Invariantní sym. polynomy

Lievova alg. $\mathfrak{g}M$ $\quad Ad_{\mathfrak{r}}{}^{\alpha}{}_{\digamma}$ $\quad ad_{\mathcal{X}}{}^{\alpha}{}_{\digamma} = \mathcal{X}^{\kappa} c_{\kappa\digamma}{}^{\alpha}$

vekt. bndl. EM $\quad T_{\mathfrak{r}}{}^{A}{}_{B}$ $\quad t_{\mathcal{X}}{}^{A}{}_{B} = \mathcal{X}^{\digamma} t_{\digamma}{}^{A}{}_{B}$

$$X^{A}{}_{B} = \mathcal{X}^{\digamma} t_{\digamma}{}^{A}{}_{B} \qquad \mathfrak{R}\ A\ A \quad \digamma\ F\ F$$

1) multili. zobr. stupně k

$$P(\mathcal{X}_1,\dots,\mathcal{X}_k) = P_{\alpha_1\dots\alpha_k}\mathcal{X}_1^{\alpha_1}\cdots\mathcal{X}_k^{\alpha_k}$$
$$P(X_1,\dots,X_k) = P^{B_1\dots B_k}_{A_1\dots A_k} X_{1}{}^{A_1}{}_{B_1}\cdots X_{k}{}^{A_k}{}_{B_k}$$
$$P_{\alpha_1\dots\alpha_k} = t_{\alpha_1}{}^{A_1}{}_{B_1}\cdots t_{\alpha_k}{}^{A_k}{}_{B_k} P^{B_1\dots B_k}_{A_1\dots A_k}$$

2) symetrie

$$P_{\alpha_1\dots\alpha_k} = P_{(\alpha_1\dots\alpha_k)}$$
$$P(X_1,\dots,X_k) = P(X_{\sigma_1},\dots,X_{\sigma_k}) \qquad \sigma \text{ permut.}$$

3) polynomy

k-mocnina $\quad P(X) = P(X,\dots,X)$
$$P(\mathcal{X}) = P(\mathcal{X},\dots,\mathcal{X})$$

$$P(X_1,\dots,X_k) = \frac{1}{k!}\frac{\partial^k}{\partial\tau_1\cdots\partial\tau_k}P(\tau_1X_1+\dots+\tau_kX_k)$$

$$P(X) = \sum_k P_k(X) \qquad P_k \text{ } k\text{-mocnina}$$

$$(P\circ Q)(X) = \underset{p}{P}(X)\,\underset{q}{Q}(X)$$

$$(P\circ Q)_{\alpha_1\dots\alpha_{p+q}} = P_{(\alpha_1\dots\alpha_p}\,Q_{\alpha_{p+1}\dots\alpha_{p+q})}$$

4) invariantnost vůči akci grupy

$$e \to m_\tau \to M$$

$$X \to \tilde{X} = T_{m_\tau}\cdot X\cdot T^{-1}_{m_\tau} \approx X + \tau[M,X]$$
$$(\mathbb{1}+\tau M)\quad(\mathbb{1}-\tau M) \qquad M^{\alpha}{}_{\digamma}$$

$$\mathcal{X}\to\tilde{\mathcal{X}} = Ad_{m_\tau}\cdot\mathcal{X} \approx \mathcal{X} + \tau\, ad_{\mathcal{M}}\mathcal{X}$$
$$\tau[\mathcal{M},\mathcal{X}]$$

$$\mathcal{M}^{\alpha} \qquad M^{A}{}_{B} = \mathcal{M}^{\alpha} t_{\alpha}{}^{A}{}_{B}$$

$$P(\tilde{X}_1,\dots,\tilde{X}_k) = P(X_1\dots X_k)$$
$$P(\tilde{X}) = P(X)$$

$$M P = 0 \qquad \mathcal{M} P = 0$$

$$0 = \tau\left[P(\mathcal{M}\cdot\mathcal{X}_1,\mathcal{X}_2,\dots\mathcal{X}_k) + P(\mathcal{X}_1,\mathcal{M}\cdot\mathcal{X}_2,\dots) + \dots P(\mathcal{X}_1,\dots,\mathcal{M}\cdot\mathcal{X}_k)\right]$$
$$0 = P_{\digamma\alpha_2\dots\alpha_k}\mathcal{M}^{\digamma}{}_{\alpha_1} + P_{\alpha_1\digamma\dots\alpha_k}\mathcal{M}^{\digamma}{}_{\alpha_2} + \dots + P_{\alpha_1\alpha_2\dots\digamma}\mathcal{M}^{\digamma}{}_{\alpha_k}$$
$$0 = \mathcal{M}P_{\alpha_1\dots\alpha_k} \qquad \mathcal{M}P = 0$$

