Křivost na vekt. bundle

Def vekt. b. $\mathcal{A}M$

D kov. der

$$F \in \mathrm{Sect}\, \Lambda^2 \otimes \mathcal{A}^1_1 M$$

$$F_{\xi,\zeta}\cdot\phi = D_\xi D_\zeta \phi - D_\zeta D_\xi \phi - D_{[\xi,\zeta]}\phi$$

$$F_{\xi,\zeta} = \xi\cdot F\cdot\zeta$$

$$F_{\xi,\zeta}{}^{\underline{A}}{}_{\underline{B}} = \xi^m \zeta^n F_{\underline{mn}}{}^{\underline{A}}{}_{\underline{B}}$$

$$(F_{\xi,\zeta}\cdot\phi)^{\underline{A}} = F_{\xi,\zeta}{}^{\underline{A}}{}_{\underline{B}}\,\phi^{\underline{B}}$$

$$\mathbb{F}_{\xi,\zeta} = D_\xi D_\zeta - D_\zeta D_\xi - D_{[\xi,\zeta]}$$

$$\mathbb{F}_{\xi,\zeta}(\phi\otimes\psi) = (\mathbb{F}_{\xi,\zeta}\phi)\otimes\psi + \phi\otimes(\mathbb{F}_{\xi,\zeta}\psi)$$

$$= D_\xi D_\zeta(\phi\otimes\psi) - D_\zeta D_\xi(\phi\otimes\psi) - D_{[\xi,\zeta]}(\phi\otimes\psi)$$

$$= D_\xi(D_\zeta\phi\otimes\psi + \phi\otimes D_\zeta\psi) - (\xi\leftrightarrow\zeta) - (D_{[\xi,\zeta]}\phi)\otimes\psi - \phi\otimes D_{[\xi,\zeta]}\psi$$

$$= D_\xi D_\zeta\phi\otimes\psi - D_\zeta D_\xi\phi\otimes\psi - D_{[\xi,\zeta]}\phi\otimes\psi$$

$$\phi\otimes D_\xi D_\zeta\psi - \phi\otimes D_\zeta D_\xi\psi - \phi\otimes D_{[\xi,\zeta]}\psi$$

$$+\underline{D_\zeta\phi\otimes D_\xi\psi} + \underline{D_\xi\phi\otimes D_\zeta\psi} - \underline{D_\xi\phi\otimes D_\zeta\psi} - \underline{D_\zeta\phi\otimes D_\xi\psi}$$

$$\mathbb{F}_{\xi,\zeta} X^{\underline{A}\dots}_{\underline{B}\dots} = F_{\xi,\zeta}{}^{\underline{A}}{}_{\underline{N}} X^{\underline{N}\dots}_{\underline{B}\dots} + \dots - F_{\xi,\zeta}{}^{\underline{N}}{}_{\underline{B}} X^{\underline{A}\dots}_{\underline{N}\dots} - \dots$$

Křivost na vekt. bundle

Def vekt. b. $A M$

D kov. der.

$F \in \mathrm{Sect}\, \Lambda^2 \otimes A^1_1\, M$

$$F_{\xi,\zeta} \cdot \phi = D_\xi D_\zeta \phi - D_\zeta D_\xi \phi - D_{[\xi,\zeta]} \phi$$

$$F_{\xi,\zeta} = \xi \cdot F \cdot \zeta$$

$$F_{\xi,\zeta}{}^{\underline{A}}{}_{\underline{B}} = \xi^{\underline{m}} \zeta^{\underline{n}} F_{\underline{mn}}{}^{\underline{A}}{}_{\underline{B}}$$

$$(F_{\xi,\zeta} \cdot \phi)^{\underline{A}} = F_{\xi,\zeta}{}^{\underline{A}}{}_{\underline{B}} \phi^{\underline{B}}$$

$$\mathbb{F}_{\xi,\zeta} = D_\xi D_\zeta - D_\zeta D_\xi - D_{[\xi,\zeta]}$$

$$\mathbb{F}_{\xi,\zeta}(\phi\otimes\psi) = (\mathbb{F}_{\xi,\zeta}\phi)\otimes\psi + \phi\otimes(\mathbb{F}_{\xi,\zeta}\psi)$$

