

# Charakteristické třídy

Primový polynom

$$\mu = [\hat{P}(F)] \in H(M)$$

- kohomol. gr.
- mezivěsí na D
- P homogenní at  $2k \rightarrow 2k$ -formy
- P nehomogenní  $\mu_{2k}$  homog. kom.

$$\int_M \underbrace{\mu \wedge \nu \wedge \dots}_{\text{stupně d}} = \int_M \hat{P}(F) \wedge \hat{Q}(F) \wedge \dots$$

$$\int_M (\hat{P}(F) + dx) \wedge \hat{Q}(F) \wedge \dots = \int_M \dots + \int_M d(\alpha \wedge \hat{Q}(F) \wedge \dots)$$

# Chernovy charakteristické třídy

EM komplex vekt. b. a herm. str.

$$\hat{C}(F) = \det \left( \mathbb{1} + \frac{i\alpha}{2\pi} F \right)$$

$$C = [\hat{C}(F)] \quad \text{nehomogen}$$

$$\hat{C}(F) = 1 + \alpha \hat{C}_1(F) + \alpha^2 \hat{C}_2(F) + \dots$$

$$C_2 = [\hat{C}_2(F)]$$

$$\det(\mathbb{1} + \lambda F) = \sum_{A_1, B_1}^N \sum_{A_2, B_2}^N \dots \sum_{A_N, B_N}^N (\mathbb{1} + \lambda F)_{B_1}^{A_1} \wedge \dots \wedge (\mathbb{1} + \lambda F)_{B_N}^{A_N}$$

$$= \sum_{\substack{2=0 \\ \dots \\ k=0}}^N \lambda^k \sum_{A_1, B_1}^N \sum_{A_2, B_2}^N \dots \sum_{A_k, B_k}^N \binom{N}{k} F_{B_1}^{A_1} \wedge \dots \wedge F_{B_k}^{A_k}$$

$$= \sum_{k=0}^N \lambda^k F_{[A_1} \wedge \dots \wedge F_{B_k]}$$

$$= 1 + \lambda F^A_A$$

$$+ \frac{1}{2} \lambda^2 (F^A_A \wedge F^B_B - F^A_{B_1} F^B_{A_1})$$

$$+ \frac{1}{3!} \lambda^3 (F^A_{A_1} F^B_{B_1} F^C_{C_1} + F^A_{B_1} F^B_{C_1} F^C_{A_1} + F^A_{C_1} F^B_{A_1} F^C_{B_1} - F^A_{A_1} F^B_{C_1} F^C_{B_1} - F^A_{B_1} F^B_{A_1} F^C_{C_1} - F^A_{C_1} F^B_{B_1} F^C_{A_1})$$

$$+ \dots - 1 + \lambda \operatorname{tr} F + \frac{1}{2} \lambda^2 ((\operatorname{tr} F)^2 - \operatorname{tr} F^2)$$

$$+ \frac{1}{3!} \lambda^3 ((\operatorname{tr} F)^3 + 2 \operatorname{tr} F^3 - 3 \operatorname{tr} F \wedge (\operatorname{tr} F^2))$$

+ ...

# Charakteristické třídy

$P$  iro sym. polynom

$$\mu = [\hat{P}(F)] \in H(M)$$

- kochanol. gr.
- mezárnivní me  $D$
- $P$  homogenní st  $k \rightarrow 2k$ -formy
- $P$  nehomogenní  $\mu_k$  homog. kom.

$$\int_M \underbrace{\mu \wedge \nu \wedge \dots}_{\text{stupně } d} = \int_M \hat{P}(F) \wedge \hat{Q}(F) \wedge \dots$$

$$\int_M (\hat{P}(F) + d\alpha) \wedge \hat{Q}(F) \wedge \dots = \int_M \dots + \int_M d(\alpha \wedge \hat{Q}(F) \wedge \dots)$$

$$P(x) = P(x, \dots, x)$$

$$P(x) = P(x_1, \dots, x_n) = P_{x_1 \dots x_n} x^{\alpha_1} \dots x^{\alpha_n}$$

$$x^{\beta} \rightarrow x^{\alpha} \quad \mathbb{E}_{\alpha}^{\beta}$$

# Chernovy charakteristické třídy

EM komplex věst. b. a herm. str.

