_1} X_{\sigma_2} \ldots X_{\sigma_k})$

$\mathrm{str}_k(X) = \mathrm{tr}(X^k)$

$\mathrm{str}_k(\mathcal{X}) \qquad \mathrm{str}_2 \propto k_{\alpha\beta}$

↳ zobecnění na $\Lambda^p \otimes EM$

$$\mathcal{X}^{\underline{\alpha}}_{\underline{m}_1 \dots \underline{m}_p} \qquad X_{\underline{m}_1 \dots \underline{m}_p}{}^{\underline{A}}{}_{\underline{B}}$$

$$\hat{P}(\mathcal{X}_1, \mathcal{X}_2, \dots) \equiv P(\mathcal{X}_1 \overset{\wedge}{,} \mathcal{X}_2 \overset{\wedge}{,} \dots)$$

$$= P_{\underline{\alpha}_1 \underline{\alpha}_2 \dots} \; \mathcal{X}_1^{\underline{\alpha}_1} \wedge \mathcal{X}_2^{\underline{\alpha}_2} \wedge \dots$$

$\wedge$ skew-symetrický $\quad \underset{p}{\omega} \wedge \underset{q}{\sigma} = (-1)^{pq} \sigma \wedge \omega$

$\hat{P}_k(\mathcal{X})$ je netriviální pro k sudé

$\hat{P}(\mathcal{X})$

Chern-Weilova věta

D na $\mathfrak{g}M$ resp. EM

křivost $\mathcal{F}^{\alpha}_{\underline{mn}}$ $\qquad F_{\underline{mn}}{}^{A}{}_{\underline{B}}$

inv. sym. pol. $\mathcal{P}$ $\qquad P$

$\Downarrow$

1) $\hat{\mathcal{P}}(\mathcal{F})$ resp. $\hat{P}(F)$ jsou uzavřené

$d\hat{\mathcal{P}}(\mathcal{F}) = 0 \qquad d\hat{P}(F) = 0$

2) $D \quad \mathcal{F} \quad F \qquad \tilde{D} \quad \tilde{\mathcal{F}} \quad \tilde{F}$

$\hat{\mathcal{P}}(\tilde{\mathcal{F}}) - \hat{\mathcal{P}}(\mathcal{F}) = d\,T\mathcal{P}(\tilde{D}, D)$

$\hat{P}(\tilde{F}) - \hat{P}(F) = d\,TP(\tilde{D}, D)$

TP forma transgrese

$$d\hat{\mathcal{P}}(\mathcal{F}) = D\wedge \hat{\mathcal{P}}(\mathcal{F}) = 0$$

hom. pol. st. k $\quad = D\wedge \mathcal{P}(\mathcal{F}, \mathcal{F}, \dots)$

$$= \underbrace{(D\wedge \mathcal{P})}_{0}(\mathcal{F}, \mathcal{F}, \dots) + \hat{\mathcal{P}}(\underbrace{D\wedge \mathcal{F}}_{0}, \mathcal{F}, \dots) + \hat{\mathcal{P}}(\mathcal{F}, \underbrace{D\wedge \mathcal{F}}_{0}, \dots) + \dots$$

$$D_m \wedge \left(\mathcal{P}_{\alpha_1\alpha_2\dots}\, \mathcal{F}^{\alpha_1}_{k_1 l_1} \wedge \mathcal{F}^{\alpha_2}_{k_2 l_2} \wedge \dots\right)$$

$\tilde{D}, D \qquad \Delta = \tilde{D} - D$

$$D_\tau = \tau\tilde{D} + (1-\tau)D = D + \tau\Delta = \tilde{D} - (1-\tau)\Delta$$

$$F_\tau = F + \tau D\wedge\Delta + \tau^2 \Delta^{\wedge 2} = \tau(F + D\wedge\Delta + \Delta^{\wedge 2}) + (1-\tau)F - \tau(1-\tau)\Delta^{\wedge 2}$$

$$= \tau\tilde{F} + (1-\tau)F - \tau(1-\tau)\Delta^{\wedge 2}$$

$$\frac{\partial}{\partial\tau} D_\tau = \Delta \qquad \frac{\partial}{\partial\tau} F_\tau = D\wedge\Delta + 2\tau\Delta^{\wedge 2} = D\wedge\Delta + [\tau\Delta \overset{\wedge}{,} \Delta] = D_\tau \wedge \Delta$$

$$\frac{\partial}{\partial\tau}\hat{P}(F_\tau) = k\,\hat{P}\left(\frac{\partial}{\partial\tau}F_\tau, F_\tau, \dots\right) = k\,\hat{P}(D_\tau\wedge\Delta, F_\tau, \dots) = k\,D_\tau \wedge \hat{P}(\Delta, F_\tau, \dots) = k\,d\hat{P}(\Delta, F_\tau, \dots)$$

hom. stupně k

$[\Delta, \Delta] \qquad \Delta_{\underline{m}}{}^{A}{}_{\underline{K}} \qquad [\Delta_{\underline{m}}, \Delta_{\underline{n}}]^{A}{}_{\underline{B}} \qquad \Delta^{\wedge 2}$

$$[\Delta_{\underline{m}}, \Delta_{\underline{n}}] = \Delta_{\underline{m}}\cdot\Delta_{\underline{n}} - \Delta_{\underline{n}}\cdot\Delta_{\underline{m}} = \underbrace{\Delta_{\underline{m}}\wedge\Delta_{\underline{n}}}_{\Delta^{\wedge 2}}$$