$$= D_\xi D_\zeta(\phi\otimes\psi) - D_\zeta D_\xi(\phi\otimes\psi) - D_{[\xi,\zeta]}(\phi\otimes\psi)$$

$$= D_\xi(D_\zeta\phi\otimes\psi + \phi\otimes D_\zeta\psi) - (\zeta\leftrightarrow\xi) - (D_{[\xi,\zeta]}\phi)\otimes\psi - \phi\otimes D_{[\xi,\zeta]}\psi$$

$$= D_\xi D_\zeta\phi\otimes\psi - D_\zeta D_\xi\phi\otimes\psi - D_{[\xi,\zeta]}\phi\otimes\psi$$

$$\phi\otimes D_\xi D_\zeta\psi - \phi\otimes D_\zeta D_\xi\psi - \phi\otimes D_{[\xi,\zeta]}\psi$$

$$+ D_\zeta\phi\otimes D_\xi\psi + D_\xi\phi\otimes D_\zeta\psi - D_\xi\phi\otimes D_\zeta\psi - D_\zeta\phi\otimes D_\xi\psi$$

$$\mathbb{F}_{\xi,\zeta} X^{\underline{A}\dots}{}_{\underline{B}\dots} = F_{\xi,\zeta}{}^{\underline{A}}{}_{\underline{N}} X^{\underline{N}\dots}{}_{\underline{B}\dots} + \dots - F_{\xi,\zeta}{}^{\underline{N}}{}_{\underline{B}} X^{\underline{A}\dots}{}_{\underline{N}\dots} - \dots$$

$$\phi \in \Lambda^p \otimes A^q_l M$$

$$\phi_{\underline{a}_1 \ldots \underline{a}_p \underline{B} \ldots}{}^{\underline{A} \ldots}$$

$${}^D d_{\underline{m}} {}^D d_{\underline{n}} \phi_{\underline{a}_1 \ldots \underline{a}_p \underline{B}}{}^{\underline{A}} = \mathbb{F}_{\underline{mn}} \wedge \phi_{\underline{a}_1 \ldots \underline{a}_p \underline{B} \ldots}{}^{\underline{A} \ldots}$$

$${}^D d \, {}^D d \equiv \mathbb{F} \wedge$$

$$= F_{\underline{mn}}{}^{\underline{A}}{}_{\underline{N}} \wedge \phi_{\underline{a}_1 \ldots \underline{a}_p \underline{B}}{}^{\underline{N} \ldots} + \ldots$$

$$- F_{\underline{mn}}{}^{\underline{N}}{}_{\underline{B}} \wedge \phi_{\underline{a}_1 \ldots \underline{a}_p \underline{N}}{}^{\underline{A}} - \ldots$$

$$\phi_{\underline{a}_1 \ldots \underline{a}_p \underline{B} \ldots}{}^{\underline{A} \ldots} = \omega_{\underline{a}_1 \ldots \underline{a}_p} \psi^{\underline{A} \ldots}_{\underline{B} \ldots}$$

$$\mathbb{F}_{\xi,\zeta} = D_\xi D_\zeta - D_\zeta D_\xi - D_{[\xi,\zeta]}$$

$$\mathbb{F}_{\xi,\zeta} = \xi^{\underline{m}} \zeta^{\underline{n}} \mathbb{F}_{\underline{mn}}$$

$$\mathbb{F}_{\underline{mn}} = D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}} + T_{\underline{mn}}^{\underline{k}} D_{\underline{k}}$$

$$\underbrace{D_\xi}_{\nabla} \underbrace{D_\zeta}_{\xi \cdot D} - D_\zeta D_\xi - D_{[\xi,\zeta]} = \xi^{\underline{m}} D_{\underline{m}} [\zeta^{\underline{n}} D_{\underline{n}}] - \zeta^{\underline{m}} D_{\underline{m}} [\xi^{\underline{n}} D_{\underline{n}}] - [\xi,\zeta]^{\underline{k}} D_{\underline{k}}$$

