Chern-Weilova věta

D na gM resp. EM

křivost $\mathcal{F}^{\alpha}_{\underline{mn}}$ $\qquad F_{\underline{mn}}{}^{A}{}_{\underline{B}}$

inv. symetr. pol. $\hat{P}$ $\qquad \hat{P}$

$\Downarrow$

1) $\hat{P}(\mathcal{F})$ resp. $\hat{P}(F)$ jsou uzavřené

$d\hat{P}(\mathcal{F}) = 0 \qquad d\hat{P}(F) = 0$

2) $D \quad \mathcal{F} \quad F \qquad \tilde{D} \quad \tilde{\mathcal{F}} \quad \tilde{F}$

$\hat{P}(\tilde{\mathcal{F}}) - \hat{P}(\mathcal{F}) = d\,TP(\tilde{D}, D)$

$\hat{P}(\tilde{F}) - \hat{P}(F) = d\,TP(\tilde{D}, D)$

TP forma transgrese

$$d\hat{P}(\mathcal{F}) = D\wedge \hat{P}(\mathcal{F}) = 0$$

hom. pol. st. k $\quad = D\wedge P(\mathcal{F}, \mathcal{F}, \dots)$

$$= \underbrace{(D\wedge P)}_{0}(\mathcal{F}, \mathcal{F}, \dots) + \hat{P}(\underbrace{D\wedge\mathcal{F}}_{0}, \mathcal{F}, \dots) + \hat{P}(\mathcal{F}, \underbrace{D\wedge\mathcal{F}}_{0}, \dots) + \dots$$

$$D_m \wedge \left(P_{\alpha_1\alpha_2\dots}\, \mathcal{F}^{\alpha_1}_{k_1 l_1} \wedge \mathcal{F}^{\alpha_2}_{k_2 l_2} \wedge \dots\right)$$

$\tilde{D}, D \qquad \Delta = \tilde{D} - D$

$$D_\tau = \tau\tilde{D} + (1-\tau)D = D + \tau\Delta = \tilde{D} - (1-\tau)\Delta$$

$$F_\tau = F + \tau D\wedge\Delta + \tau^2 \Delta^{\wedge 2} = \tau(F + D\wedge\Delta + \Delta^{\wedge 2}) + (1-\tau)F - \tau(1-\tau)\Delta^{\wedge 2}$$

$$= \tau\tilde{F} + (1-\tau)F - \tau(1-\tau)\Delta^{\wedge 2}$$

$$\frac{\partial}{\partial\tau} D_\tau = \Delta \qquad \frac{\partial}{\partial\tau} F_\tau = D\wedge\Delta + 2\tau\Delta^{\wedge 2} = D\wedge\Delta + [\tau\Delta \overset{\wedge}{,} \Delta] = D_\tau \wedge \Delta$$

$$\frac{\partial}{\partial\tau}\hat{P}(F_\tau) = k\,\hat{P}\left(\frac{\partial}{\partial\tau}F_\tau, F_\tau, \dots\right) = k\,\hat{P}(D_\tau\wedge\Delta, F_\tau, \dots) = k\, D_\tau\wedge\hat{P}(\Delta, F_\tau, \dots) = k\, d\,\hat{P}(\Delta, F_\tau, \dots)$$

hom. stupně k

$[\Delta, \Delta] \qquad \Delta_{\underline{m}}{}^{A}{}_{\underline{K}} \qquad [\Delta_{\underline{m}}, \Delta_{\underline{n}}]^{A}{}_{\underline{B}} \qquad \Delta^{\wedge 2}$

$$[\Delta_{\underline{m}}, \Delta_{\underline{n}}] = \Delta_{\underline{m}}\cdot\Delta_{\underline{n}} - \Delta_{\underline{n}}\cdot\Delta_{\underline{m}} = \underbrace{\Delta_{\underline{m}}\wedge\Delta_{\underline{n}}}_{\Delta^{\wedge 2}}$$

$$[A_{\underline{m}}, B_{\underline{n}}] = A_{\underline{m}}\cdot B_{\underline{n}} - B_{\underline{n}}\cdot A_{\underline{m}} \neq A_{\underline{m}}\wedge B_{\underline{n}}$$

$$A_{\underline{m}}\wedge B_{\underline{n}} = A_{\underline{m}}\cdot B_{\underline{n}} - A_{\underline{n}}\cdot B_{\underline{m}}$$