$$[A_{\underline{m}}, B_{\underline{n}}] = A_{\underline{m}}\cdot B_{\underline{n}} - B_{\underline{n}}\cdot A_{\underline{m}} \neq A_{\underline{m}}\wedge B_{\underline{n}}$$

$$A_{\underline{m}}\wedge B_{\underline{n}} = A_{\underline{m}}\cdot B_{\underline{n}} - A_{\underline{n}}\cdot B_{\underline{m}}$$

$$[A_{\underline{m}} \overset{\wedge}{,} B_{\underline{n}}] = A_{\underline{m}}\wedge B_{\underline{n}} - B_{\underline{n}}\wedge A_{\underline{m}} =$$

$$= A_{\underline{m}}\cdot B_{\underline{n}} - A_{\underline{n}}\cdot B_{\underline{m}} - B_{\underline{n}}\cdot A_{\underline{m}} + B_{\underline{m}}\cdot A_{\underline{n}}$$

$$[\Delta_{\underline{m}}, \Delta_{\underline{n}}] = \Delta_{\underline{m}}\cdot\Delta_{\underline{n}} - \Delta_{\underline{n}}\cdot\Delta_{\underline{m}} = \Delta_{\underline{m}}\wedge\Delta_{\underline{n}}$$

$$[\Delta_{\underline{m}} \overset{\wedge}{,} \Delta_{\underline{n}}] = 2[\Delta_{\underline{m}}, \Delta_{\underline{n}}] = 2\Delta_{\underline{m}}\wedge\Delta_{\underline{n}}$$

$$P'(\Delta, X) = \sum_{k} k\,P_k(\Delta, X, X, \dots)$$

$$= \sum_{k}\left(P_k(\Delta, X, X, \dots) + P_k(X, \Delta, X, \dots) + P_k(X, X, \Delta, \dots)\right)$$

$$\frac{\partial}{\partial\tau}\hat{P}(F_\tau) = d\hat{P}'(\Delta, F_\tau)$$

$$\hat{P}(\tilde{F}) - \hat{P}(F) = d\underbrace{\int_0^1 \hat{P}'(\Delta, F_\tau)\,d\tau}_{TP(\tilde{D}, D)}$$

$$TP(\tilde{D}, D) = \int_0^1 P'(\Delta, F_\tau)\,d\tau$$

Chern-Weilova věta

D na $\mathfrak{g}M$ resp. $\mathbb{E}M$

křivost $\hat{F}_{\underline{mn}}{}^{\underline{\alpha}}$ $\qquad F_{\underline{mn}}{}^{\underline{A}}{}_{\underline{B}}$

inv. sym. pol. P $\qquad P$

$\Downarrow$

1) $\hat{P}(\hat{F})$ resp. $\hat{P}(F)$ jsou uzavřené

$$d\hat{P}(\hat{F}) = 0 \qquad d\hat{P}(F) = 0$$

2) $D \quad \hat{F} \quad F \qquad \tilde{D} \quad \tilde{\hat{F}} \quad \tilde{F}$

$$\hat{P}(\tilde{\hat{F}}) - \hat{P}(\hat{F}) = d\,TP(\tilde{D}, D)$$

$$\hat{P}(\tilde{F}) - \hat{P}(F) = d\,TP(\tilde{D}, D)$$

TP forma transgrese

$$d\hat{P}(\hat{F}) = D \wedge \hat{P}(\hat{F}) = 0$$

hom poly st $\hat{k}$ $\quad = D \wedge P(\hat{F}, \hat{F}, \ldots)$

$$= \underbrace{(D \wedge P)}_{0}(\hat{F}, \hat{F}, \ldots) + \hat{P}(\underbrace{D \wedge \hat{F}}_{0}, \hat{F}, \ldots) + \hat{P}(\hat{F}, \underbrace{D \wedge \hat{F}}_{0}, \ldots) + \ldots$$

$$D_m \wedge \left(P_{\alpha_1 \alpha_2 \ldots} \hat{F}^{\alpha_1}_{k_1 l_1} \wedge \hat{F}^{\alpha_2}_{k_2 l_2} \wedge \ldots \right)$$

$$[\Delta,\Delta] \qquad \Delta_{\underline{m}}{}^{\underline{A}}{}_{\underline{K}} \qquad [\Delta_{\underline{m}},\Delta_{\underline{n}}]^{\underline{A}}{}_{\underline{B}} \qquad \Delta^{\wedge 2}$$

$$[\Delta_{\underline{m}},\Delta_{\underline{n}}] = \Delta_{\underline{m}}\cdot\Delta_{\underline{n}} - \Delta_{\underline{n}}\cdot\Delta_{\underline{m}} = \overbrace{\Delta_{\underline{m}}\wedge\Delta_{\underline{n}}}$$

$$\tilde{D},\, D \qquad \Delta = \tilde{D} - D$$

$$D_\tau = \tau\tilde{D} + (1-\tau)D = D + \tau\Delta = \tilde{D} - (1-\tau)\Delta$$

$$F_\tau = F + \tau D\wedge\Delta + \tau^2\Delta^{\wedge 2} = \tau\left(F + D\wedge\Delta + \Delta^{\wedge 2}\right) + (1-\tau)F - \tau(1-\tau)\Delta^{\wedge 2}$$

$$= \tau\tilde{F} + (1-\tau)F - \tau(1-\tau)\Delta^{\wedge 2}$$

$$\frac{\partial}{\partial\tau}D_\tau = \Delta \qquad \frac{\partial}{\partial\tau}F_\tau = D\wedge\Delta + 2\tau\Delta^{\wedge 2} = D\wedge\Delta + [\tau\Delta,\Delta] = D_\tau\wedge\Delta$$

