

Krivost na vekt. bundle

Def vekt. b. AM

D kov. der

$$F \in \text{Sect } \Lambda^2 A^1, M$$

$$F_{\xi, \eta} \cdot \phi = D_{\xi} D_{\eta} \phi - D_{\eta} D_{\xi} \phi - D_{[\xi, \eta]} \phi$$

$$F_{\xi, \eta} = \xi \cdot F \cdot \eta$$

$$F_{\xi, \eta}^A = \xi^B \eta^C F_{BC}^A$$

$$(F_{\xi, \eta} \cdot \phi)^A = F_{\xi, \eta}^A \phi^B$$

$$F_{\xi, \eta} = D_{\xi} D_{\eta} - D_{\eta} D_{\xi} - D_{[\xi, \eta]}$$

$$F_{\xi, \eta}(\phi \otimes \psi) = (F_{\xi, \eta} \phi) \otimes \psi + \phi \otimes (F_{\xi, \eta} \psi)$$

$$= D_{\xi} D_{\eta}(\phi \otimes \psi) - D_{\eta} D_{\xi}(\phi \otimes \psi) - D_{[\xi, \eta]}(\phi \otimes \psi)$$

$$= D_{\xi}(D_{\eta} \phi \otimes \psi + \phi \otimes D_{\eta} \psi) - (D_{\eta} D_{\xi} \phi) \otimes \psi - \phi \otimes D_{[\xi, \eta]} \psi$$

$$= D_{\xi} D_{\eta} \phi \otimes \psi - D_{\eta} D_{\xi} \phi \otimes \psi - D_{[\xi, \eta]} \phi \otimes \psi$$

$$\phi \otimes D_{\eta} D_{\xi} \psi - \phi \otimes D_{\xi} D_{\eta} \psi - \phi \otimes D_{[\xi, \eta]} \psi$$

$$+ D_{\xi} \phi \otimes D_{\eta} \psi + D_{\eta} \phi \otimes D_{\xi} \psi - D_{\xi} \phi \otimes D_{\eta} \psi - D_{\eta} \phi \otimes D_{\xi} \psi$$

$$F_{\xi, \eta}^A X_B^{\dots} = F_{\xi, \eta}^A X_B^N + \dots - F_{\xi, \eta}^N X_B^A - \dots$$

Krivost na vekt. bundle

Def vekt. b. AM

D kov. der

$$F \in \text{sect } \Lambda^2 \otimes A^1_M$$

$$F_{\xi, \eta} \cdot \phi = D_{\xi} D_{\eta} \phi - D_{\eta} D_{\xi} \phi - D_{[\xi, \eta]} \phi$$

$$F_{\xi, \eta} = \xi \cdot F \cdot \eta$$

$$F_{\xi, \eta}^A = \sum_{m, n} \xi^m \eta^n F_{mn}^A$$

$$(F_{\xi, \eta} \cdot \phi)^A = F_{\xi, \eta}^A \phi^B$$

$$F_{\xi, \eta} = D_{\xi} D_{\eta} - D_{\eta} D_{\xi} - D_{[\xi, \eta]}$$

$$F_{\xi, \eta} (\phi \otimes \psi) = (F_{\xi, \eta} \phi) \otimes \psi + \phi \otimes (F_{\xi, \eta} \psi)$$

$$= D_{\xi} D_{\eta} (\phi \otimes \psi) - D_{\eta} D_{\xi} (\phi \otimes \psi) - D_{[\xi, \eta]} (\phi \otimes \psi)$$

$$= D_{\xi} (D_{\eta} \phi \otimes \psi + \phi \otimes D_{\eta} \psi) - (\eta \leftrightarrow \xi) - (D_{[\xi, \eta]} \phi) \otimes \psi - \phi \otimes D_{[\xi, \eta]} \psi$$

$$= D_{\xi} D_{\eta} \phi \otimes \psi - D_{\eta} D_{\xi} \phi \otimes \psi - D_{[\xi, \eta]} \phi \otimes \psi$$

$$\phi \otimes D_{\xi} D_{\eta} \psi - \phi \otimes D_{\eta} D_{\xi} \psi - \phi \otimes D_{[\xi, \eta]} \psi$$