$$[A_{\underline{m}} \overset{\wedge}{,} B_{\underline{n}}] = A_{\underline{m}}\wedge B_{\underline{n}} - B_{\underline{n}}\wedge A_{\underline{m}} =$$

$$= A_{\underline{m}}\cdot B_{\underline{n}} - A_{\underline{n}}\cdot B_{\underline{m}} - B_{\underline{n}}\cdot A_{\underline{m}} + B_{\underline{m}}\cdot A_{\underline{n}}$$

$$[\Delta_{\underline{m}}, \Delta_{\underline{n}}] = \Delta_{\underline{m}}\cdot\Delta_{\underline{n}} - \Delta_{\underline{n}}\cdot\Delta_{\underline{m}} = \Delta_{\underline{m}}\wedge\Delta_{\underline{n}}$$

$$[\Delta_{\underline{m}} \overset{\wedge}{,} \Delta_{\underline{n}}] = 2[\Delta_{\underline{m}}, \Delta_{\underline{n}}] = 2\Delta_{\underline{m}}\wedge\Delta_{\underline{n}}$$

$$P'(\Delta, X) = \sum_k k\, P_k(\Delta, X, X, \dots)$$

$$= \sum_k \left(P_k(\Delta, X, X, \dots) + P_k(X, \Delta, X, \dots) + P_k(X, X, \Delta, \dots)\right)$$

$$\frac{\partial}{\partial\tau}\hat{P}(F_\tau) = d\,\hat{P}'(\Delta, F_\tau)$$

$$\hat{P}(\tilde{F}) - \hat{P}(F) = d\underbrace{\int_0^1 \hat{P}'(\Delta, F_\tau)\, d\tau}_{TP(\tilde{D}, D)}$$

$$TP(\tilde{D}, D) = \int_0^1 P'(\Delta, F_\tau)\, d\tau$$

Chern-Weilova věta

D na gM resp. EM

křivost $\mathcal{F}_{\underline{mn}}{}^{\underline{\alpha}}$ $\qquad$ $F_{\underline{mn}}{}^{\underline{A}}{}_{\underline{B}}$

inv. sym. pol. P $\qquad$ P

$\Downarrow$

1) $\hat{P}(\mathcal{F})$ resp. $\hat{P}(F)$ jsou uzavřené

$$d\hat{P}(\mathcal{F})=0 \qquad d\hat{P}(F)=0$$

2) $D \quad \mathcal{F} \quad F \qquad \tilde{D} \quad \tilde{\mathcal{F}} \quad \tilde{F}$

$$\hat{P}(\tilde{\mathcal{F}})-\hat{P}(\mathcal{F}) = d\,TP(\tilde{D},D)$$

$$\hat{P}(\tilde{F})-\hat{P}(F) = d\,TP(\tilde{D},D)$$

TP forma transgrese

$$d\hat{P}(\hat{f}) = D\wedge \hat{P}(\hat{f}) = 0$$

hom poly st $\mathbb{k}$ $= D\wedge P(\hat{f}, \hat{f}, \dots)$

$$= \underbrace{(D\wedge P)}_{0}(\hat{f}, \hat{f}, \dots) + \hat{P}(\underbrace{D\wedge \hat{f}}_{0}, \hat{f}, \dots) + \hat{P}(\hat{f}, \underbrace{D\wedge \hat{f}}_{0}, \dots) + \dots$$

$$D_m \wedge \left(P_{\underline{\alpha_1 \alpha_2} \dots} \hat{f}^{\alpha_1}_{k_1 l_1} \wedge \hat{f}^{\alpha_2}_{k_2 l_2} \wedge \dots \right)$$

$$[\Delta,\Delta] \qquad \Delta_{\underline{m}}{}^{\underline{A}}{}_{\underline{K}} \qquad [\Delta_{\underline{m}},\Delta_{\underline{n}}]^{\underline{A}}{}_{\underline{B}} \qquad \Delta^{\wedge 2}$$

$$[\Delta_{\underline{m}},\Delta_{\underline{n}}] = \Delta_{\underline{m}}\cdot\Delta_{\underline{n}} - \Delta_{\underline{n}}\cdot\Delta_{\underline{m}} = \overbrace{\Delta_{\underline{m}}\wedge\Delta_{\underline{n}}}$$