$$\hat{C}(F) = \det \left( \mathbb{1} + \frac{i\alpha}{2\pi} F \right)$$

$$c = [ \hat{C}(F) ] \quad \text{nehomogení}$$

$$\hat{C}(F) = 1 + \alpha \hat{C}_1(F) + \alpha^2 \hat{C}_2(F) + \dots$$

$$c_2 = [ \hat{C}_2(F) ]$$

$$\det(\mathbb{1} + \lambda F) = \sum_{A_1, \dots, A_N}^{[N]} \sum_{B_1, \dots, B_N}^{[N]} (\mathbb{1} + \lambda F)_{B_1}^{A_1} \wedge \dots \wedge (\mathbb{1} + \lambda F)_{B_N}^{A_N}$$

$$= \sum_{\ell=0}^N \lambda^\ell \sum_{A_1, \dots, A_\ell}^{[N]} \sum_{B_1, \dots, B_\ell}^{[N]} \binom{N}{\ell} F_{B_1}^{A_1} \wedge \dots \wedge F_{B_\ell}^{A_\ell}$$

$$= \sum_{\ell=0}^N \lambda^\ell F_{[A_1} \wedge \dots \wedge F_{A_\ell]}$$

$$= 1 + \lambda F^A_A$$

$$+ \frac{1}{2} \lambda^2 (F^A_A \wedge F^B_B - F^A_B \wedge F^B_A)$$

$$+ \frac{1}{3!} \lambda^3 (F^A_A \wedge F^B_B \wedge F^C_C + F^A_B \wedge F^B_C \wedge F^C_A + F^A_C \wedge F^B_A \wedge F^C_B$$

$$- F^A_A \wedge F^B_C \wedge F^C_B - F^A_B \wedge F^B_A \wedge F^C_C - F^A_C \wedge F^B_B \wedge F^C_A)$$

+ ...

$$= 1 + \lambda \operatorname{tr} F + \frac{1}{2} \lambda^2 ((\operatorname{tr} F)^{\wedge 2} - \operatorname{tr} F^{\wedge 2})$$

$$+ \frac{1}{3!} \lambda^3 ((\operatorname{tr} F)^{\wedge 3} + 2 \operatorname{tr} F^{\wedge 3} - 3 (\operatorname{tr} F) \wedge (\operatorname{tr} F^{\wedge 2}))$$

+ ...

# Chernova charakteristika

$$\hat{Ch}(F) = \text{tr} \exp\left(\frac{i\lambda}{2\pi} F\right) = \sum_{q=0}^{\infty} \frac{\lambda^q}{q!} \text{tr} F^{Aq} = \sum_q \hat{Ch}_q(F)$$

$$ch = [\hat{Ch}(F)] \quad ch_q = [\hat{Ch}_q(F)]$$

$$\hat{Ch}_q(F) = \frac{\lambda^q}{q!} \text{tr}(F^{Aq}) = \frac{\lambda^q}{q!} \text{str}_q(F, \dots, F)$$

$$\text{str}_q(A_1, \dots, A_q) = \sum_{\mu \in S_q} \text{tr}(A_{\sigma_1} \dots A_{\sigma_q})$$

$$\text{str}_{q \times 1 \dots \times 1} = t_{\begin{pmatrix} A_1 \\ \alpha_1 \end{pmatrix} \beta_1} t_{\begin{pmatrix} A_2 \\ \alpha_2 \end{pmatrix} \beta_2} \dots t_{\begin{pmatrix} A_q \\ \alpha_q \end{pmatrix} \beta_q}$$

$$\log \det(1 + \lambda F) = \text{tr} \log(1 + \lambda F)$$

$$\text{tr}(\lambda F - \frac{1}{2} \lambda^2 F^{A2} + \frac{1}{3} \lambda^3 F^{A3} - \dots)$$