$$+ \underbrace{D_{\eta} \phi \otimes D_{\xi} \psi} + \underbrace{D_{\xi} \phi \otimes D_{\eta} \psi} - \underbrace{D_{\xi} \phi \otimes D_{\eta} \psi} - \underbrace{D_{\eta} \phi \otimes D_{\xi} \psi}$$

$$F_{\xi, \eta} X_{\underline{B}}^{\underline{A}} \dots = F_{\xi, \eta}^{\underline{A}} X_{\underline{B}}^{\underline{N}} \dots + \dots - F_{\xi, \eta}^{\underline{N}} X_{\underline{B}}^{\underline{A}} \dots - \dots$$

$$\phi \in \Lambda^p \otimes A^2 M$$

$$\phi_{a_1 \dots a_p \underline{B} \dots}$$

$$D_{\underline{m}}^D D_{\underline{n}}^D \phi_{a_1 \dots a_p \underline{B} \dots} = F_{\underline{mn}} \wedge \phi_{a_1 \dots a_p \underline{B} \dots}$$

$$D_{\underline{m}}^D D_{\underline{n}}^D = F_{\underline{mn}} \wedge$$

$$= F_{\underline{mn}}^A \wedge \phi_{a_1 \dots a_p \underline{B} \dots} + \dots$$

$$- F_{\underline{mn}}^N \wedge \phi_{a_1 \dots a_p \underline{B} \dots}$$

$$\phi_{a_1 \dots a_p \underline{B} \dots} = \omega_{a_1 \dots a_p} \psi_{\underline{B} \dots}$$

$$F_{\xi, \zeta} = D_{\xi} D_{\zeta} - D_{\zeta} D_{\xi} - D_{[\xi, \zeta]}$$

$$F_{\xi, \zeta} = \xi^B \zeta^D F_{\underline{mn}}$$

$$F_{\underline{mn}} = D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}} + T_{\underline{mn}}^k D_k$$

$$D_{\xi} D_{\zeta} - D_{\zeta} D_{\xi} - D_{[\xi, \zeta]} = \xi^B \zeta^D [D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}}] - [\xi, \zeta]^k D_k$$

$$= \xi^B \zeta^D [D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}}] + \underbrace{\left(\xi^B (\nabla_{\underline{m}} \zeta^k) - \zeta^B (\nabla_{\underline{m}} \xi^k) - [\xi, \zeta]^k \right)}_{T_{\underline{mn}}^k} D_k$$

$$F_{\underline{mn}} \phi_{\underline{B} \dots} = D_{\underline{m}} (D_{\underline{n}} \phi_{\underline{B} \dots}) - D_{\underline{n}} (D_{\underline{m}} \phi_{\underline{B} \dots}) + T_{\underline{mn}}^k D_k \phi_{\underline{B} \dots}$$

$$D_{\underline{m}}^D D_{\underline{n}}^D M = [F, M]$$

$$D_{\underline{m}}^D D_{\underline{n}}^D \phi^A = F_{\underline{mn}}^A \phi^B \quad D_{\underline{m}}^D \phi = F \cdot \phi$$

$$D_{\underline{m}}^D D_{\underline{n}}^D M^A_{\underline{B}} = F_{\underline{mn}}^A M^N_{\underline{B}} - M^A_{\underline{N}} F_{\underline{mn}}^N$$

$$F_{\xi, \zeta} = D_{\xi} D_{\zeta} - D_{\zeta} D_{\xi} - D_{[\xi, \zeta]}$$

$$F_{\xi, \zeta} = \sum^B \sum^D F_{mn}$$

$$F_{mn} = D_m D_n - D_n D_m + T_{mn}^k D_k$$

$$\begin{aligned} D_{\xi} D_{\zeta} - D_{\zeta} D_{\xi} - D_{[\xi, \zeta]} &= \sum^B D_{\xi} [\sum^D D_{\zeta}] - \sum^B D_{\zeta} [\sum^D D_{\xi}] - [\xi, \zeta]^k D_k \\ \Delta \quad \uparrow \quad \uparrow \quad \xi \cdot D &= \sum^B \sum^D [D_m D_n - D_n D_m] + \underbrace{\left(\sum^B (\nabla_m \xi^k) - \sum^B (\nabla_n \xi^k) \right) D - [\xi, \zeta]^k}_{T_{mn}^k} D_k \end{aligned}$$

