Charakteristické třídy

P inv. sym. polynom

$$p = [\hat{P}(F)] \in H(M)$$

- kohomol. gr.
- nezávisí na D
- P homogenní st. $k \to 2k$-forma
- P nehomogenní $\quad p_k$ homog. kompon.

$$\underbrace{\int_M p \wedge q \wedge \dots}_{\text{stejné } \partial} = \int_M \hat{P}(F) \wedge \hat{Q}(F) \wedge \dots$$

$$\int_M (\hat{P}(F) + d\alpha) \wedge \hat{Q}(F) \wedge \dots = \int \dots + \int_M \overbrace{d(\alpha \wedge \hat{Q}(F) \wedge \dots)}^{0}$$

Chernovy charakteristické třídy

$E\,M$ komplex. vekt. b. s herm. str.

$$\hat{C}(F) = \widehat{\det}\left(\mathbb{1} + \frac{i\alpha}{2\pi} F\right)$$

$$C = [\hat{C}(F)] \qquad \text{nehomogenní}$$

$$\hat{C}(F) = 1 + \alpha \hat{C}_1(F) + \alpha^2 \hat{C}_2(F) + \dots$$

$$C_k = [\hat{C}_k(F)]$$

$$\widehat{\det}(\mathbb{1} + \lambda F) = {}^{[N]}\delta^{B_1 \dots B_N}_{A_1 \dots A_N} (\mathbb{1} + \lambda F)^{A_1}{}_{B_1} \wedge \dots \wedge (\mathbb{1} + \lambda F)^{A_N}{}_{B_N}$$

$$= \sum_{k=0}^{N} \lambda^k\, {}^{[N]}\delta^{B_1 \dots B_k C_1 \dots C_{N-k}}_{A_1 \dots A_k C_1 \dots C_{N-k}} \binom{N}{k} F^{A_1}{}_{B_1} \wedge \dots \wedge F^{A_k}{}_{B_k}$$

$$= \sum_{k=0}^{N} \lambda^k F^{A_1}{}_{[A_1} \wedge \dots \wedge F^{A_k}{}_{A_k]}$$

$$= 1 + \lambda F^A{}_A$$

$$+ \frac{1}{2} \lambda^2 \left(F^A{}_A \wedge F^B{}_B - F^A{}_B \wedge F^B{}_A \right)$$

$$+ \frac{1}{3!} \lambda^3 \big(F^A{}_A \wedge F^B{}_B \wedge F^C{}_C + F^A{}_B \wedge F^B{}_C \wedge F^C{}_A + F^A{}_C \wedge F^B{}_A \wedge F^C{}_B$$

$$- F^A{}_A \wedge F^B{}_C \wedge F^C{}_B - F^A{}_B \wedge F^B{}_A \wedge F^C{}_C - F^A{}_C \wedge F^B{}_B \wedge F^C{}_A \big)$$

$$+ \dots$$

$$= 1 + \lambda \operatorname{tr} F + \frac{1}{2} \lambda^2 \left((\operatorname{tr} F)^{\wedge 2} - \operatorname{tr} F^{\wedge 2} \right)$$

$$+ \frac{1}{3!} \lambda^3 \left((\operatorname{tr} F)^{\wedge 3} + 2 \operatorname{tr} F^{\wedge 3} - 3 (\operatorname{tr} F) \wedge (\operatorname{tr} F^{\wedge 2}) \right)$$

$$+ \dots$$

Charakteristické třídy

P inv. sym. polynom

$$p = [\hat{P}(F)] \in H(M)$$

- kohomol. gr.
- nezávisí na D
- P homogenní st $k \to 2k$-forma
- P nehomogenní $\quad p_k$ homog. komp.

$$\int_M \underbrace{p \wedge q \wedge \dots}_{\text{stupeň } d} = \int_M \hat{P}(F) \wedge \hat{Q}(F) \wedge \dots$$

$$\int_M (\hat{P}(F) + d\alpha) \wedge \hat{Q}(F) \wedge \dots = \int \dots + \int_M d(\alpha \wedge \hat{Q}(F) \wedge \dots) \to 0$$

$$P(X) = P(X, \ldots, X)$$

$$\mathcal{P}(\mathcal{X}) = \mathcal{P}(\mathcal{X}, \ldots, \mathcal{X}) = P_{\underline{\alpha}_1 \ldots \underline{\alpha}_n} \mathcal{X}^{\underline{\alpha}_1} \ldots \mathcal{X}^{\underline{\alpha}_n}$$

$$\mathcal{X}^{\underline{\kappa}} \to \mathcal{X}^{\alpha} \qquad E^{\underline{\kappa}}_{\alpha}$$

Chernovy charakteristické třídy

$E M$ komplex. vekt. b. s herm. str.