$$\frac{\partial}{\partial\tau}\hat{P}(F_\tau) = k\,\hat{P}\left(\frac{\partial}{\partial\tau}F_\tau, F_\tau, \dots\right) = k\,\hat{P}(D_\tau\wedge\Delta, F_\tau, \dots) = k\,D_\tau\wedge\hat{P}(\Delta, F_\tau, \dots) = k\,d\,\hat{P}(\Delta, F_\tau, \dots)$$

hom. stupně k

$$[\Delta,\Delta] \qquad \Delta_{\underline{m}}{}^{\underline{A}}{}_{\underline{K}} \qquad [\Delta_{\underline{m}},\Delta_{\underline{n}}]^{\underline{A}}{}_{\underline{B}} \qquad \Delta^{12}$$

$$[\Delta_{\underline{m}},\Delta_{\underline{n}}] = \Delta_{\underline{m}}\cdot\Delta_{\underline{n}} - \Delta_{\underline{n}}\cdot\Delta_{\underline{m}} = \overbrace{\Delta_{\underline{m}}\wedge\Delta_{\underline{n}}}$$

$$[A_{\underline{m}},B_{\underline{n}}] = A_{\underline{m}}\cdot B_{\underline{n}} - B_{\underline{n}}\cdot A_{\underline{m}} \neq A_{\underline{m}}\wedge B_{\underline{n}}$$

$$A_{\underline{m}}\wedge B_{\underline{n}} = A_{\underline{m}}\cdot B_{\underline{n}} - A_{\underline{n}}\cdot B_{\underline{m}}$$

$$[A_{\underline{m}}\overset{\wedge}{,} B_{\underline{n}}] = A_{\underline{m}}\wedge B_{\underline{n}} - B_{\underline{n}}\wedge A_{\underline{m}} =$$

$$= A_{\underline{m}}\cdot B_{\underline{n}} - A_{\underline{n}}\cdot B_{\underline{m}} - B_{\underline{n}}\cdot A_{\underline{m}} + B_{\underline{m}}\cdot A_{\underline{n}}$$

$$[\Delta_{\underline{m}},\Delta_{\underline{n}}] = \Delta_{\underline{m}}\cdot\Delta_{\underline{n}} - \Delta_{\underline{n}}\cdot\Delta_{\underline{m}} = \Delta_{\underline{m}}\wedge\Delta_{\underline{n}}$$

$$[\Delta_{\underline{m}}\overset{\wedge}{,}\Delta_{\underline{n}}] = 2[\Delta_{[\underline{m}},\Delta_{\underline{n}]}] = 2\,\Delta_{\underline{m}}\wedge\Delta_{\underline{n}}$$

$$P'(\Delta, x) = \sum_{q} q\, P_q(\Delta, x, x, \dots)$$

$$= \sum_{q} \left(P_q(\Delta, x, x, \dots) + P_q(x, \Delta, x, \dots) + P_q(x, x, \Delta, \dots) \right)$$

$$\frac{\partial}{\partial \tau} \hat{P}(F_\tau) = d\, \hat{P}'(\Delta, F_\tau)$$

$$\hat{P}(\tilde{F}) - \hat{P}(F) = \underbrace{d \int_0^1 \hat{P}'(\Delta, F_\tau)\, d\tau}_{TP(\hat{\tilde{D}}, D)}$$

$$TP(\tilde{D}, D) = \int_0^1 P'(\Delta, F_\tau)\, d\tau$$

Charakteristické třídy

P inv. sym. polynom

$$p = [\hat{P}(F)] \in H(M)$$

- kohomol. gr.
- nezávisí na D
- P homogenní st. k $\to$ $2k$-forma
- P nehomogenní $\quad p_k$ homog. komp.

$$\underbrace{\int_M p \wedge q \wedge \dots}_{\text{nezávisí na } \alpha} = \int_M \hat{P}(F) \wedge \hat{Q}(F) \wedge \dots$$

$$\int_M (\hat{P}(F) + d\alpha) \wedge \hat{Q}(F) \wedge \dots = \int \dots + \int_M d(\alpha \wedge \hat{Q}(F) \wedge \dots) \to 0$$

Chernovy charakteristické třídy

EM komplex. vekt. b. s herm. str.

$$\hat{C}(F) = \hat{\det}\left(\mathbb{1} + \frac{i\alpha}{2\pi} F\right)$$

$$C = [\hat{C}(F)] \qquad \text{nehomogenní}$$

$$\hat{C}(F) = 1 + \alpha \hat{C}_1(F) + \alpha^2 \hat{C}_2(F) + \dots$$

$$C_k = [\hat{C}_k(F)]$$

$$\hat{\det}(\mathbb{1} + \lambda F) = {}^{[N]}\delta^{B_1 \dots B_N}_{A_1 \dots A_N} (\mathbb{1} + \lambda F)^{A_1}{}_{B_1} \wedge \dots \wedge (\mathbb{1} + \lambda F)^{A_N}{}_{B_N}$$

$$= \sum_{k=0}^{N} \lambda^k \, {}^{[N]}\delta^{B_1 \dots B_k C_1 \dots C_{N-k}}_{A_1 \dots A_k C_1 \dots C_{N-k}} \binom{N}{k} F^{A_1}{}_{B_1} \wedge \dots \wedge F^{A_k}{}_{B_k}$$