$$= \sum_{q=1}^{\infty} (-1)^{q-1} \frac{\lambda^q}{q} \text{tr} F^{Aq}$$

$$= \sum_{q=1}^{\infty} (-1)^{q-1} (q-1)! \hat{Ch}_q(F)$$

$$= \hat{Ch}_1 - \hat{Ch}_2 + 2 \hat{Ch}_3 - \dots$$

$$\Rightarrow \log(1 + \sum_{q=1}^{\infty} \hat{C}_q) =$$

$$= (\hat{C}_1 + \hat{C}_2 + \hat{C}_3 + \dots) - \frac{1}{2} (\hat{C}_1 + \hat{C}_2 + \dots)^{A2} + \frac{1}{3} (\hat{C}_1 + \dots)^{A3} - \dots$$

$$= \hat{C}_1 + (\hat{C}_2 - \frac{1}{2} \hat{C}_1^{A2}) + (\hat{C}_3 - \hat{C}_2 \hat{C}_1 + \frac{1}{3} \hat{C}_1^{A3}) + \dots$$

$$\hat{Ch}_1 = \hat{C}_1 \quad \hat{Ch}_2 = -\hat{C}_2 + \frac{1}{2} \hat{C}_1^{A2}$$

$$\hat{Ch}_3 = \frac{1}{2} \hat{C}_3 - \frac{1}{2} \hat{C}_2 \hat{C}_1 + \frac{1}{6} \hat{C}_1^{A3}$$

$$\hat{C}(F) = 1 + \lambda F^A_A$$

$$+ \frac{1}{2} \lambda^2 (F^A_A \wedge F^B_B - F^A_{B1} F^B_A)$$

$$+ \frac{1}{3!} \lambda^3 (F^A_{A1} F^B_{B1} F^C_C + F^D_{B1} F^B_{C1} F^C_A + F^A_{C1} F^B_{A1} F^C_B - F^A_{A1} F^B_{C1} F^C_B - F^A_{B1} F^B_{A1} F^C_C - F^A_{C1} F^B_{B1} F^C_A) + \dots$$

$$= 1 + \lambda \text{tr} F + \frac{1}{2} \lambda^2 ((\text{tr} F)^{A2} - \text{tr} F^{A2})$$

$$+ \frac{1}{3!} \lambda^3 ((\text{tr} F)^{A3} + 2 \text{tr} F^{A3} - 3 (\text{tr} F) \wedge (\text{tr} F^{A2}))$$

$$\hat{C}_1 = \hat{Ch}_1$$

$$\hat{C}_2 = -\hat{Ch}_2 + \frac{1}{2} \hat{Ch}_1^{A2}$$

$$\hat{C}_3 = 2 \hat{Ch}_3 - \hat{Ch}_2 \wedge \hat{Ch}_1 + \frac{1}{6} \hat{Ch}_1^{A3}$$

# Chernova karakteristika

$$\hat{\text{Ch}}(F) = \text{tr} \exp\left(\frac{i\alpha}{2\pi} F\right)$$
$$= \sum_{q=0}^{\infty} \frac{\lambda^q}{q!} \text{tr} F^{Aq} = \sum_q \text{Ch}_q(F)$$

$$\text{ch} = [\hat{\text{Ch}}(F)] \quad \text{ch}_q = [\hat{\text{Ch}}_q(F)]$$

$$\hat{\text{Ch}}_q(F) = \frac{\lambda^q}{q!} \text{tr}(F^{Aq}) = \frac{\lambda^q}{q!} \text{str}_q(F, \dots, F)$$

$$\text{str}_q(A_1, \dots, A_q) = \sum_{\sigma \in S_q} \text{tr}(A_{\sigma_1} \cdots A_{\sigma_q})$$

$$\text{str}_{q, \alpha_1, \dots, \alpha_q} = t_{(\alpha_1)}^{A_1} t_{(\alpha_2)}^{A_2} \cdots t_{(\alpha_q)}^{A_q}$$