$$= \xi^{\underline{m}} \zeta^{\underline{n}} [D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}}] + \underbrace{\left(\xi^{\underline{m}} (\nabla_{\underline{m}} \zeta^{\underline{k}}) - \zeta^{\underline{m}} (\nabla_{\underline{m}} \xi^{\underline{k}}) D - [\xi,\zeta]^{\underline{k}} \right)}_{\xi^{\underline{m}} \zeta^{\underline{n}} T_{\underline{mn}}^{\underline{k}}} D_{\underline{k}}$$

$$\mathbb{F}_{\underline{mn}} \phi_{\underline{B} \ldots}^{\underline{A} \ldots} = D_{\underline{m}} (D_{\underline{n}} \phi^{\underline{A}}_{\underline{B}}) - D_{\underline{n}} (D_{\underline{m}} \phi^{\underline{A}}_{\underline{B}}) + T_{\underline{mn}}^{\underline{k}} (D_{\underline{k}} \phi^{\underline{A}}_{\underline{B}})$$

$$= {}^D d_{\underline{m}} {}^D d_{\underline{n}} \phi^{\underline{A} \ldots}_{\underline{B} \ldots}$$

$$\Lambda^1 A^q_l M$$

$${}^D d \, {}^D d M = [F, M]$$

$${}^D d_{\underline{m}} {}^D d_{\underline{n}} M^{\underline{A}}_{\underline{B}} = F_{\underline{mn}}{}^{\underline{A}}{}_{\underline{N}} M^{\underline{N}}_{\underline{B}} - M^{\underline{A}}_{\underline{N}} F_{\underline{mn}}{}^{\underline{N}}{}_{\underline{B}}$$

$${}^D d_{\underline{m}} {}^D d_{\underline{n}} \phi^{\underline{A}} = F_{\underline{mn}}{}^{\underline{A}}{}_{\underline{B}} \phi^{\underline{B}} \qquad {}^D d \, {}^D d \phi = F \cdot \phi$$

$$\mathbb{F}_{\xi,\zeta} = D_\xi D_\zeta - D_\zeta D_\xi - D_{[\xi,\zeta]}$$

$$\mathbb{F}_{\xi,\zeta} = \xi^{\underline{m}}\zeta^{\underline{n}}\,\mathbb{F}_{\underline{mn}}$$

$$\mathbb{F}_{\underline{mn}} = D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}} + T^{\underline{k}}_{\underline{mn}} D_{\underline{k}}$$

$$D_\xi D_\zeta - D_\zeta D_\xi - D_{[\xi,\zeta]} = \xi^{\underline{m}} D_{\underline{m}}\left[\zeta^{\underline{n}} D_{\underline{n}}\right] - \zeta^{\underline{m}} D_{\underline{m}}\left[\xi^{\underline{n}} D_{\underline{n}}\right] - [\xi,\zeta]^{\underline{k}} D_{\underline{k}}$$

∇ $\qquad \xi\cdot D$

$$= \xi^{\underline{m}}\zeta^{\underline{n}}\left[D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}}\right] + \underbrace{\left(\xi^{\underline{m}}(\nabla_{\underline{m}}\zeta^{\underline{k}}) - \zeta^{\underline{m}}(\nabla_{\underline{m}}\xi^{\underline{k}}) D - [\xi,\zeta]^{\underline{k}}\right)}_{\xi^{\underline{m}}\zeta^{\underline{m}}T^{\underline{k}}_{\underline{mn}}} D_{\underline{k}}$$

$$\mathbb{F}_{\underline{mn}}\,\phi^{\underline{A}\dots}_{\underline{B}\dots} = D_{\underline{m}}\, D_{\underline{n}}\phi^{\underline{A}\dots}_{\underline{B}\dots} - D_{\underline{n}}\, D_{\underline{m}}\phi^{\underline{A}\dots}_{\underline{B}\dots} + T^{\underline{k}}_{\underline{mn}}\, D_{\underline{k}}\phi^{\underline{A}\dots}_{\underline{B}\dots}$$

$$\Lambda^1 A^2_{L} M$$

$$= {}^D d_{\underline{m}}\,{}^D d_{\underline{n}}\,\phi^{\underline{A}\dots}_{\underline{B}\dots}$$