$$= \sum_{k=0}^{N} \lambda^k F^{A_1}{}_{[A_1} \wedge \dots \wedge F^{A_k}{}_{B_k]}$$

$$= 1 + \lambda F^A{}_A$$

$$+ \frac{1}{2} \lambda^2 \left(F^A{}_A \wedge F^B{}_B - F^A{}_B \wedge F^B{}_A \right)$$

$$+ \frac{1}{3!} \lambda^3 \left(F^A{}_A \wedge F^B{}_B \wedge F^C{}_C + F^A{}_B \wedge F^B{}_C \wedge F^C{}_A + F^A{}_C \wedge F^B{}_A \wedge F^C{}_B \right.$$

$$\left. - F^A{}_A \wedge F^B{}_C \wedge F^C{}_B - F^A{}_B \wedge F^B{}_A \wedge F^C{}_C - F^A{}_C \wedge F^B{}_B \wedge F^C{}_A \right)$$

$$+ \dots$$

$$= 1 + \lambda \operatorname{tr} F + \frac{1}{2} \lambda^2 \left((\operatorname{tr} F)^{\wedge 2} - \operatorname{tr} F^{\wedge 2} \right)$$

$$+ \frac{1}{3!} \lambda^3 \left((\operatorname{tr} F)^{\wedge 3} + 2 \operatorname{tr} F^{\wedge 3} - 3 (\operatorname{tr} F) \wedge (\operatorname{tr} F^{\wedge 2}) \right)$$

$$+ \dots$$

Charakteristické třídy

P inv. sym. polynom.

$$p = [\hat{P}(F)] \in H(M)$$

- kohomol. tř.
- nezávisí na D
- P homogenní st $k \to 2k$-forma
- P nehomogenní $\quad p_k$ homog. komp.

$$\int_M \underbrace{p \wedge q \wedge \ldots}_{\text{stupeň } d} = \int_M \hat{P}(F) \wedge \hat{Q}(F) \wedge \ldots$$

$$\int_M (\hat{P}(F) + d\alpha) \wedge \hat{Q}(F) \wedge \ldots = \int \ldots + \int_M \cancelto{0}{d(\alpha \wedge \hat{Q}(F) \wedge \ldots)}$$

$$P(X) = P(X, \dots, X)$$

$$\mathcal{P}(\mathcal{X}) = \mathcal{P}(\mathcal{X}, \dots, \mathcal{X}) = P_{\underline{\alpha}_1 \dots \underline{\alpha}_n} \mathcal{X}^{\underline{\alpha}_1} \cdots \mathcal{X}^{\underline{\alpha}_n}$$

$$\mathcal{X}^{\underline{\mu}} \to \mathcal{X}^{\alpha} \qquad E_{\alpha}^{\underline{\mu}}$$

Chernovy charakteristické třídy

E M komplex. vekt. b. s herm. str.

$$\hat{C}(F) = \hat{\det}\Big(\mathbb{1} + \underbrace{\frac{i\alpha}{2\pi}}_{\lambda} F\Big)$$

$$c = \big[\hat{C}(F)\big] \qquad \text{nehomogenní}$$

$$\hat{C}(F) = 1 + \alpha \hat{C}_1(F) + \alpha^2 \hat{C}_2(F) + \dots$$

$$c_k = \big[\hat{C}_k(F)\big]$$

$$\hat{\det}(\mathbb{1} + \lambda F) = {}^{[N]}\delta^{B_1 \dots B_N}_{A_1 \dots A_N} (\mathbb{1} + \lambda F)^{A_1}{}_{B_1} \wedge \dots \wedge (\mathbb{1} + \lambda F)^{A_N}{}_{B_N}$$

$$= \sum_{k=0}^{N} \lambda^k \, {}^{[N]}\delta^{B_1 \dots B_k C_1 \dots C_{N-k}}_{A_1 \dots A_k C_1 \dots C_{N-k}} \binom{N}{k} F^{A_1}{}_{B_1} \wedge \dots \wedge F^{A_k}{}_{B_k}$$

$$= \sum_{k=0}^{N} \lambda^k F^{A_1}{}_{[A_1} \wedge \dots \wedge F^{A_k}{}_{A_k]}$$

$$= 1 + \lambda F^A{}_A$$

$$+ \frac{1}{2}\lambda^2 \left(F^A{}_A \wedge F^B{}_B - F^A{}_B \wedge F^B{}_A \right)$$

$$+ \frac{1}{3!}\lambda^3 \Big(F^A{}_A \wedge F^B{}_B \wedge F^C{}_C + F^A{}_B \wedge F^B{}_C \wedge F^C{}_A + F^A{}_C \wedge F^B{}_A \wedge F^C{}_B$$

$$- F^A{}_A \wedge F^B{}_C \wedge F^C{}_B - F^A{}_B \wedge F^B{}_A \wedge F^C{}_C - F^A{}_C \wedge F^B{}_B \wedge F^C{}_A \Big)$$

$$+ \cdots$$

$$= 1 + \lambda\, \mathrm{tr} F + \frac{1}{2}\lambda^2 \left((\mathrm{tr} F)^{\wedge 2} - \mathrm{tr} F^{\wedge 2} \right)$$

$$+ \frac{1}{3!}\lambda^3 \left((\mathrm{tr} F)^{\wedge 3} + 2\, \mathrm{tr} F^{\wedge 3} - 3 (\mathrm{tr} F) \wedge (\mathrm{tr} F^{\wedge 2}) \right)$$

$$+ \cdots$$

Chernova charakteristika

$$\hat{Ch}(F) = \mathrm{tr}\exp\Big(\underbrace{\tfrac{i\alpha}{2\pi}}_{\lambda}F\Big)$$

