

Invariantni sym. polynomy

Liouv. alg. $\mathfrak{g}M$ Ad_{g^x} $ad_{x^k} = X^k c_{F^k}$

velst. budl. EM T_{A^B} $t_{x^A}^B = X^k t_{F^k}^A$

$X^A_B = X^k t_{F^k}^A$ $\mathfrak{A} \ A \ A$
 $\mathfrak{F} \ F \ F$

1) multipli. zobor. stupne \mathfrak{z}

$P(x_1, \dots, x_n) = P_{\alpha_1, \dots, \alpha_n} x_1^{\alpha_1} \dots x_n^{\alpha_n}$

$P(x_1, \dots, x_n) = P_{A_1, \dots, A_n} x_{B_1}^{A_1} \dots x_{B_n}^{A_n}$

$P_{\alpha_1, \dots, \alpha_n} = t_{\alpha_1, B_1}^{A_1} \dots t_{\alpha_n, B_n}^{A_n} P_{A_1, \dots, A_n}$

2) symetrie

$P_{\alpha_1, \dots, \alpha_n} = P_{\alpha_{\sigma(1)}, \dots, \alpha_{\sigma(n)}}$
 $P(x_1, \dots, x_n) = P(x_{\sigma(1)}, \dots, x_{\sigma(n)})$ σ permut.

3) polynomy

\mathfrak{z} -mocina $P(x) = P(x_1, \dots, x_n)$
 $P(x) = P(x_1, \dots, x_n)$

$P(x_1, \dots, x_n) = \frac{1}{\mathfrak{z}!} \frac{\partial^{\mathfrak{z}}}{\partial \tau_1 \dots \partial \tau_{\mathfrak{z}}} P(\tau_1 x_1 + \dots + \tau_{\mathfrak{z}} x_{\mathfrak{z}})$

$P(x) = \sum_{\mathfrak{z}} P_{\mathfrak{z}}(x)$

$(P \circ Q)(x) = P(x) Q(x)$

$(P \circ Q)_{\alpha_1, \dots, \alpha_{p+q}} = P_{\alpha_1, \dots, \alpha_p} Q_{\alpha_{p+1}, \dots, \alpha_{p+q}}$

4) invariant-ost vnci dca gary
 $X \rightarrow \tilde{X} = T_{m_{\tau}} \cdot X \cdot T_{m_{\tau}}^{-1} \approx X + \tau[M, X]$
 $(1 + \tau M) \quad (1 - \tau M) \quad M^{\mathfrak{z}}$

$X \rightarrow \tilde{X} = Ad_{m_{\tau}} \cdot X \approx X + \tau ad_M X$
 $\tau[M, X]$

$M^{\mathfrak{z}} \quad M^A_B = M^{\mathfrak{z}} t_{A^B}$

$P(\tilde{x}_1, \dots, \tilde{x}_n) = P(x_1, \dots, x_n)$

$P(\tilde{x}) = P(x)$

$M P = 0 \quad M P = 0$

$0 = \tau [P(M \cdot x_1, x_2, \dots, x_n) + P(x_1, M \cdot x_2, \dots) + \dots + P(x_1, \dots, M \cdot x_n)]$

$0 = P_{\alpha_1, \dots, \alpha_n} M^{\alpha_1} + P_{\alpha_1, \dots, \alpha_n} M^{\alpha_2} + \dots + P_{\alpha_1, \dots, \alpha_n} M^{\alpha_n}$

$0 = M P_{\alpha_1, \dots, \alpha_n} \quad M P = 0$

Invariantni sym. polynomy

Linearna alga \mathfrak{g} M Ad_h^α $ad_X^\alpha = X^k c_{k\alpha}^\alpha$

vekt. bund. EM $T_{h, E}^A$ $t_X^A = X^k t_{F, E}^A$

$$X^A_B = X^k t_{F, E}^A \quad \begin{matrix} \exists & A & A \\ \neq & F & F \end{matrix}$$

1) multi. izob. stupnje α

$$P(x_1, \dots, x_n) = P_{\alpha_1, \dots, \alpha_n} x_1^{\alpha_1} \dots x_n^{\alpha_n}$$

$$P(X_1, \dots, X_n) = P_{A_1, \dots, A_n}^{B_1, \dots, B_n} X_{1, B_1}^{A_1} \dots X_{n, B_n}^{A_n}$$

$$P_{\alpha_1, \dots, \alpha_n} = t_{\alpha_1, B_1}^{A_1} \dots t_{\alpha_n, B_n}^{A_n} P_{A_1, \dots, A_n}^{B_1, \dots, B_n}$$

2) simetrije

$$P_{\alpha_1, \dots, \alpha_n} = P_{\alpha_1, \dots, \alpha_n}$$

$$P(X_1, \dots, X_n) = P(X_{B_1}, \dots, X_{B_n})$$

σ permut.