$${}^D d\,{}^D d M = [F, M]$$

$${}^D d_{\underline{m}}\,{}^D d_{\underline{n}}\,\phi^{\underline{A}} = F_{\underline{mn}}{}^{\underline{A}}{}_{\underline{B}}\,\phi^{\underline{B}} \qquad {}^D d\,{}^D d\,\phi = F\cdot\phi \qquad {}^D d_{\underline{m}}\,{}^D d_{\underline{n}} M^{\underline{A}}{}_{\underline{B}} = F_{\underline{mn}}{}^{\underline{A}}{}_{\underline{N}} M^{\underline{N}}{}_{\underline{B}} - M^{\underline{A}}{}_{\underline{N}} F_{\underline{mn}}{}^{\underline{N}}{}_{\underline{B}}$$

$$\phi \in \Lambda^p \otimes A^q_\ell M$$

$$\phi_{\underline{a}_1 \dots \underline{a}_p \, \underline{B} \dots}{}^{\underline{A} \dots}$$

$${}^{D}\!d_{\underline{m}} \, {}^{D}\!d_{\underline{n}} \, \phi_{\underline{a}_1 \dots \underline{a}_p \, \underline{B}}{}^{\underline{A}} = \mathbb{F}_{\underline{mn}} \wedge \phi_{\underline{a}_1 \dots \underline{a}_p \, \underline{B} \dots}{}^{\underline{A} \dots}$$

$${}^{D}\!d \, {}^{D}\!d = \mathbb{F} \wedge$$

$$= F_{\underline{mn}}{}^{\underline{A}}{}_{\underline{N}} \wedge \phi_{\underline{a}_1 \dots \underline{a}_p \, \underline{B}}{}^{\underline{N} \dots} + \dots$$

$$- F_{\underline{mn}}{}^{\underline{N}}{}_{\underline{B}} \wedge \phi_{\underline{a}_1 \dots \underline{a}_p \, \underline{N}}{}^{\underline{A}} - \dots$$

$$\phi_{\underline{a}_1 \, \underline{a}_p \, \underline{B} \dots}{}^{\underline{A} \dots} = \omega_{\underline{a}_1 \dots \underline{a}_p} \, \psi^{\underline{A} \dots}_{\underline{B} \dots}$$

$$X^{\underline{a}\ldots\underline{A}\ldots}{}_{\underline{b}\ldots\underline{B}\ldots}$$

$$D \;\cdots\cdots\; D_{ma}{}^{AM} \qquad F_{\underline{mn}}{}^{\underline{A}}{}_{\underline{B}}$$

$$\nabla_{ma}{}^{TM} \qquad T_{\underline{mn}}{}^{\underline{k}} \qquad R_{\underline{mn}}{}^{\underline{a}}{}_{\underline{b}}$$

$$\left[D_{\underline{m}}D_{\underline{n}} - D_{\underline{n}}D_{\underline{m}} + T_{\underline{mn}}{}^{\underline{k}}D_{\underline{k}}\right] X^{\underline{a}\ldots\underline{K}\ldots}{}_{\underline{b}\ldots\underline{L}\ldots}$$

$$= \underset{\substack{\uparrow \\ R_{\underline{mn}}{}^{\underline{a}}{}_{\underline{b}}}}{R_{\underline{mn}}} X^{\underline{a}\ldots\underline{K}\ldots}{}_{\underline{b}\ldots\underline{L}\ldots} + \underset{\substack{\uparrow \\ F_{\underline{mn}}{}^{\underline{A}}{}_{\underline{B}}}}{F_{\underline{mn}}} X^{\underline{a}\ldots\underline{K}\ldots}{}_{\underline{b}\ldots\underline{L}\ldots}$$

$D,\ \tilde{D}$ $\quad \tilde{D}_{\underline{m}}\phi^{\underline{A}} - D_{\underline{m}}\phi^{\underline{A}} = A_{\underline{m}}{}^{\underline{A}}{}_{\underline{B}}\,\phi^{\underline{B}}$ $\quad D$ на TM $\quad T=0$

$$\tilde{F}_{\underline{mn}} - F_{\underline{mn}} = {}^{D}d_{\underline{m}} A_{\underline{n}} + [A_{\underline{m}}, A_{\underline{n}}] = D_{\underline{m}} \wedge A_{\underline{n}} + [A_{\underline{m}}, A_{\underline{n}}]$$