$$F_{mn} \phi^A \dots = D_m (D_n \phi^A) - D_n (D_m \phi^A) + T_{mn}^k (D_k \phi^A) \quad \wedge^1 A^2 M$$

$$= D_m^D D_n^D \phi^A$$

$$D_m^D D_n^D M = [F, M]$$

$$D_m^D D_n^D \phi^A = F_{mn}^A \phi^B \quad D_m^D D_n^D \phi = F \cdot \phi$$

$$D_m^D D_n^D M^A = F_{mn}^A M^N - M^A_{\quad N} F_{mn}^N$$

$$\phi \in \Lambda^p \otimes A^q \quad M$$

$$\phi_{\underline{a}_1 \dots \underline{a}_p \underline{B} \dots}$$

$$\begin{matrix} D \\ \downarrow \\ \underline{c}_m \end{matrix} \begin{matrix} D \\ \downarrow \\ \underline{c}_n \end{matrix} \phi_{\underline{a}_1 \dots \underline{a}_p \underline{B} \dots} = \underline{F}_{\underline{mn}} \wedge \phi_{\underline{a}_1 \dots \underline{a}_p \underline{B} \dots}$$

$$\begin{matrix} D \\ \downarrow \\ \underline{c} \end{matrix} \begin{matrix} D \\ \downarrow \\ \underline{c} \end{matrix} = \underline{F} \wedge$$

$$= \underline{F}_{\underline{mn}}^A \wedge \phi_{\underline{a}_1 \dots \underline{a}_p \underline{B} \dots} + \dots$$

$$- \underline{F}_{\underline{mn}}^B \wedge \phi_{\underline{a}_1 \dots \underline{a}_p \underline{N} \dots} - \dots$$

$$\phi_{\underline{a}_1 \dots \underline{a}_p \underline{B} \dots} = \omega_{\underline{a}_1 \dots \underline{a}_p} \psi_{\underline{B} \dots}$$

$$X \begin{matrix} a & \dots & A & \dots \\ b & \dots & B & \dots \end{matrix}$$

$$D \dots D \text{ ma AM}$$

$$\nabla \text{ ma TM}$$

$$T_{mn}^k$$

$$\begin{matrix} F_{mn}^A \\ R_{mn}^a \\ \quad \quad b \end{matrix}$$

$$\left[D_m D_n - D_n D_m + T_{mn}^k D_k \right] X \begin{matrix} a & \dots & K & \dots \\ b & \dots & L & \dots \end{matrix}$$

$$= R_{mn} X \begin{matrix} a & \dots & K & \dots \\ b & \dots & L & \dots \end{matrix} + F_{mn} X \begin{matrix} a & \dots & K & \dots \\ b & \dots & L & \dots \end{matrix}$$

$$\begin{matrix} \uparrow \\ R_{mn}^a \\ \quad \quad b \end{matrix} \qquad \begin{matrix} \uparrow \\ F_{mn}^A \\ \quad \quad B \end{matrix}$$

$$D, \tilde{D} \quad \tilde{D}_m \phi^A - D_m \phi^A = A_{m\ B}^A \phi^B \quad D \text{ ma TH} \quad T=0$$

$$\begin{aligned} \tilde{F}_{mn} - F_{mn} &= \tilde{D}_m A_n - D_m A_n + [A_m, A_n] = \tilde{D}_m \wedge A_n + [A_m, A_n] \\ &= D_m A_n - D_n A_m + A_m \cdot A_n - A_n \cdot A_m \\ &= \tilde{D}_m A_n - [A_m, A_n] = \tilde{D}_m \wedge A_n - [A_m, A_n] \end{aligned}$$

