Pontrjaginovy charakt. třídy

$A \to M$ reálný vekt. bundle s metrikou

kal. gr. $O(A,H)$

$$F = -F^T$$

$$\mathrm{tr}\, F = 0 \qquad Q = F \wedge F^T = -F^{\wedge 2}$$

$$\hat{\mathrm{Pt}}(F) = \hat{\det}{}^{\frac{1}{2}}\left(\mathbb{1} + \frac{Q}{(2\pi)^2}\right) =$$

$$= \hat{\det}\left(\mathbb{1} - \frac{F}{2\pi}\right) = \hat{\det}\left(\mathbb{1} + \frac{F}{2\pi}\right)$$

$$p = [\hat{\mathrm{Pt}}(F)] \qquad \mathrm{tr}\, F^{\wedge 2k+1} = 0$$

$$\hat{\mathrm{Pt}}(F) = \sum_{k=0}^{\infty} \hat{\mathrm{Pt}}_{2k}(F) \qquad \hat{\mathrm{Pt}}_{2k+1}(F) = 0$$

Eulerova charakt. třída

TM tečný bundle

g_{ab} $\quad \nabla_a$ $\quad R_{ab}{}^m{}_n$

$\dim M = 2m$

$$\hat{\mathrm{Pf}}\left(\frac{R}{2\pi}\right) \qquad e = \left[\hat{\mathrm{Pf}}\left(\frac{R}{2\pi}\right)\right]$$

$$\mathrm{Pf}(X) = \frac{1}{2^m m!} \varepsilon_{a_1 b_1 \dots a_m b_m} X^{a_1 b_1} \dots X^{a_m b_m}$$

$$= X^{12} X^{34} \dots X^{2m-1\, 2m} = \xi^1 \xi^2 \dots \xi^m \qquad \begin{bmatrix} 0 & \xi_1 & & & \\ -\xi_1 & 0 & & & \\ & & 0 & \xi_2 & \\ & & -\xi_2 & 0 & \\ & & & & \ddots \end{bmatrix}$$

$$= (\det X)^{\frac{1}{2}}$$

$$\hat{\mathrm{Pf}}(R)_{a_1 b_1 \dots a_m b_m} = \frac{1}{2^m m!} \varepsilon_{k_1 l_1 \dots k_m l_m} R_{a_1 b_1}{}^{k_1 l_1} \wedge \dots \wedge R_{a_m b_m}{}^{k_m l_m}$$

$$= \frac{1}{2^m m!} \frac{1}{(2m)!} \varepsilon^{m_1 n_1 \dots m_m n_m} \varepsilon_{k_1 l_1 \dots k_m l_m} R_{m_1 n_1}{}^{k_1 l_1} \wedge \dots \wedge R_{m_m n_m}{}^{k_m l_m} \varepsilon_{a_1 b_1 \dots}$$

$$= \frac{1}{2^m m!} R_{k_1 l_1}{}^{k_1 l_1} \wedge \dots \wedge R_{k_m l_m}{}^{k_m l_m} \varepsilon_{a_1 b_1 \dots a_m b_m}$$

$$= \frac{(2m)!}{2^{2m} m!} R_{[k_1 l_1}{}^{k_1 l_1} \dots R_{k_m l_m]}{}^{k_m l_m} \varepsilon_{a_1 b_1 \dots a_m b_m}$$

Gauss-Bonnetova věta

$$\int_M e = \chi(M) = \mathrm{index}_{\mathrm{deRham}}(M)$$

$d \quad \delta \quad \Delta$

$d + \delta$

Pontrjaginovy charakt. třídy

$A\,M$ reálný vekt. bundle s metrikou

kal. gr. $O(A, H)$

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$$p = [\hat{P}t(F)] \qquad \mathrm{tr}\, F^{\wedge 2k+1} = 0$$

$$\hat{P}t(F) = \sum_{k=0}^{\infty} \hat{P}t_{2k}(F) \qquad \hat{P}t_{2k+1}(F) = 0$$