3) polynomy

k -mocina

$$P(X) = P(X_1, \dots, X_k)$$

$$P(X) = P(X_1, \dots, X_k)$$

$$P(X_1, \dots, X_k) = \frac{1}{k!} \frac{\partial^k}{\partial \tau_1 \dots \partial \tau_k} P(\tau_1 X_1 + \dots + \tau_k X_k)$$

$$P(X) = \sum_k P_k(X)$$

P_k k -mocina

$$(P \circ Q)(X) = P(X) Q(X)$$

$$(P \circ Q)_{\alpha_1, \dots, \alpha_{p+q}} = P_{(\alpha_1, \dots, \alpha_p)} Q_{(\alpha_{p+1}, \dots, \alpha_{p+q})}$$

M
 e \nearrow m_τ 4) invariant - ost n i e g g g

$$X \rightarrow \tilde{X} = T_{m_\tau} \cdot X \cdot T_{m_\tau}^{-1} \approx X + \tau [M, X]$$

$(\mathbb{1} + \tau M) \quad (\mathbb{1} - \tau M) \quad M^x_f$

$$\mathcal{X} \rightarrow \tilde{\mathcal{X}} = \text{Ad}_{m_\tau} \cdot \mathcal{X} \approx \mathcal{X} + \tau \text{ad}_M \mathcal{X}$$

$\tau [M, \mathcal{X}]$

$$M^a \quad M^A_B = M^a t^A_B$$

$$P(\tilde{X}_1, \tilde{X}_2) = P(X_1, X_2)$$

$$P(\tilde{X}) = P(X)$$

$$M P = 0 \quad M P = 0$$

$$0 = \tau [P(M \cdot X_1, X_2, \dots, X_n) + P(X_1, M \cdot X_2, \dots) + \dots + P(X_1, \dots, M \cdot X_n)]$$

$$0 = P_{\alpha_2 \dots \alpha_n} M^F_{\alpha_1} + P_{\alpha_1 \alpha_2 \dots \alpha_n} M^F_{\alpha_2} + \dots + P_{\alpha_1 \alpha_2 \dots \alpha_n} M^F_{\alpha_n}$$

$$0 = M P_{\alpha_1 \dots \alpha_n} \quad M P = 0$$

$$P(\tilde{X}_1, \tilde{X}_2) = P(X_1, X_2)$$

$$0 = P([M, X_1], X_2, \dots) + P(X_1, [M, X_2], \dots) + \dots$$

$$= P_{\substack{K A_2 \dots \\ L B_2 \dots}} (M_N^L X_{1K}^N - X_{1N}^L M_K^N) X_{2B_2}^{B_2} + \dots$$

$$+ P_{\substack{A_1 K \dots \\ B_1 L \dots}} X_{1A_1}^{B_1} (M_N^L X_{2K}^N - X_{2N}^L M_K^N) + \dots$$

$$0 = P_{\substack{A_1 A_2 \dots \\ L B_2 \dots}} M_{B_1}^L - P_{\substack{K A_2 \dots \\ B_1 B_2 \dots}} M_K^{B_1} + P_{\substack{A_1 A_2 \dots \\ B_1 L \dots}} M_{B_2}^L - P_{\substack{A_1 K \dots \\ B_1 B_2 \dots}} M_K^{B_2}$$

$$0 = M P_{\substack{A_1 A_2 \dots \\ B_1 B_2 \dots}} \quad M P = 0$$

Invariantní sym. polynomy

5) konstantnost polynomu

$$D_m C_{\mathbb{F}^k}^k = 0 \quad D_m t_{\mathbb{R}^A}^A = 0$$

$$DP = 0 \quad DP = 0$$

$$\partial P = 0 \quad DP = \partial P + AP$$

$$A_{\mathbb{R}^k}^k = \mathbb{R}^k C_{\mathbb{F}^k}^k$$

$$\text{str}_2(X_1, \dots, X_k) = \frac{1}{2!} \sum_G \text{tr}(X_{G_1} X_{G_2} \dots X_{G_k})$$

$$\text{str}_2(X) = \text{tr}(X^2)$$

$$\text{str}_2(X) \quad \text{str}_2 \propto K_{\mathbb{R}^k}$$

6) zobecnění na $\Lambda^p \otimes EM$

$$\chi_{m_1, \dots, m_p}^{\alpha} \quad X_{m_1, \dots, m_p}^A \in \mathbb{R}$$

$$\hat{P}(\chi_1, \chi_2, \dots) \equiv P(\chi_1, \chi_2, \dots)$$

$$= P_{\alpha_1, \alpha_2, \dots} \chi_1^{\alpha_1} \wedge \chi_2^{\alpha_2} \wedge \dots$$