$$= D_{\underline{m}} A_{\underline{n}} - D_{\underline{n}} A_{\underline{m}} + A_{\underline{m}}\cdot A_{\underline{n}} - A_{\underline{n}}\cdot A_{\underline{m}}$$

$$= {}^{\tilde{D}}d_{\underline{m}} A_{\underline{n}} - [A_{\underline{m}}, A_{\underline{n}}] = \tilde{D}_{\underline{m}} \wedge A_{\underline{n}} - [A_{\underline{m}}, A_{\underline{n}}]$$

$$\tilde{F}_{\underline{mn}}\cdot\phi = \tilde{D}_{\underline{m}}\tilde{D}_{\underline{n}}\phi - \tilde{D}_{\underline{n}}\tilde{D}_{\underline{m}}\phi = \tilde{D}_{\underline{m}}(D_{\underline{n}}\phi + A_{\underline{n}}\cdot\phi) - \tilde{D}_{\underline{n}}(D_{\underline{m}}\phi + A_{\underline{m}}\cdot\phi)$$

$$= D_{\underline{m}} D_{\underline{n}}\phi - D_{\underline{n}} D_{\underline{m}}\phi + D_{\underline{m}}(A_{\underline{n}}\cdot\phi) - D_{\underline{n}}(A_{\underline{m}}\cdot\phi)$$

$$A_{\underline{m}}\cdot D_{\underline{n}}\phi - A_{\underline{n}}\cdot D_{\underline{m}}\phi + A_{\underline{m}}\cdot A_{\underline{n}}\cdot\phi - A_{\underline{n}}\cdot A_{\underline{m}}\cdot\phi$$

$$= F_{\underline{mn}}\cdot\phi + (D_{\underline{m}} A_{\underline{n}} - D_{\underline{n}} A_{\underline{m}} + A_{\underline{m}}\cdot A_{\underline{n}} - A_{\underline{n}}\cdot A_{\underline{m}})\cdot\phi$$

$$+ A_{\underline{n}}\cdot D_{\underline{m}}\phi - A_{\underline{m}}\cdot D_{\underline{n}}\phi + A_{\underline{m}}\cdot D_{\underline{n}}\phi - A_{\underline{n}}\cdot D_{\underline{m}}\phi$$

Bianchiho identity

$$\overset{D}{d} F = 0$$

$$\overset{D}{d}_a F_{bc}{}^{M}{}_{N} = 0$$

rozšíření na TM $\quad T = 0$

$$D \wedge F = 0$$

$$D_{[a} F_{bc]}{}^{M}{}_{N} = 0$$

$$\overset{D}{d}(F \cdot \phi) = \overset{D}{d}\,\overset{D}{d}\,\overset{D}{d}\phi = \underline{F \wedge \overset{D}{d}\phi}$$

$$= (\overset{D}{d}F)\cdot\phi + \underline{F \wedge \overset{D}{d}\phi}$$

$$\Rightarrow \overset{D}{d}F = 0$$

Zákon zachování

M Rieman. varieta

$g_{ab} \quad \nabla_a \quad T = 0 \quad R$

AM vekt. bundl

D_a kov. der. F

$$\underbrace{D_m D_n F^{mn\,K}{}_{L}}_{J^{m\,K}{}_{L}} = 0$$

$$2 D_m D_n F^{mn} = [D_m D_n - D_n D_m] F^{mn}$$

$$= \mathbb{R}_{mn} F^{mn} + \mathbb{F}_{mn} F^{mn}$$

$$= R_{mn}{}^{m}{}_{k} F^{kn} + R_{mn}{}^{n}{}_{k} F^{mk} + [F_{mn}, F^{mn}]$$

$$= 2\,\cancel{Ric_{nk} F^{kn}} + \cancel{F_{mn} F^{mn}} - \cancel{F^{mn} F_{mn}}$$

$$= 0$$

Bianchiho identity

$$\overset{D}{d} F = 0$$

$$\overset{D}{d}_{\underline{a}} F_{\underline{bc}}{}^{\underline{M}}{}_{\underline{N}} = 0$$

rozšíření na $\mathbb{T}M$ $\qquad T = 0$

$$D \wedge F = 0$$

$$D_{[\underline{a}} F_{\underline{bc}]}{}^{\underline{M}}{}_{\underline{N}} = 0$$