$$= \sum_{k=0}^{\infty} \frac{\lambda^k}{k!}\,\mathrm{tr}\,F^{\wedge k} = \sum_k \hat{Ch}_k(F)$$

$$ch = [\hat{Ch}(F)] \qquad ch_k = [\hat{Ch}_k(F)]$$

$$\hat{Ch}_k(F) = \frac{\lambda^k}{k!}\,\mathrm{tr}(F^{\wedge k}) = \frac{\lambda^k}{k!}\,\mathrm{str}_k(F,\dots,F)$$

$$\mathrm{str}_k(A_1,\dots,A_k) = \sum_{\mathrm{perm}\,\sigma} \mathrm{tr}(A_{\sigma_1}\cdots A_{\sigma_k})$$

$$\mathrm{str}_{k\,\alpha_1\dots\alpha_k} = t_{(\alpha_1}{}^{A_1}{}_{A_2}\, t_{\alpha_2}{}^{A_2}{}_{A_3}\cdots t_{\alpha_k)}{}^{A_k}{}_{A_1}$$

$$\hat{\log}\,\hat{\det}(1+\lambda F) = \mathrm{tr}\,\hat{\log}(1+\lambda F)$$

$$\mathrm{tr}\big(\lambda F - \tfrac12\lambda^2F^{\wedge 2} + \tfrac13\lambda^3F^{\wedge 3} - \dots\big)$$

$$= \sum_{k=1}^{\infty}(-1)^{k-1}\frac{\lambda^k}{k}\,\mathrm{tr}\,F^{\wedge k}$$

$$= \sum_{k=1}^{\infty}(-1)^{k-1}(k-1)!\,\hat{Ch}_k(F)$$

$$= \hat{Ch}_1 - \hat{Ch}_2 + 2\hat{Ch}_3 - \dots$$

$$\to = \hat{\log}\Big(1+\sum_{k=1}^{\infty}\hat{C}_k\Big) =$$

$$= (\hat C_1+\hat C_2+\hat C_3+\dots) - \tfrac12(\hat C_1+\hat C_2+\dots)^{\wedge 2} + \tfrac13(\hat C_1+\dots)^{\wedge 3} + \dots$$

$$= \hat C_1 + \big(\hat C_2 - \tfrac12\hat C_1^{\wedge 2}\big) + \big(\hat C_3 - \hat C_2\wedge\hat C_1 + \tfrac13\hat C_1^{\wedge 3}\big) + \dots$$

$$\hat{Ch}_1 = \hat C_1 \qquad \hat{Ch}_2 = -\hat C_2 + \tfrac12\hat C_1^{\wedge 2}$$

$$\hat{Ch}_3 = \tfrac12\hat C_3 - \tfrac12\hat C_2\wedge\hat C_1 + \tfrac16\hat C_1^{\wedge 3}$$

$$\hat C(F) = 1 + \lambda F^A{}_A$$

$$+ \tfrac12\lambda^2\big(F^A{}_A\wedge F^B{}_B - F^A{}_B\wedge F^B{}_A\big)$$

$$+ \tfrac1{3!}\lambda^3\big(F^A{}_A\wedge F^B{}_B\wedge F^C{}_C + F^A{}_B\wedge F^B{}_C\wedge F^C{}_A + F^A{}_C\wedge F^B{}_A\wedge F^C{}_B$$

$$- F^A{}_A\wedge F^B{}_C\wedge F^C{}_B - F^A{}_B\wedge F^B{}_A\wedge F^C{}_C - F^A{}_C\wedge F^B{}_B\wedge F^C{}_A\big)$$

$$+ \dots$$

$$= 1 + \lambda\,\mathrm{tr}F + \tfrac12\lambda^2\big((\mathrm{tr}F)^{\wedge 2} - \mathrm{tr}F^{\wedge 2}\big)$$

$$+ \tfrac1{3!}\lambda^3\big((\mathrm{tr}F)^{\wedge 3} + 2\,\mathrm{tr}F^{\wedge 3} - 3(\mathrm{tr}F)\wedge(\mathrm{tr}F^{\wedge 2})\big)$$

$$+ \dots$$

$$\hat C_1 = \hat{Ch}_1$$

$$\hat C_2 = -\hat{Ch}_2 + \tfrac12\hat{Ch}_1^{\wedge 2}$$

$$\hat C_3 = 2\hat{Ch}_3 - \hat{Ch}_2\wedge\hat{Ch}_1 + \tfrac16\hat{Ch}_1^{\wedge 3}$$

Chernova charakteristika

$$\hat{ch}(F) = \mathrm{tr}\exp\Big(\underbrace{\tfrac{i\alpha}{2\pi}}_{\lambda}F\Big)$$

$$= \sum_{k=0}^{\infty} \frac{\lambda^k}{k!}\,\mathrm{tr}\,F^{\wedge k} = \sum_{k} \hat{ch}_k(F)$$

$$ch = \big[\hat{ch}(F)\big] \qquad ch_k = \big[\hat{ch}_k(F)\big]$$

$$\hat{ch}_k(F) = \frac{\lambda^k}{k!}\,\mathrm{tr}\big(F^{\wedge k}\big) = \frac{\lambda^k}{k!}\,\mathrm{str}_k(F,\dots,F)$$

$$\mathrm{str}_k(A_1,\dots,A_k) = \sum_{\text{perm }\sigma} \mathrm{tr}\big(A_{\sigma_1}\cdot\ldots\cdot A_{\sigma_k}\big)$$