Invariantní sym. polynomy

Lieova alg. $\mathfrak{g}M$ $\quad \mathrm{Ad}_{h}{}^{\alpha}{}_{\beta} \quad \mathrm{ad}_{x}{}^{\alpha}{}_{\beta} = x^{\kappa} c_{\kappa\beta}{}^{\alpha}$

vekt. bundl. $\mathbb{E}M$ $\quad T_{h}{}^{A}{}_{B} \quad t_{x}{}^{A}{}_{B} = x^{\kappa} t_{\kappa}{}^{A}{}_{B}$

$$X^{A}{}_{B} = x^{\kappa} t_{\kappa}{}^{A}{}_{B} \qquad \begin{matrix} \mathfrak{A} & A & A \\ \mathfrak{F} & F & F \end{matrix}$$

1) multilin. zobr. stupně k

$$P(x_1, \dots, x_k) = P_{\alpha_1 \dots \alpha_k} x_1^{\alpha_1} \cdots x_k^{\alpha_k}$$

$$P(X_1, \dots, X_k) = P^{B_1 \dots B_k}_{A_1 \dots A_k} X_1{}^{A_1}{}_{B_1} \cdots X_k{}^{A_k}{}_{B_k}$$

$$P_{\alpha_1 \dots \alpha_k} = t_{\alpha_1}{}^{A_1}{}_{B_1} \cdots t_{\alpha_k}{}^{A_k}{}_{B_k} P^{B_1 \dots B_k}_{A_1 \dots A_k}$$

2) symetrické

$$P_{\alpha_1 \dots \alpha_k} = P_{(\alpha_1 \dots \alpha_k)}$$

$$P(X_1, \dots, X_k) = P(X_{\sigma_1}, \dots, X_{\sigma_k}) \qquad \sigma \text{ permut.}$$

3) polynomy

k-mocniowe

$$P(X) = P(X, \ldots, X)$$

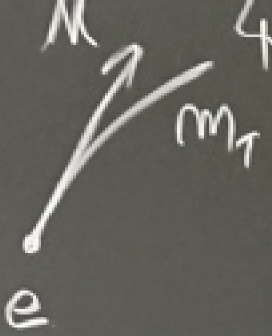
$$\mathcal{P}(\mathcal{X}) = \mathcal{P}(\mathcal{X}, \ldots, \mathcal{X})$$

$$P(X_1, \ldots, X_k) = \frac{1}{k!} \frac{\partial^k}{\partial \tau_1 \ldots \partial \tau_k} P(\tau_1 X_1 + \ldots + \tau_k X_k)$$

$$P(X) = \sum_k P_k(X)$$ P_k k-mocniowy

$$(P \circ Q)(X) = \underset{p}{P}(X)\, \underset{q}{Q}(X)$$

$$(P \circ Q)_{\underline{\alpha}_1 \ldots \underline{\alpha}_{p+q}} = P_{(\underline{\alpha}_1 \ldots \underline{\alpha}_p} Q_{\underline{\alpha}_{p+1} \ldots \underline{\alpha}_{p+q})}$$



4) invariant-ost vůči akci grupy

$$X \to \tilde{X} = T_{m_\tau} \cdot X \cdot T^{-1}_{m_\tau} \approx X + \tau[M, X]$$

$$(\mathbb{1} + \tau M) \quad (\mathbb{1} - \tau M) \qquad \mathcal{M}^\alpha{}_\beta$$

$$\mathcal{X} \to \tilde{\mathcal{X}} = \mathrm{Ad}_{m_\tau} \cdot \mathcal{X} \approx \mathcal{X} + \tau\, \mathrm{ad}_{\mathcal{M}}\, \mathcal{X}$$

$$\tau[\mathcal{M}, \mathcal{X}]$$

$$\mathcal{M}^\alpha \qquad M^A{}_B = \mathcal{M}^\alpha t_\alpha{}^A{}_B$$

$$P(\tilde{X}_1, \dots, \tilde{X}_k) = P(X_1 \dots X_k)$$

$$P(\tilde{X}) = P(X)$$

$$M P = 0 \qquad \mathbb{M} P = 0$$

$$0 = \tau\left[P(\mathcal{M} \cdot \mathcal{X}_1, \mathcal{X}_2, \dots \mathcal{X}_k) + P(\mathcal{X}_1, \mathcal{M} \cdot \mathcal{X}_2, \dots) + \dots P(\mathcal{X}_1, \dots, \mathcal{M} \cdot \mathcal{X}_k)\right]$$