$$\hat{C}(F) = \hat{\det}\left(\mathbb{1} + \frac{i\alpha}{2\pi} F \right)$$

$$C = \left[\hat{C}(F) \right]$$ nehomogenní

$$\hat{C}(F) = 1 + \alpha \hat{C}_1(F) + \alpha^2 \hat{C}_2(F) + \dots$$

$$C_k = \left[\hat{C}_k(F) \right]$$

$$\hat{\det}(\mathbb{1} + \lambda F) = {}^{[N]}\delta^{B_1 \dots B_N}_{A_1 \dots A_N} (\mathbb{1} + \lambda F)^{A_1}{}_{B_1} \wedge \dots \wedge (\mathbb{1} + \lambda F)^{A_N}{}_{B_N}$$

$$= \sum_{k=0}^{N} \lambda^k \, {}^{[N]}\delta^{B_1 \dots B_k C_1 \dots C_{N-k}}_{A_1 \dots A_k C_1 \dots C_{N-k}} \binom{N}{k} F^{A_1}{}_{B_1} \wedge \dots \wedge F^{A_k}{}_{B_k}$$

$$= \sum_{k=0}^{N} \lambda^k F^{A_1}{}_{[A_1} \wedge \dots \wedge F^{A_k}{}_{A_k]}$$

$$= 1 + \lambda F^{A}{}_{A}$$

$$+ \frac{1}{2}\lambda^{2}\left(F^{A}{}_{A} \wedge F^{B}{}_{B} - F^{A}{}_{B} \wedge F^{B}{}_{A}\right)$$

$$+ \frac{1}{3!}\lambda^{3}\Big(F^{A}{}_{A} \wedge F^{B}{}_{B} \wedge F^{C}{}_{C} + F^{A}{}_{B} \wedge F^{B}{}_{C} \wedge F^{C}{}_{A} + F^{A}{}_{C} \wedge F^{B}{}_{A} \wedge F^{C}{}_{B}$$

$$- F^{A}{}_{A} \wedge F^{B}{}_{C} \wedge F^{C}{}_{B} - F^{A}{}_{B} \wedge F^{B}{}_{A} \wedge F^{C}{}_{C} - F^{A}{}_{C} \wedge F^{B}{}_{B} \wedge F^{C}{}_{A}\Big)$$

$$+ \cdots$$

$$= 1 + \lambda \,\mathrm{tr} F + \frac{1}{2}\lambda^{2}\left((\mathrm{tr} F)^{\wedge 2} - \mathrm{tr} F^{\wedge 2}\right)$$

$$+ \frac{1}{3!}\lambda^{3}\left((\mathrm{tr} F)^{\wedge 3} + 2\,\mathrm{tr} F^{\wedge 3} - 3(\mathrm{tr} F) \wedge (\mathrm{tr} F^{\wedge 2})\right)$$

$$+ \cdots$$

Chernova charakteristika

$$\hat{Ch}(F) = \operatorname{tr} \exp\Big(\underbrace{\tfrac{i\alpha}{2\pi}}_{\lambda} F\Big)$$

$$= \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} \operatorname{tr} F^{\wedge k} = \sum_k \hat{Ch}_k(F)$$

$$ch = [\hat{Ch}(F)] \qquad ch_k = [\hat{Ch}_k(F)]$$

$$\hat{Ch}_k(F) = \frac{\lambda^k}{k!} \operatorname{tr}(F^{\wedge k}) = \frac{\lambda^k}{k!} \operatorname{str}_k(F, \dots, F)$$

$$\operatorname{str}_k(A_1, \dots, A_k) = \sum_{\text{perm } \sigma} \operatorname{tr}(A_{\sigma_1} \cdots A_{\sigma_k})$$

$$\operatorname{str}_{k\,\alpha_1 \dots \alpha_k} = t_{(\alpha_1}{}^{B_1}{}_{B_2}\, t_{\alpha_2}{}^{B_2}{}_{B_3} \cdots t_{\alpha_k)}{}^{B_k}{}_{B_1}$$

$$\hat{\log} \hat{\det}(\mathbb{1} + \lambda F) = \operatorname{tr} \hat{\log}(\mathbb{1} + \lambda F)$$

$$\operatorname{tr}\Big(\lambda F - \frac{1}{2}\lambda^2 F^{\wedge 2} + \frac{1}{3}\lambda^3 F^{\wedge 3} - \dots\Big)$$

$$= \sum_{k=1}^{\infty} (-1)^{k-1} \frac{\lambda^k}{k} \operatorname{tr} F^{\wedge k}$$