Eulerova charakt. třída

TM tečný bundle

$g_{\underline{ab}}$ $\quad \nabla_{\underline{a}}$ $\quad R_{\underline{ab}}{}^{\underline{m}}{}_{\underline{n}}$

$\dim M = 2m$

$$\hat{Pf}\left(\frac{R}{2\pi}\right) \qquad e = \left[\hat{Pf}\left(\frac{R}{2\pi}\right)\right]$$

$$Pf(X) = \frac{1}{2^m m!}\,\varepsilon_{\underline{a_1 b_1} \ldots \underline{a_m b_m}} X^{\underline{a_1 b_1}} \cdots X^{\underline{a_m b_m}}$$

$$= X^{12} X^{34} \cdots X^{2m-1\,m} = \xi^1 \xi^2 \cdots \xi^m \qquad \begin{bmatrix} 0 & \xi_1 & & & \\ -\xi_1 & 0 & & & \\ & & 0 & \xi_2 & \\ & & -\xi_2 & 0 & \\ & & & & \ddots \end{bmatrix}$$

$$= (\det X)^{\frac{1}{2}}$$

$$\hat{\mathrm{Pf}}(R)_{a_1b_1\cdots a_nb_n} = \frac{1}{2^n n!}\,\varepsilon_{k_1l_1\cdots k_nl_n}\, R_{a_1b_1}{}^{k_1l_1}\wedge\cdots\wedge R_{a_nb_n}{}^{k_nl_n}$$

$$= \frac{1}{2^n n!}\,\frac{1}{(2n)!}\,\varepsilon^{m_1n_1\cdots m_nn_n}\,\varepsilon_{k_1l_1\cdots k_nl_n}\, R_{m_1n_1}{}^{k_1l_1}\wedge\cdots\wedge R_{m_nn_n}{}^{k_nl_n}\,\varepsilon_{a_1b_1\cdots a_nb_n}$$

$$= \frac{1}{2^n n!}\, R_{k_1l_1}{}^{k_1l_2}\wedge\cdots\wedge R_{k_nl_n}{}^{k_nl_n}\quad \varepsilon_{a_1b_1\cdots a_nb_n}$$

$$= \frac{(2n)!}{2^{2n}n!}\, R_{[k_1l_1}{}^{k_1l_1}\cdots\cdots R_{k_nl_n]}{}^{k_nl_n}\quad \varepsilon_{a_1b_1\cdots a_nb_n}$$

Gauss-Bonnet inverse vete

$$\int_M e = \chi(M) = \mathrm{index}_{\mathrm{deRham}}(M)$$

$d \quad \delta \quad \Delta$

$d - \delta$

$m=1 \quad \dim M = 2$

$$2\pi e_{ab} = \frac{1}{2} R_{kl}{}^{kl} \, \varepsilon_{ab} = K \, \varepsilon_{ab}$$

$$\frac{1}{2\pi}\int_M K \varepsilon = \chi(M) = b_0 - b_1 + b_2$$

$$\overset{\text{sféra}}{=} 1 - 0 + 1 = 2$$

$m=2 \quad \dim M = 4$

$$(2\pi)^2 e_{abcd} = \frac{1}{8}\left(R_{kl}{}^{kl} \wedge R_{mn}{}^{mn}\right) \varepsilon_{abcd}$$

$$= \frac{1}{8}\left(R_{kl}{}^{kl} R_{mn}{}^{mn} + R_{mn}{}^{kl} R_{kl}{}^{mn} - 4 R_{km}{}^{kl} R_{ln}{}^{mn}\right)$$

$$= \frac{1}{8}\left(R^2 - 4 Ric^2 + \mathbb{R}^2\right) \varepsilon_{abcd}$$

$$R_{klmn} R^{klmn} \qquad Ric_{kl} Ric^{kl} \qquad \mathbb{R}\mathbb{R}$$

$$\frac{1}{8}\frac{1}{(2\pi)^2}\int_M \left(R^2 - 4 Ric^2 + \mathbb{R}^2\right) \varepsilon =$$

$$= \chi(M) = b_0 - b_1 + b_2 - b_3 + b_4$$

Lovelock gravitace

$$S_{Lov} = \sum_k \int_M \alpha_k \mathcal{L}_k \, g^{\frac{1}{2}}$$

$$k < m = \dim M / 2$$

$\varepsilon_{abcd} \quad \dim 4 \quad m=2 \qquad \mathcal{L}_0 \quad \mathcal{L}_1$