$$0 = P_{\mu \alpha_2 \dots \alpha_k} \mathcal{M}^\mu{}_{\alpha_1} + P_{\alpha_1 \mu \dots \alpha_k} \mathcal{M}^\mu{}_{\alpha_2} + \dots + P_{\alpha_1 \alpha_2 \dots \mu} \mathcal{M}^\mu{}_{\alpha_k}$$

$$0 = \mathbb{M} P_{\alpha_1 \dots \alpha_k} \qquad \mathbb{M} P = 0$$

$$P(\tilde{X}_1, \ldots, \tilde{X}_k) = P(X_1 \ldots X_k)$$

$$0 = P([M, X_1], X_2, \ldots) + P(X_1, [M, X_2], \ldots) + \ldots$$

$$= P^{\underline{K}\,\underline{A}_2 \ldots}_{\underline{L}\,\underline{B}_2 \ldots} \left(M^{\underline{L}}_{\underline{N}} X_{1\,\underline{K}}^{\underline{N}} - X_{1\,\underline{N}}^{\underline{L}} M^{\underline{N}}_{\underline{K}} \right) X_{2\,\underline{A}_2}^{\underline{B}_2} \ldots +$$

$$+ P^{\underline{A}_1 \underline{K} \ldots}_{\underline{B}_1 \underline{L} \ldots} X_{1\,\underline{A}_1}^{\underline{B}_1} \left(M^{\underline{L}}_{\underline{N}} X_{2\,\underline{K}}^{\underline{N}} - X_{2\,\underline{N}}^{\underline{L}} M^{\underline{N}}_{\underline{K}} \right) \ldots + \ldots$$

$$0 = P^{\underline{A}_1 \underline{A}_2 \ldots}_{\underline{L}\,\underline{B}_2 \ldots} M^{\underline{L}}_{\underline{B}_1} - P^{\underline{K}\,\underline{A}_2 \ldots}_{\underline{B}_1 \underline{B}_2 \ldots} M^{\underline{A}_1}_{\underline{K}} + P^{\underline{A}_1 \underline{A}_2}_{\underline{B}_1 \underline{L}} M^{\underline{L}}_{\underline{B}_2} - P^{\underline{A}_1 \underline{K} \ldots}_{\underline{B}_1 \underline{B}_2 \ldots} M^{\underline{A}_2}_{\underline{K}}$$

$$0 = \mathbb{M}\, P^{\underline{A}_1 \underline{A}_2 \ldots}_{\underline{B}_1 \underline{B}_2 \ldots} \qquad \mathbb{M} P = 0$$

Invariantní sym. polynomy

5) Konstantnost polynomu

$$D_{m} c_{\kappa\beta}{}^{\alpha} = 0 \qquad D_{m} t_{\alpha}{}^{A}{}_{B} = 0$$

$$DP = 0 \qquad DP = 0$$

$$\partial P = 0 \qquad DP = \partial P + \mathbb{A} P$$

$$A^{\alpha}{}_{\beta} = \mathcal{A}^{\kappa} c_{\kappa\beta}{}^{\alpha}$$

$$\mathrm{str}_k(X_1, \dots, X_k) = \frac{1}{k!} \sum_{\sigma} \mathrm{tr}(X_{\sigma_1} X_{\sigma_2} \dots X_{\sigma_k})$$

$$\mathrm{str}_k(X) = \mathrm{tr}(X^k)$$

$$\mathrm{str}_k(\mathcal{X}) \qquad \mathrm{str}_2 \propto k_{\alpha\beta}$$

G zobecnění na $\Lambda^p \otimes EM$

$$\mathcal{X}^{\alpha}_{m_1 \dots m_p} \qquad X_{m_1 \dots m_p}{}^{A}{}_{B}$$

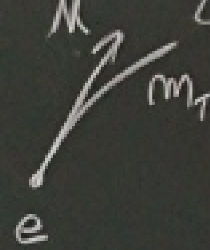
$$\hat{P}(\mathcal{X}_1, \mathcal{X}_2, \dots) \equiv P(\mathcal{X}_1 \wedge \mathcal{X}_2 \wedge \dots)$$

$$= P_{\alpha_1 \alpha_2 \dots} \mathcal{X}_1^{\alpha_1} \wedge \mathcal{X}_2^{\alpha_2} \wedge \dots$$