$$= \sum_{k=1}^{\infty} (-1)^{k-1} (k-1)!\, \hat{Ch}_k(F)$$

$$= \hat{Ch}_1 - \hat{Ch}_2 + 2\hat{Ch}_3 - \dots$$

$$\hookrightarrow = \hat{\log}\Big(1 + \sum_{k=1}^{\infty} \hat{C}_k\Big) =$$

$$= (\hat{C}_1 + \hat{C}_2 + \hat{C}_3 + \dots) - \frac{1}{2}(\hat{C}_1 + \hat{C}_2 + \dots)^{\wedge 2} + \frac{1}{3}(\hat{C}_1 + \dots)^{\wedge 3} + \dots$$

$$= \hat{C}_1 + \Big(\hat{C}_2 - \frac{1}{2}\hat{C}_1^{\wedge 2}\Big) + \Big(\hat{C}_3 - \hat{C}_2 \wedge \hat{C}_1 + \frac{1}{3}\hat{C}_1^{\wedge 3}\Big) + \dots$$

$$\hat{Ch}_1 = \hat{C}_1 \qquad \hat{Ch}_2 = -\hat{C}_2 + \frac{1}{2}\hat{C}_1^{\wedge 2}$$

$$\hat{Ch}_3 = \frac{1}{2}\hat{C}_3 - \frac{1}{2}\hat{C}_2 \wedge \hat{C}_1 + \frac{1}{6}\hat{C}_1^{\wedge 3}$$

$$\hat{C}(F) = 1 + \lambda F^A{}_A$$

$$+ \frac{1}{2}\lambda^2\big(F^A{}_A \wedge F^B{}_B - F^A{}_B \wedge F^B{}_A\big)$$

$$+ \frac{1}{3!}\lambda^3\big(F^A{}_A \wedge F^B{}_B \wedge F^C{}_C + F^A{}_B \wedge F^B{}_C \wedge F^C{}_A + F^A{}_C \wedge F^B{}_A \wedge F^C{}_B$$

$$- F^A{}_A \wedge F^B{}_C \wedge F^C{}_B - F^A{}_B \wedge F^B{}_A \wedge F^C{}_C - F^A{}_C \wedge F^B{}_B \wedge F^C{}_A\big)$$

$$+ \dots$$

$$= 1 + \lambda \operatorname{tr} F + \frac{1}{2}\lambda^2\big((\operatorname{tr} F)^{\wedge 2} - \operatorname{tr} F^{\wedge 2}\big)$$

$$+ \frac{1}{3!}\lambda^3\big((\operatorname{tr} F)^{\wedge 3} + 2\operatorname{tr} F^{\wedge 3} - 3(\operatorname{tr} F) \wedge (\operatorname{tr} F^{\wedge 2})\big)$$

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Chernova charakteristika

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$$str_k(A_1,\dots,A_k) = \sum_{\text{perm } \sigma} \mathrm{tr}\big(A_{\sigma_1}\cdots A_{\sigma_k}\big)$$

$$str_{k\,\alpha_1\dots\alpha_k} = t_{(\alpha_1}{}^{A_1}{}_{A_2}\, t_{\alpha_2}{}^{A_2}{}_{A_3} \cdots t_{\alpha_k)}{}^{A_k}{}_{A_1}$$

$$\hat{C}(F) = 1 + \lambda F^{A}{}_{A}$$

$$+ \frac{1}{2}\lambda^2\left(F^{A}{}_{A} \wedge F^{B}{}_{B} - F^{A}{}_{B} \wedge F^{B}{}_{A}\right)$$

$$+ \frac{1}{3!}\lambda^3\left(F^{A}{}_{A} \wedge F^{B}{}_{B} \wedge F^{C}{}_{C} + F^{A}{}_{B} \wedge F^{B}{}_{C} \wedge F^{C}{}_{A} + F^{A}{}_{C} \wedge F^{B}{}_{A} \wedge F^{C}{}_{B}\right.$$

$$\left. - F^{A}{}_{A} \wedge F^{B}{}_{C} \wedge F^{C}{}_{B} - F^{A}{}_{B} \wedge F^{B}{}_{A} \wedge F^{C}{}_{C} - F^{A}{}_{C} \wedge F^{B}{}_{B} \wedge F^{C}{}_{A}\right)$$

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$$\hat{\log}\,\hat{\det}(\mathbb{1}+\lambda F) = \mathrm{tr}\,\hat{\log}(\mathbb{1}+\lambda F)$$

$$\mathrm{tr}\left(\lambda F - \frac{1}{2}\lambda^2 F^{\wedge 2} + \frac{1}{3}\lambda^3 F^{\wedge 3} - \dots\right)$$

$$= \sum_{k=1}^{\infty} (-1)^{k-1} \frac{\lambda^k}{k} \,\mathrm{tr}\, F^{\wedge k}$$

$$= \sum_{k=1}^{\infty} (-1)^{k-1} (k-1)!\, \hat{Ch}_k(F)$$

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