$\qquad\qquad\qquad\qquad\qquad\qquad \Lambda \quad \mathcal{H}$

$m=3 \qquad \mathcal{L}_0 \quad \mathcal{L}_1 \quad \mathcal{L}_2$

$$\hat{Pf}(R)_{a_1 b_1 \dots a_n b_n} = \frac{1}{2^n n!} \varepsilon_{k_1 l_1 \dots k_n l_n} R_{a_1 b_1}{}^{k_1 l_1} \wedge \dots \wedge R_{a_n b_n}{}^{k_n l_n}$$

$$= \frac{1}{2^n n!} \frac{1}{(2n)!} \varepsilon^{m_1 n_1 \dots m_n n_n} \varepsilon_{k_1 l_1 \dots k_n l_n} R_{m_1 n_1}{}^{k_1 l_1} \wedge \dots \wedge R_{m_n n_n}{}^{k_n l_n} \varepsilon_{a_1 b_1 \dots a_n b_n}$$

$$= \frac{1}{2^n n!} R_{k_1 l_1}{}^{k_1 l_1} \wedge \dots \wedge R_{k_n l_n}{}^{k_n l_n} \varepsilon_{a_1 b_1 \dots a_n b_n}$$

$$= \underbrace{\frac{(2m)!}{2^{2m} m!} R_{[k_1 l_1}{}^{k_1 l_1} \dots R_{k_n l_n]}{}^{k_n l_n}}_{\mathcal{L}_m} \varepsilon_{a_1 b_1 \dots a_n b_n}$$

$$n = 1 \qquad \dim M = 2$$

$$2\pi \Omega_{ab} = \frac{1}{2} R_{kl}{}^{kl} \, \varepsilon_{ab} = K \, \varepsilon_{ab}$$

$$\frac{1}{2\pi} \int_M K \varepsilon = \chi(M) = b_0 - b_1 + b_2$$

$$\overset{\text{sféra}}{=} 1 - 0 + 1 = 2$$

$m = 2 \quad \dim M = 4$

$$(2\pi)^2 e_{abcd} = \frac{1}{8}\left(R_{kl}{}^{kl} \wedge R_{mn}{}^{mn}\right) \varepsilon_{abcd}$$

$$= \frac{1}{8}\left(R_{kl}{}^{kl} R_{mn}{}^{mn} + R_{mn}{}^{kl} R_{kl}{}^{mn} - 4 R_{km}{}^{kl} R_{ln}{}^{mn}\right) \varepsilon_{abcd}$$

$$= \frac{1}{8}\left(R^2 - 4 Ric^2 + \mathbb{R}^2\right) \varepsilon_{abcd}$$

$$R_{klmn} R^{klmn} \quad Ric_{kl} Ric^{kl} \quad \mathbb{R}\mathbb{R}$$

$$\frac{1}{8}\frac{1}{(2\pi)^2}\int_M \left(R^2 - 4Ric^2 + \mathbb{R}^2\right) \varepsilon =$$

$$= \chi(M) = b_0 - b_1 + b_2 - b_3 + b_4$$

Lovelock gravitace

$$S_{Lov} = \sum_k \int_M \alpha_k \mathcal{L}_k \, g^{\frac{1}{2}}$$

$$\underbrace{\frac{(2m)!}{2^{2m} m!} R_{[k_1 l_1}{}^{k_1 l_1} \cdots\cdots R_{k_m l_m]}{}^{k_m l_m}}_{\mathcal{L}_m}$$

$k < m = \dim M/2$

$\dim 4 \quad m=2 \qquad \mathcal{L}_0 \quad \mathcal{L}_1$

$\qquad\qquad\qquad\qquad \Lambda \quad \mathcal{H}$

$\qquad m=3 \qquad \mathcal{L}_0 \quad \mathcal{L}_1 \quad \mathcal{L}_2$