$$\begin{aligned}
 \hat{C}(F) &= 1 + \lambda F^A_A \\
 &+ \frac{1}{2} \lambda^2 (F^A_A \wedge F^B_B - F^A_B \wedge F^B_A) \\
 &+ \frac{1}{3!} \lambda^3 (F^A_A \wedge F^B_B \wedge F^C_C + F^A_B \wedge F^B_C \wedge F^C_A + F^A_C \wedge F^B_A \wedge F^C_B \\
 &\quad - F^A_A \wedge F^B_C \wedge F^C_B - F^A_B \wedge F^B_A \wedge F^C_C - F^A_C \wedge F^B_A \wedge F^C_A) \\
 &+ \dots \\
 &= 1 + \lambda \operatorname{tr} F + \frac{1}{2} \lambda^2 ((\operatorname{tr} F)^{\wedge 2} - \operatorname{tr} F^{\wedge 2}) \\
 &+ \frac{1}{3!} \lambda^3 ((\operatorname{tr} F)^{\wedge 3} + 2 \operatorname{tr} F^{\wedge 3} - 3(\operatorname{tr} F) \wedge (\operatorname{tr} F^{\wedge 2}))
 \end{aligned}$$

$$\begin{aligned}
 \hat{C}_1 &= \hat{C} h_1 \\
 \hat{C}_2 &= -\hat{C} h_2 + \frac{1}{2} \hat{C} h_1^{\wedge 2} \\
 \hat{C}_3 &= 2 \hat{C} h_3 - \hat{C} h_2 \wedge \hat{C} h_1 + \frac{1}{6} \hat{C} h_1^{\wedge 3}
 \end{aligned}$$



$$\hat{\log} \det(\mathbb{1} + \lambda F) = \text{tr} \hat{\log}(\mathbb{1} + \lambda F)$$

$$\text{tr} \left( \lambda F - \frac{1}{2} \lambda^2 F^{\wedge 2} + \frac{1}{3} \lambda^3 F^{\wedge 3} - \dots \right)$$

$$= \sum_{k=1}^{\infty} (-1)^{k-1} \frac{\lambda^k}{k} \text{tr} F^{\wedge k}$$

$$= \sum_{k=1}^{\infty} (-1)^{k-1} (k-1)! \hat{\mathcal{C}}_k(F)$$

$$= \hat{\mathcal{C}}_1 - \hat{\mathcal{C}}_2 + 2 \hat{\mathcal{C}}_3 - \dots$$

$$\Rightarrow \hat{\log} \left( 1 + \sum_{k=1}^{\infty} \hat{\mathcal{C}}_k \right) =$$

$$= (\hat{\mathcal{C}}_1 + \hat{\mathcal{C}}_2 + \hat{\mathcal{C}}_3 + \dots) - \frac{1}{2} (\hat{\mathcal{C}}_1 + \hat{\mathcal{C}}_2 + \dots)^{\wedge 2} + \frac{1}{3} (\hat{\mathcal{C}}_1 + \dots)^{\wedge 3} - \dots$$

$$= \hat{\mathcal{C}}_1 + \left( \hat{\mathcal{C}}_2 - \frac{1}{2} \hat{\mathcal{C}}_1^{\wedge 2} \right) + \left( \hat{\mathcal{C}}_3 - \hat{\mathcal{C}}_2^{\wedge 1} \hat{\mathcal{C}}_1 + \frac{1}{3} \hat{\mathcal{C}}_1^{\wedge 3} \right) + \dots$$

$$\hat{\mathcal{C}}_1 = \hat{\mathcal{C}}_1 \quad \hat{\mathcal{C}}_2 = -\hat{\mathcal{C}}_2 + \frac{1}{2} \hat{\mathcal{C}}_1^{\wedge 2}$$

$$\hat{\mathcal{C}}_3 = \frac{1}{2} \hat{\mathcal{C}}_3 - \frac{1}{2} \hat{\mathcal{C}}_2^{\wedge 1} \hat{\mathcal{C}}_1 + \frac{1}{6} \hat{\mathcal{C}}_1^{\wedge 3}$$