$$\overset{D}{d}(F \cdot \phi) = \overset{D}{d}\,\overset{D}{d}\,\overset{D}{d} \phi = \underline{F \wedge \overset{D}{d} \phi}$$

$$= (\overset{D}{d} F) \cdot \phi + \underline{F \wedge \overset{D}{d} \phi}$$

$$\Rightarrow \overset{D}{d} F = 0$$

Zákon zachování

M Rieman. varieta

$g_{\underline{ab}}$ $\quad \nabla_{\underline{a}}$ $\quad T=0$ $\quad R$

AM vekt. bundl

$D_{\underline{m}}$ kov. der. $\quad F$

$$D_{\underline{m}} \underbrace{D_{\underline{n}} F^{\underline{mn}\,\underline{K}}{}_{\underline{L}}}_{J^{\underline{m}\,\underline{K}}{}_{\underline{L}}} = 0$$

Zákon zachování

M Rieman. varieta

$g_{\underline{ab}}$ $\nabla_{\underline{a}}$ $T=0$ R

AM vekt. bundl

$D_{\underline{m}}$ kov. der. F

$$D_{\underline{m}} \underbrace{D_{\underline{n}} F^{\underline{mn}\,\underline{k}}{}_{\underline{l}}}_{J^{\underline{m}\,\underline{k}}{}_{\underline{l}}} = 0$$

$$2D_{\underline{m}} D_{\underline{n}} F^{\underline{mn}} = [D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}}] F^{\underline{mn}}$$

$$= \mathbb{R}_{\underline{mn}} F^{\underline{mn}} + \mathbb{F}_{\underline{mn}} F^{\underline{mn}}$$

$$= R_{\underline{mn}}{}^{\underline{m}}{}_{\underline{k}} F^{\underline{kn}} + R_{\underline{mn}}{}^{\underline{n}}{}_{\underline{k}} F^{\underline{mk}} + [F_{\underline{mn}}, F^{\underline{mn}}]$$

$$= 2 \cancel{Ric_{\underline{nk}} F^{\underline{kn}}} + F_{\underline{mn}} \cancel{F^{\underline{mn}}} - \cancel{F^{\underline{mn}} F_{\underline{mn}}}$$

$$= 0$$

D na AM

∇ na TM $\quad T^a{}_{mn}$

D rozšíření D na TM

$$D(A\otimes\phi) = \underbrace{(DA)}_{\nabla A}\otimes\phi + A\otimes\underbrace{(D\phi)}_{D\phi}$$

$\uparrow TM \quad \uparrow AM$

$${}^{D}d_{a_0}\phi^{A}{}_{a_1\dots a_p} = D_{a_0}\wedge\phi^{A}{}_{a_1\dots a_p} + T^{n}{}_{a_0a_1}\wedge\phi^{A}{}_{|n|a_2\dots a_p}$$

$$= (p+1)\,D_{[a_0}\phi^{A}{}_{a_1\dots a_p]} + \binom{p+1}{2}T^{n}{}_{[a_0a_1}\phi^{A}{}_{|n|a_2\dots a_p]}$$

$\Lambda^p\otimes A^k_l M$

$\Lambda^p\otimes T^k_l M$

$d \quad \nabla$

$${}^{\nabla}d_{a_0}\omega_{a_1\dots a_p}{}^{m\dots}{}_{n\dots} = \nabla_{a_0}\wedge\omega_{a_1\dots a_p}{}^{m}{}_{n} + T^{k}{}_{a_0a_1}\wedge\omega_{|k|a_2\dots a_p}{}^{m}{}_{n}$$

$\delta^m_a \in \Lambda^1\otimes TM$

$${}^{\nabla}d_a\delta^m_b = \underbrace{\nabla_a\wedge\delta^m_b}_{0} + T^k{}_{ab}\,\delta^m_k = T^m{}_{ab}$$

$\nabla = \partial + \Gamma \qquad \Gamma_a{}^m{}_n \in \Lambda^1\otimes T^1_1 M$