$$\begin{aligned} \tilde{F}_{mn} \cdot \phi &= \tilde{D}_m \tilde{D}_n \phi - \tilde{D}_n \tilde{D}_m \phi = \tilde{D}_m (\underline{D}_n \phi + \underline{A}_n \phi) - \tilde{D}_n (\underline{D}_m \phi + \underline{A}_m \phi) \\ &= \underline{D}_m \underline{D}_n \phi - \underline{D}_n \underline{D}_m \phi + \underline{D}_m (\underline{A}_n \phi) - \underline{D}_n (\underline{A}_m \phi) \\ &\quad \underline{A}_m \cdot \underline{D}_n \phi - \underline{A}_n \cdot \underline{D}_m \phi + \underline{A}_m \cdot \underline{A}_n \phi - \underline{A}_n \cdot \underline{A}_m \phi \\ &= F_{mn} \cdot \phi + (D_m A_n - D_n A_m + A_m \cdot A_n - A_n \cdot A_m) \cdot \phi \\ &\quad + \underline{A}_n \cdot \underline{D}_m \phi - \underline{A}_m \cdot \underline{D}_n \phi + \underline{A}_m \cdot \underline{D}_n \phi - \underline{A}_n \cdot \underline{D}_m \phi \end{aligned}$$

Bianchiho identity

$$D d F = 0$$

$$D_{[a} F_{bc]}^M = 0$$

pozadíem' na $\mathbb{T}M$ $T=0$

$$D \wedge F = 0$$

$$D_{[a} F_{bc]}^M = 0$$

$$D(F \cdot \phi) = \underbrace{D d \phi}_{= F \wedge \phi} = F \wedge \phi$$

$$= (D F) \cdot \phi + \underbrace{F \wedge \phi}_{= 0}$$

$$\Rightarrow D F = 0$$

Zákon zachování

M Riemann. varietu

g_{ab} $\nabla_a T = 0$ R

AM vekt. bundl

D_a kov. der. F

$$D_m \underbrace{D_n F^{mnk}}_J = 0$$

$$2D_m D_n F^{mn} = [D_m D_n - D_n D_m] F^{mn}$$

$$= R_{mn} F^{mn} + F_{mn} F^{mn}$$

$$= R_{mn}^k F^{kn} + R_{mn}^n F^{mk} + [F_{mn}, F^{mn}]$$

$$= 2R_{nk} F^{kn} + F_{mn} F^{mn} - F_{mn} F^{mn}$$

$$= 0$$

Bianchiho identity

$$D d F = 0$$

$$D_{[a} F_{bc]}^M = 0$$

pozor si je m' me $\mathbb{T}M$ $T=0$

$$D \wedge F = 0$$

$$D_{[a} F_{bc]}^M = 0$$

$$D d(F \cdot \phi) = D d d \phi = \underline{F \wedge d \phi}$$

$$= (D F) \cdot \phi + \underline{F \wedge d \phi}$$

$$\Rightarrow D F = 0$$

Zákon zachování

M Riemann. variete

$$g_{ab} \quad \nabla_a \quad T=0 \quad R$$

AM vekt. bundl

$$D_m \quad \text{kov. der.} \quad F$$

$$D_m \underbrace{D_n F^{mn}{}_{\underline{k}}}_{J^{mn}{}_{\underline{k}}{}_{\underline{l}}} = 0$$

Zákon zachování

M Riemann. variete

$$g_{ab} \quad \nabla_a \quad T=0 \quad R$$

AM vekt. bundl

$$D_m \quad \text{kov. des.} \quad F$$

$$D_m \underbrace{D_n F^{mn k}}_J^{mn k} = 0$$

$$2D_m D_n F^{mn} = [D_m D_n - D_n D_m] F^{mn}$$

$$= R_{mn} F^{mn} + F_{mn} F^{mn}$$

$$= R_{mn}^{\quad k} F^{kn} + R_{mn}^{\quad n} F^{mk} + [F_{mn}, F^{mn}]$$

$$= 2 \cancel{R_{nk}} F^{kn} + \cancel{F_{mn}} F^{mn} - \cancel{F^{kn}} \cancel{F_{mn}}$$

$$= 0$$