$$\tilde{D},\, D \qquad \Delta = \tilde{D} - D$$

$$D_\tau = \tau\tilde{D} + (1-\tau)D = D + \tau\Delta = \tilde{D} - (1-\tau)\Delta$$

$$F_\tau = F + \tau D\wedge\Delta + \tau^2\Delta^{\wedge 2} = \tau\left(F + D\wedge\Delta + \Delta^{\wedge 2}\right) + (1-\tau)F - \tau(1-\tau)\Delta^{\wedge 2}$$

$$= \tau\tilde{F} + (1-\tau)F - \tau(1-\tau)\Delta^{\wedge 2}$$

$$\frac{\partial}{\partial\tau}D_\tau = \Delta \qquad \frac{\partial}{\partial\tau}F_\tau = D\wedge\Delta + 2\tau\Delta^{\wedge 2} = D\wedge\Delta + [\tau\Delta,\Delta] = D_\tau\wedge\Delta$$

$$\frac{\partial}{\partial\tau}\hat{P}(F_\tau) = k\,\hat{P}\left(\frac{\partial}{\partial\tau}F_\tau, F_\tau, \dots\right) = k\,\hat{P}(D_\tau\wedge\Delta, F_\tau, \dots) = k\,D_\tau\wedge\hat{P}(\Delta, F_\tau, \dots) = k\,d\,\hat{P}(\Delta, F_\tau, \dots)$$

hom. [illegible] k

$$[\Delta,\Delta] \qquad \Delta_{\underline{m}}{}^{\underline{A}}{}_{\underline{K}} \qquad [\Delta_{\underline{m}},\Delta_{\underline{n}}]^{\underline{A}}{}_{\underline{B}} \qquad \Delta^{12}$$

$$[\Delta_{\underline{m}},\Delta_{\underline{n}}] = \Delta_{\underline{m}}\cdot\Delta_{\underline{n}} - \Delta_{\underline{n}}\cdot\Delta_{\underline{m}} = \Delta_{\underline{m}}\wedge\Delta_{\underline{n}}$$

$$[A_{\underline{m}},B_{\underline{n}}] = A_{\underline{m}}\cdot B_{\underline{n}} - B_{\underline{n}}\cdot A_{\underline{m}} \neq A_{\underline{m}}\wedge B_{\underline{n}}$$

$$A_{\underline{m}}\wedge B_{\underline{n}} = A_{\underline{m}}\cdot B_{\underline{n}} - A_{\underline{n}}\cdot B_{\underline{m}}$$

$$[A_{\underline{m}} \overset{\wedge}{,} B_{\underline{n}}] = A_{\underline{m}}\wedge B_{\underline{n}} - B_{\underline{n}}\wedge A_{\underline{m}} =$$

$$= A_{\underline{m}}\cdot B_{\underline{n}} - A_{\underline{n}}\cdot B_{\underline{m}} - B_{\underline{n}}\cdot A_{\underline{m}} + B_{\underline{m}}\cdot A_{\underline{n}}$$

$$[\Delta_{\underline{m}},\Delta_{\underline{n}}] = \Delta_{\underline{m}}\cdot\Delta_{\underline{n}} - \Delta_{\underline{n}}\cdot\Delta_{\underline{m}} = \Delta_{\underline{m}}\wedge\Delta_{\underline{n}}$$

$$[\Delta_{\underline{m}} \overset{\wedge}{,} \Delta_{\underline{n}}] = 2[\Delta_{[\underline{m}},\Delta_{\underline{n}]}] = 2\,\Delta_{\underline{m}}\wedge\Delta_{\underline{n}}$$

$$P'(\Delta, x) = \sum_k k\, P_k(\Delta, x, x, \dots)$$

$$= \sum_k \left(P_k(\Delta, x, x, \dots) + P_k(x, \Delta, x, \dots) + P_k(x, x, \Delta, \dots) \right)$$

$$\frac{\partial}{\partial \tau} \hat{P}(F_\tau) = d\, \hat{P}'(\Delta, F_\tau)$$

$$\hat{P}(\tilde{F}) - \hat{P}(F) = d \underbrace{\int_0^1 \hat{P}'(\Delta, F_\tau)\, d\tau}_{TP(\hat{D}, D)}$$

$$TP(\tilde{D}, D) = \int_0^1 P'(\Delta, F_\tau)\, d\tau$$