$$\mathrm{str}_{k\,\underline{\alpha}_1\dots\underline{\alpha}_k} = t_{(\underline{\alpha}_1}{}^{\underline{A}_1}{}_{\underline{A}_2}\, t_{\underline{\alpha}_2}{}^{\underline{A}_2}{}_{\underline{A}_3} \cdots t_{\underline{\alpha}_k)}{}^{\underline{A}_k}{}_{\underline{A}_1}$$

$$\hat{C}(F) = 1 + \lambda F^A{}_A$$
$$+ \frac{1}{2}\lambda^2\left(F^A{}_A \wedge F^B{}_B - F^A{}_B \wedge F^B{}_A\right)$$
$$+ \frac{1}{3!}\lambda^3\left(F^A{}_A \wedge F^B{}_B \wedge F^C{}_C + F^A{}_B \wedge F^B{}_C \wedge F^C{}_A + F^A{}_C \wedge F^B{}_A \wedge F^C{}_B\right.$$
$$\left. - F^A{}_A \wedge F^B{}_C \wedge F^C{}_B - F^A{}_B \wedge F^B{}_A \wedge F^C{}_C - F^A{}_C \wedge F^B{}_B \wedge F^C{}_A\right)$$
$$+ \cdots$$
$$= 1 + \lambda \operatorname{tr} F + \frac{1}{2}\lambda^2\left((\operatorname{tr} F)^{\wedge 2} - \operatorname{tr} F^{\wedge 2}\right)$$
$$+ \frac{1}{3!}\lambda^3\left((\operatorname{tr} F)^{\wedge 3} + 2 \operatorname{tr} F^{\wedge 3} - 3(\operatorname{tr} F) \wedge (\operatorname{tr} F^{\wedge 2})\right)$$
$$+$$
$$\hat{C}_1 = \hat{Ch}_1$$
$$\hat{C}_2 = -\hat{Ch}_2 + \frac{1}{2}\hat{Ch}_1^{\wedge 2}$$
$$\hat{C}_3 = 2\hat{Ch}_3 - \hat{Ch}_2 \wedge \hat{Ch}_1 + \frac{1}{6}\hat{Ch}_1^{\wedge 3}$$

$$\hat{\log}\,\hat{\det}(\mathbb{1}+\lambda F) = \mathrm{tr}\,\hat{\log}(\mathbb{1}+\lambda F)$$

$$\mathrm{tr}\left(\lambda F - \frac{1}{2}\lambda^2 F^{\wedge 2} + \frac{1}{3}\lambda^3 F^{\wedge 3} - \dots\right)$$

$$= \sum_{k=1}^{\infty} (-1)^{k-1} \frac{\lambda^k}{k}\,\mathrm{tr}\,F^{\wedge k}$$

$$= \sum_{k=1}^{\infty} (-1)^{k-1} (k-1)!\, \hat{Ch}_k(F)$$

$$= \hat{Ch}_1 - \hat{Ch}_2 + 2\hat{Ch}_3 - \dots$$

$$= \hat{\log}\left(1 + \sum_{k=1}^{\infty} \hat{C}_k\right) =$$

$$= (\hat{C}_1 + \hat{C}_2 + \hat{C}_3 + \dots) - \frac{1}{2}(\hat{C}_1 + \hat{C}_2 + \dots)^{\wedge 2} + \frac{1}{3}(\hat{C}_1 + \dots)^{\wedge 3} +$$

$$= \hat{C}_1 + \left(\hat{C}_2 - \frac{1}{2}\hat{C}_1^{\wedge 2}\right) + \left(\hat{C}_3 - \hat{C}_2 \wedge \hat{C}_1 + \frac{1}{3}\hat{C}_1^{\wedge 3}\right) + \dots$$

$$\hat{Ch}_1 = \hat{C}_1 \qquad \hat{Ch}_2 = -\hat{C}_2 + \frac{1}{2}\hat{C}_1^{\wedge 2}$$

$$\hat{Ch}_3 = \frac{1}{2}\hat{C}_3 - \frac{1}{2}\hat{C}_2 \wedge \hat{C}_1 + \frac{1}{6}\hat{C}_1^{\wedge 3}$$

Pontrjaginovy charakt. třídy

$A\,M$ reálný vekt. bundl s metrikou

str. gr. $O(A, M)$

$$F = -F^T$$

$$\operatorname{tr} F = 0 \qquad Q = F \wedge F^T = -F^{\wedge 2}$$

$$\hat{Pt}(F) = \hat{\det}^{\frac{1}{2}}\left(\mathbb{1} + \frac{Q}{(2\pi)^2}\right) =$$

$$= \hat{\det}\left(\mathbb{1} - \frac{F}{2\pi}\right) = \hat{\det}\left(\mathbb{1} + \frac{F}{2\pi}\right)$$

$$p = \left[\hat{Pt}(F)\right] \qquad \operatorname{tr} F^{\wedge 2k+1} = 0$$

$$\hat{Pt}(F) = \sum_{k=0}^{\infty} \hat{Pt}_{2k}(F) \qquad \hat{Pt}_{2k+1}(F) = 0$$