$R_{ab}{}^m{}_n \in \Lambda^2\otimes T^1_1 M$

$$R_{ab}{}^m{}_n = {}^{\partial}d_a\Gamma_b{}^m{}_n + [\Gamma_a,\Gamma_b]^m{}_n$$

$$= \partial_a\Gamma_b{}^m{}_n - \partial_b\Gamma_a{}^m{}_n + \underbrace{\Gamma_a{}^m{}_k\Gamma_b{}^k{}_n - \Gamma_b{}^m{}_k\Gamma_a{}^k{}_n}_{\Gamma_a\wedge\Gamma_b}$$

${}^{\nabla}d R = 0 \qquad T = 0$

$\Downarrow$

$\nabla\wedge R = 0$

$\nabla_{[a}R_{bc]}{}^m{}_n = 0$

$$\Lambda^p \otimes \mathbb{A}^k_l M$$

$$\Lambda^p \otimes \mathbb{T}^k_l M$$

$$d \qquad \nabla$$

$$\overset{\nabla}{d}_{\underline{a}_0} \omega_{\underline{a}_1 \cdots \underline{a}_p}{}^{\underline{m} \cdots}_{\underline{n} \cdots} = \nabla_{\underline{a}_0} \wedge \omega_{\underline{a}_1 \cdots \underline{a}_p}{}^{\underline{k}}_{\underline{n}} + T^{\underline{k}}_{\underline{a}_0 \underline{a}_1} \wedge \omega_{\underline{k} | \underline{a}_2 \cdots \underline{a}_p}{}^{\underline{m}}_{\underline{n}}$$

$$d \qquad \nabla$$

$$\overset{\nabla}{d}_{\underline{a}_0}\omega_{\underline{a}_1\cdots\underline{a}_p}{}^{\underline{m}\cdots}{}_{\underline{n}\cdots} = \nabla_{\underline{a}_0}\wedge\omega_{\underline{a}_1\cdots\underline{a}_p}{}^{\underline{k}}{}_{\underline{n}} + T_{\underline{a}_0\underline{a}_1}{}^{\underline{k}}\wedge\omega_{|\underline{k}|\underline{a}_2\cdots\underline{a}_p}{}^{\underline{m}}{}_{\underline{n}}$$

$$\delta_{\underline{a}}^{\underline{m}} \in \Lambda^1\otimes TM$$

$$\overset{\nabla}{d}_{\underline{a}}\delta_{\underline{b}}^{\underline{m}} = \underbrace{\nabla_{\underline{a}}\wedge\delta_{\underline{b}}^{\underline{m}}}_{0} + T_{\underline{ab}}^{\underline{k}}\,\delta_{\underline{k}}^{\underline{m}} = T_{\underline{ab}}^{\underline{m}}$$

$$\nabla = \partial + \Gamma \qquad \Gamma_{a}{}^{m}{}_{n} \in \Lambda^1 \otimes T^1_1 M$$

$$R_{ab}{}^{m}{}_{n} \in \Lambda^2 \otimes T^1_1 M$$

$$R_{ab}{}^{m}{}_{n} = \partial_a \Gamma_{b}{}^{m}{}_{n} + [\Gamma_a, \Gamma_b]^{m}{}_{n}$$

$$= \partial_a \Gamma_{b}{}^{m}{}_{n} - \partial_b \Gamma_{a}{}^{m}{}_{n} + \underbrace{\Gamma_{a}{}^{m}{}_{k} \Gamma_{b}{}^{k}{}_{n} - \Gamma_{b}{}^{m}{}_{k} \Gamma_{a}{}^{k}{}_{n}}_{\Gamma_a \wedge \Gamma_b}$$

$$\nabla = \partial + \Gamma \qquad \Gamma_{\underline{a}\,\underline{n}}{}^{\underline{m}} \in \Lambda^1 \otimes T^1_1 M$$

$$R_{\underline{a}\underline{b}}{}^{\underline{m}}{}_{\underline{n}} \in \Lambda^2 \otimes T^1_1 M$$

$$\overset{\nabla}{d} R = 0 \qquad T = 0$$

$$\Downarrow$$

$$\nabla \wedge R = 0$$

$$\nabla_{[\underline{a}} R_{\underline{b}\underline{c}]}{}^{\underline{m}}{}_{\underline{n}} = 0$$