$\wedge$ skew-symetrický $\underset{p}{\omega} \wedge \underset{q}{\sigma} = (-1)^{pq} \sigma \wedge \omega$

$\hat{P}_k(\mathcal{X})$ je netriviální pro k sudé

$$\hat{P}(\mathcal{X})$$



4) invariantnost vůči akci grupy

$$X \to \tilde{X} = T_{m_\tau} \cdot X \cdot T^{-1}_{m_\tau} \approx X + \tau[M, X]$$

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$$\mathcal{X} \to \tilde{\mathcal{X}} = \mathrm{Ad}_{m_\tau} \mathcal{X} \approx \mathcal{X} + \tau\, \mathrm{ad}_{\mathcal{M}} \mathcal{X}$$

$$\tau[\mathcal{M}, \mathcal{X}]$$

$$\mathcal{M}^{\alpha} \qquad M^{A}{}_{B} = \mathcal{M}^{\alpha} t_{\alpha}{}^{A}{}_{B}$$

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$$P(\tilde{X}) = P(X) \qquad P(\tilde{\mathcal{X}}) = P(\mathcal{X})$$

$$\mathbb{M} P = 0 \qquad \mathbb{M} P = 0$$

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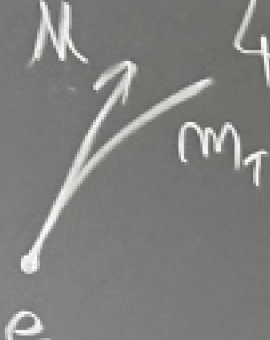

4) invariantnost vůči akci grupy

$$X \to \tilde{X} = \underset{(\mathbb{1}+\tau M)}{T_{m_\tau}} \cdot X \cdot \underset{(\mathbb{1}-\tau M)}{T^{-1}_{m_\tau}} \approx X + \tau[M, X]$$

$$M^{\alpha}{}_{\beta}$$

$$\mathcal{X} \to \tilde{\mathcal{X}} = Ad_{m_\tau} \cdot \mathcal{X} \approx \mathcal{X} + \tau\, ad_{\mathcal{M}}\, \mathcal{X}$$

$$\tau[\mathcal{M}, \mathcal{X}]$$

$$\mathcal{M}^{\alpha} \qquad M^{A}{}_{B} = \mathcal{M}^{\alpha} t_{\alpha}{}^{A}{}_{B}$$

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Invariantní sym. polynomy

5) konstantnost polynomu

$$D_{\underline{m}} c_{\mu\beta}{}^{\alpha} = 0 \qquad D_{\underline{m}} t_{\underline{\alpha}}{}^{\underline{A}}{}_{\underline{B}} = 0$$

$$DP = 0 \qquad DP = 0$$

$$\partial P = 0 \qquad DP = \partial P + \mathbb{A} P$$

$$A^{\alpha}{}_{\beta} = \mathcal{R}^{\kappa} c_{\kappa\beta}{}^{\alpha}$$

$$\mathrm{str}_k(X_1, \ldots, X_k) = \frac{1}{k!} \sum_{\sigma} \mathrm{tr}(X_{\sigma_1} X_{\sigma_2} \cdots X_{\sigma_k})$$

$$\mathrm{str}_k(X) = \mathrm{tr}(X^k)$$

$$\mathrm{str}_k(\mathcal{X}) \qquad \mathrm{str}_2 \propto k_{\alpha\beta}$$

↳ zobecnění na $\Lambda^p \otimes EM$

$$\mathcal{X}^{\underline{\alpha}}_{\underline{m}_1 \dots \underline{m}_p} \qquad X_{\underline{m}_1 \dots \underline{m}_p}{}^{\underline{A}}{}_{\underline{B}}$$

$$\hat{P}(\mathcal{X}_1, \mathcal{X}_2, \dots) \equiv P(\mathcal{X}_1 \overset{\wedge}{,} \mathcal{X}_2 \overset{\wedge}{,} \dots)$$

$$= P_{\underline{\alpha}_1 \underline{\alpha}_2 \dots} \, \mathcal{X}_1^{\underline{\alpha}_1} \wedge \mathcal{X}_2^{\underline{\alpha}_2} \wedge \dots$$

$\wedge$ skew-symetrický $\quad \underset{p}{\omega} \wedge \underset{q}{\sigma} = (-1)^{pq} \sigma \wedge \omega$

$\hat{P}_k(\mathcal{X})$ je netriviální pro k sudé

$\hat{P}(\mathcal{X})$