\wedge skew-symetricky $\omega \wedge \sigma = (-1)^{pq} \sigma \wedge \omega$

$\hat{P}_2(X)$ je metrický pro k sudé

$$\hat{P}(X)$$

4) invariantnost vůči akci grupy

$$X \rightarrow \tilde{X} = T_{m_\tau} \cdot X \cdot T_{m_\tau}^{-1} \approx X + \tau [M, X]$$

$(\mathbb{1} + \tau M) \quad (\mathbb{1} - \tau M) \quad M_{\mathbb{F}^k}$

$$\chi \rightarrow \tilde{\chi} = \text{Ad}_{m_\tau} \chi \approx \chi + \tau (\text{ad}_M) \chi$$

$\tau [M, \chi]$

$$M_{\mathbb{R}^k} \quad M_{\mathbb{R}^k}^A = M_{\mathbb{F}^k}^k t_{\mathbb{R}^k}^A$$

$$P(\tilde{\chi}_1, \tilde{\chi}_2) = P(\chi_1, \chi_2)$$

$$P(\tilde{X}) = P(X) \quad P(\tilde{\chi}) = P(\chi)$$

$$M P = 0 \quad M P = 0$$

$$M_{\mathbb{R}^k}^A = M_{\mathbb{F}^k}^k t_{\mathbb{R}^k}^A \quad M_{\mathbb{R}^k}^k = M_{\mathbb{F}^k}^k C_{\mathbb{R}^k}^k$$

M
 e \nearrow m_τ 4) invariant - ost n'ici edici grupy

$$X \rightarrow \tilde{X} = T_{m_\tau} \cdot X \cdot T_{m_\tau}^{-1} \approx X + \tau [M, X]$$

$(\mathbb{1} + \tau M)$ $(\mathbb{1} - \tau M)$ M^α

$$\mathcal{X} \rightarrow \tilde{\mathcal{X}} = \text{Ad}_{m_\tau} \cdot \mathcal{X} \approx \mathcal{X} + \tau \text{ad}_M \mathcal{X}$$

$\tau [M, \mathcal{X}]$

$$M^\alpha \quad M_{\underline{B}}^{\underline{A}} = M^\alpha t_{\alpha \underline{B}}^{\underline{A}}$$

$$P(\tilde{X}_1, \tilde{X}_2) = P(X_1, X_2)$$

$$P(\tilde{X}) = P(X)$$

$$P(\tilde{\mathcal{X}}) = P(\mathcal{X})$$

$$M \parallel P = 0$$

$$M \parallel P = 0$$

$$M_{\underline{B}}^{\underline{A}} = M^k t_{\underline{F} \underline{B}}^{\underline{A}}$$

$$M_{\underline{B}}^{\underline{A}} = M^F c_{\underline{F} \underline{B}}^{\underline{A}}$$

Invariantni sym. polynomy

5) konstantnost polynomu

$$D_m C_{\mathbb{F}}^{\mathbb{R}} = 0 \quad D_m t_{\mathbb{R}}^{\mathbb{D}} = 0$$

$$DP = 0 \quad DP = 0$$

$$\partial P = 0 \quad DP = \partial P + \mathbb{A}P$$

$$A_{\mathbb{F}}^{\mathbb{R}} = \mathbb{R}^{\mathbb{K}} C_{\mathbb{F}}^{\mathbb{R}}$$

$$\text{str}_2(X_1, \dots, X_k) = \frac{1}{k!} \sum_G \text{tr}(X_{G_1} X_{G_2} \dots X_{G_k})$$

$$\text{str}_2(X) = \text{tr}(X^2)$$

$$\text{str}_2(X) \quad \text{str}_2 \propto K_{\mathbb{R}\mathbb{B}}$$

6) zobecnění na $\Lambda^p \otimes EM$

$$\mathcal{X}_{m_1 \dots m_p}^\alpha$$

$$X_{m_1 \dots m_p}^{\mathbb{A}} \quad \mathbb{B}$$

$$\hat{P}(\mathcal{X}_1, \mathcal{X}_2, \dots) \equiv P(\mathcal{X}_1 \uparrow \mathcal{X}_2 \uparrow \dots)$$

$$= P_{\alpha_1 \alpha_2 \dots} \mathcal{X}_1^{\alpha_1} \wedge \mathcal{X}_2^{\alpha_2} \wedge \dots$$

\wedge skew-symetrický $\omega \wedge \sigma = (-1)^{pq} \sigma \wedge \omega$
 $\begin{matrix} \omega & \sigma \\ p & q \end{matrix}$

$\hat{P}_\alpha(\mathcal{X})$ je metrický pro \mathbb{K} sudé

$$\hat{P}(\mathcal{X})$$