Eulerova charakt. třída

TM tečný bundle

$$g_{ab} \qquad \nabla_a \qquad R_{ab}{}^m{}_n$$

$$\dim M = 2m$$

$$\hat{Pf}\left(\frac{R}{2\pi}\right) \qquad e = \left[\hat{Pf}\left(\frac{R}{2\pi}\right)\right]$$

$$Pf(X) = \frac{1}{2^m m!} \varepsilon_{a_1 b_1 \ldots a_m b_m} X^{a_1 b_1} \ldots X^{a_m b_m}$$

$$= X^{12} X^{34} \ldots X^{2m-1\, m} = \xi^1 \xi^2 \ldots \xi^m \qquad \begin{bmatrix} 0 & \xi_1 & & & \\ -\xi_1 & 0 & & & \\ & & 0 & \xi_2 & \\ & & -\xi_2 & 0 & \\ & & & & \ddots \end{bmatrix}$$

$$= (\det X)^{\frac{1}{2}}$$

$$\hat{Pf}(R)_{a_1 b_1 \ldots a_m b_m} = \frac{1}{2^m m!} \varepsilon_{k_1 l_1 \ldots k_m l_m} R_{a_1 b_1}{}^{k_1 l_1} \wedge \ldots \wedge R_{a_m b_m}{}^{k_m l_m}$$

$$= \frac{1}{2^m m!} \frac{1}{(2m)!} \varepsilon^{m_1 n_1 \ldots m_m n_m} \varepsilon_{k_1 l_1 \ldots k_m l_m} R_{m_1 n_1}{}^{k_1 l_1} \wedge \ldots \wedge R_{m_m n_m}{}^{k_m l_m} \varepsilon_{a_1 b_1 \ldots}$$

$$= \frac{1}{2^m m!} R_{k_1 l_1}{}^{k_1 l_1} \wedge \ldots \wedge R_{k_m l_m}{}^{k_m l_m} \varepsilon_{a_1 b_1 \ldots a_m b_m}$$

$$= \frac{(2m)!}{2^{2m} m!} R_{[k_1 l_1}{}^{k_1 l_1} \ldots R_{k_m l_m]}{}^{k_m l_m} \varepsilon_{a_1 b_1 \ldots a_m b_m}$$

Gauss-Bonnetova věta

$$\int_M e = \chi(M) = \operatorname{index}_{deRham}(M)$$

$$d \quad \delta \quad \Delta$$

$$d + \delta$$

Pontrjaginovy charakt. třídy

A M reálný vekt. bundle s metrikou

kal. gr. $O(A, H)$

$$F = -F^T$$

$$\mathrm{tr}\, F = 0 \qquad Q = F \wedge F^T = -F^{\wedge 2}$$

$$\hat{Pt}(F) = \hat{\det}^{\frac{1}{2}}\left(\mathbb{1} + \frac{Q}{(2\pi)^2}\right) =$$

$$= \hat{\det}\left(\mathbb{1} - \frac{F}{2\pi}\right) = \hat{\det}\left(\mathbb{1} + \frac{F}{2\pi}\right)$$

$$p = [\hat{Pt}(F)] \qquad \mathrm{tr}\, F^{\wedge 2k+1} = 0$$

$$\hat{Pt}(F) = \sum_{k=0}^{\infty} \hat{Pt}_{2k}(F) \qquad \hat{Pt}_{2k+1}(F) = 0$$

Eulerova charakt. třída

TM tečný bundle

$g_{\underline{ab}}$ $\nabla_{\underline{a}}$ $R_{\underline{ab}}{}^{\underline{m}}{}_{\underline{n}}$

$\dim M = 2m$

$$\hat{Pf}\left(\frac{R}{2\pi}\right) \qquad e = \left[\hat{Pf}\left(\frac{R}{2\pi}\right)\right]$$

$$Pf(X) = \frac{1}{2^m m!}\, \varepsilon_{\underline{a_1 b_1} \dots \underline{a_m b_m}} X^{\underline{a_1 b_1}} \cdots X^{\underline{a_m b_m}}$$

$$= X^{12} X^{34} \cdots X^{2m-1\, m} = \xi^1 \xi^2 \cdots \xi^m \qquad \begin{bmatrix} 0 & \xi_1 & & & \\ -\xi_1 & 0 & & & \\ & & 0 & \xi_2 & \\ & & -\xi_2 & 0 & \\ & & & & \ddots \end{bmatrix}$$

$$= (\det X)^{\frac{1}{2}}$$

$$\hat{\mathrm{Pf}}(R)_{a_1b_1\ldots a_nb_n} = \frac{1}{2^n n!}\,\varepsilon_{k_1l_1\ldots k_nl_n}\, R_{a_1b_1}{}^{k_1l_1}\wedge\ldots\wedge R_{a_nb_n}{}^{k_nl_n}$$

$$= \frac{1}{2^n n!}\,\frac{1}{(2n)!}\,\varepsilon^{m_1n_1\ldots m_nn_n}\,\varepsilon_{k_1l_1\ldots k_nl_n}\, R_{m_1n_1}{}^{k_1l_1}\wedge\ldots\wedge R_{m_nn_n}{}^{k_nl_n}\,\varepsilon_{a_1b_1\ldots a_nb_n}$$

$$= \frac{1}{2^n n!}\, R_{k_1l_1}{}^{k_1l_2}\wedge\ldots\wedge R_{k_nl_n}{}^{k_nl_n}\quad \varepsilon_{a_1b_1\ldots a_nb_n}$$

$$= \frac{(2n)!}{2^{2n} n!}\, R_{[k_1l_1}{}^{k_1l_1}\ldots\ldots R_{k_nl_n]}{}^{k_nl_n}\quad \varepsilon_{a_1b_1\ldots a_nb_n}$$

Gauss-Bonnet învârte vete

$$\int_M e = \chi(M) = \text{index}_{\text{deRham}}(M)$$

$$d \quad \delta \quad \Delta$$

$$d - \delta$$

$m=1 \quad \dim M = 2$

$$2\pi e_{ab} = \frac{1}{2} R_{kl}{}^{kl} \quad \varepsilon_{ab} = K\, \varepsilon_{ab}$$

$$\frac{1}{2\pi}\int_M K\,\varepsilon = \chi(M) = b_0 - b_1 + b_2$$

$$\overset{\text{sféra}}{=} 1 - 0 + 1 = 2$$

$m=2 \quad \dim M = 4$

$$(2\pi)^2 e_{abcd} = \frac{1}{8}\left(R_{kl}{}^{kl} \wedge R_{mn}{}^{mn}\right) \varepsilon_{abcd}$$

$$= \frac{1}{8}\left(R_{kl}{}^{kl} R_{mn}{}^{mn} + R_{mn}{}^{kl} R_{kl}{}^{mn} - 4 R_{km}{}^{kl} R_{ln}{}^{mn}\right)$$

$$= \frac{1}{8}\left(R^2 - 4 Ric^2 + \mathbb{R}^2\right) \varepsilon_{abcd}$$

$$R_{klmn} R^{klmn} \quad Ric_{kl} Ric^{kl} \quad RR$$

$$\frac{1}{8}\frac{1}{(2\pi)^2}\int_M \left(R^2 - 4Ric^2 + \mathbb{R}^2\right)\varepsilon =$$

$$= \chi(M) = b_0 - b_1 + b_2 - b_3 + b_4$$

Lovelock. gravitace

$$S_{Lov} = \sum_k \int_M \alpha_k \mathcal{L}_k\, g^{\frac{1}{2}}$$

$$k < m = \dim M/2$$

$\varepsilon_{abcd} \quad \dim 4 \quad m=2 \qquad \mathcal{L}_0 \quad \mathcal{L}_1$

$\Lambda \quad \mathcal{H}$

$m=3 \qquad \mathcal{L}_0 \quad \mathcal{L}_1 \quad \mathcal{L}_2$

$$\hat{Pf}(R)_{a_1b_1\dots a_nb_n} = \frac{1}{2^n n!}\varepsilon_{k_1l_1\dots k_nl_n} R_{a_1b_1}{}^{k_1l_1} \wedge \dots \wedge R_{a_nb_n}{}^{k_nl_n}$$

$$= \frac{1}{2^n n!}\frac{1}{(2n)!}\varepsilon^{m_1n_1\dots m_nn_n}\varepsilon_{k_1l_1\dots k_nl_n} R_{m_1n_1}{}^{k_1l_1} \wedge \dots \wedge R_{m_nn_n}{}^{k_nl_n}\varepsilon_{a_1b_1\dots a_nb_n}$$

$$= \frac{1}{2^n n!} R_{k_1l_1}{}^{k_1l_1} \wedge \dots \wedge R_{k_nl_n}{}^{k_nl_n} \quad \varepsilon_{a_1b_1\dots a_nb_n}$$

$$= \underbrace{\frac{(2m)!}{2^{2m} m!} R_{[k_1l_1}{}^{k_1l_1} \dots R_{k_nl_n]}{}^{k_nl_n}}_{\mathcal{L}_m} \quad \varepsilon_{a_1b_1\dots a_nb_n}$$

$$m = 1 \qquad \dim M = 2$$

$$2\pi\,\Omega_{ab} = \frac{1}{2} R_{kl}{}^{kl}\,\varepsilon_{ab} = K\,\varepsilon_{ab}$$

$$\frac{1}{2\pi}\int_M K\,\varepsilon = \chi(M) = b_0 - b_1 + b_2$$

Sféra

$$= 1 - 0 + 1 = 2$$

$m=2 \quad \dim M = 4$

$$(2\pi)^2 e_{abcd} = \frac{1}{8}\left(R_{kl}{}^{kl} \wedge R_{mn}{}^{mn}\right) \varepsilon_{abcd}$$

$$= \frac{1}{8}\left(R_{kl}{}^{kl} R_{mn}{}^{mn} + R_{mn}{}^{kl} R_{kl}{}^{mn} - 4 R_{km}{}^{kl} R_{ln}{}^{mn}\right) \varepsilon_{abcd}$$

$$= \frac{1}{8}\left(R^2 - 4 Ric^2 + \mathbb{R}^2\right) \varepsilon_{abcd}$$

$$R_{klmn} R^{klmn} \quad Ric_{kl} Ric^{kl} \quad \mathbb{R}\mathbb{R}$$

$$\frac{1}{8}\frac{1}{(2\pi)^2}\int_M \left(R^2 - 4 Ric^2 + \mathbb{R}^2\right) \varepsilon =$$

$$= \chi(M) = b_0 - b_1 + b_2 - b_3 + b_4$$

Lovelock gravitace

$$S_{Lov} = \sum_k \int_M \alpha_k \mathcal{L}_k \, g^{\frac{1}{2}}$$

$$\underbrace{\frac{(2m)!}{2^{2m} m!} R_{[k_1 l_1}{}^{k_1 l_1} \cdots \cdots R_{k_m l_m]}{}^{k_m l_m}}_{\mathcal{L}_m}$$

$k < m = \dim M/2$

$\dim 4 \quad m = 2 \quad \mathcal{L}_0 \quad \mathcal{L}_1$

$\Lambda \quad \mathcal{H}$

$m = 3 \quad \mathcal{L}_0 \quad \mathcal{L}_1 \quad \mathcal{L}_2$