Geometrie Lieových grup

$T_e M$ Lieova alg. $[\,,\,]$ $C_{\alpha\beta}{}^{\kappa}$ $k_{\alpha\beta}$ $C_{\alpha\beta\gamma} = C_{[\alpha\beta\gamma]}$

$\overset{L}{E}_\alpha$ $\overset{L}{E}{}^\alpha$ $\overset{R}{E}_\alpha$ $\overset{R}{E}{}^\alpha$

1) $\forall A$ levoinv. tens. pole

$$\overset{L}{\nabla} A = 0$$

$$A = \underbrace{A^{\alpha\dots}_{\beta\dots}}_{\text{konst.}} \overset{L}{E}_\alpha \dots \overset{L}{E}{}^\beta \dots$$

$$\overset{L}{\nabla}\overset{L}{E}_\alpha = 0 \quad \overset{L}{\nabla}\overset{L}{E}{}^\alpha = 0$$

$$\overset{L}{T}_{\alpha\beta}{}^{\kappa} = -C_{\alpha\beta}{}^{\kappa} \quad \overset{L}{R}_{\alpha\beta}{}^{\kappa}{}_{\gamma} = 0$$

2) $\forall A$ pravoinv. tens. pole

$$\overset{R}{\nabla} A = 0$$

$$\overset{R}{\nabla}\overset{R}{E}_\alpha = 0$$

$$\overset{R}{T}_{\alpha\beta}{}^{\kappa} = C_{\alpha\beta}{}^{\kappa} \quad \overset{R}{R}_{\alpha\beta}{}^{\kappa}{}_{\gamma} = 0$$

$$\overset{R}{\nabla} - \overset{L}{\nabla} = C \qquad C_{\alpha\kappa}{}^{\gamma}$$

$$0 = \overset{R}{\nabla}_\alpha r_n^\kappa = \underline{\overset{L}{\nabla}_\alpha r_n^\kappa} + A_{\alpha\gamma}{}^{\kappa} r_n^\gamma = (A_{\alpha\gamma}{}^{\kappa} - C_{\alpha\gamma}{}^{\kappa}) r_n^\gamma$$

$$\overset{L}{T}_{\alpha\beta}{}^{\kappa} = \overset{L}{T}_{\kappa\lambda}{}^{\kappa} \overset{L}{E}{}^{\kappa}_{\alpha} \overset{L}{E}{}^{\lambda}_{\beta} =$$

$$= \overset{L}{\nabla}_{\overset{L}{E}_\alpha} \overset{L}{E}{}^{\kappa}_{\beta} - \overset{L}{\nabla}_{\overset{L}{E}_\beta} \overset{L}{E}{}^{\kappa}_{\alpha} - [\overset{L}{E}_\alpha, \overset{L}{E}_\beta]^\kappa$$

$$= -C_{\kappa\lambda}{}^{\kappa} \overset{L}{E}{}^{\kappa}_{\alpha} \overset{L}{E}{}^{\lambda}_{\beta} = -C_{\alpha\beta}{}^{\kappa}$$

$$\overset{L}{\nabla}_{l_m} r_n = \overset{L}{\nabla}_{l_m} r_n - \underbrace{\overset{L}{\nabla}_{r_n} l_m}_{0} - \underbrace{[l_m, r_n]}_{0}$$

$$= l_m \cdot \overset{L}{T} \cdot r_n = -l_m \cdot C \cdot r_n$$

$$\overset{L}{\nabla} r_n = -C \cdot r_n$$

Geometrie Lieových grup

T_eM Lieova alg. $[\,,\,]$ $c_{\alpha\beta}{}^{\gamma}$ $k_{\alpha\beta}$ $c_{\alpha\beta\gamma} = c_{[\alpha\beta\gamma]}$

$\overset{L}{E}_{\alpha}$ $\overset{L}{E}{}^{\alpha}$ $\overset{R}{E}_{\alpha}$ $\overset{R}{E}{}^{\alpha}$

Geometrie Lieových grup

$T_e M$ Lieova alg. $[\,,\,]$ $c_{\alpha\beta}{}^{\kappa}$ $k_{\alpha\beta}$ $c_{\alpha\beta\gamma} = c_{[\alpha\beta\gamma]}$

$\overset{L}{E}_\alpha$ $\overset{L}{E}{}^\alpha$ $\overset{R}{E}_\alpha$ $\overset{R}{E}{}^\alpha$

1) ∀ A levoinv. tens. pole

$$\overset{L}{\nabla} A = 0$$

$$A = \underbrace{A^{\alpha\dots}{}_{\beta\dots}}_{\text{konst.}} \overset{L}{E}_\alpha \dots \overset{L}{E}{}^\beta \dots$$

$$\overset{L}{\nabla}\overset{L}{E}_\alpha = 0 \qquad \overset{L}{\nabla}\overset{L}{E}{}^\alpha = 0$$

$$\overset{L}{T}_{\alpha\beta}{}^{\kappa} = -c_{\alpha\beta}{}^{\kappa} \qquad \overset{L}{R}_{\alpha\beta}{}^{\kappa}{}_{\gamma} = 0$$

2) ∀ A pravoin. tens. pole

$$\overset{R}{\nabla} A = 0$$

$$\overset{R}{\nabla}\overset{R}{E}_\alpha = 0$$

$$\overset{R}{T}_{\alpha\beta}{}^{\kappa} = c_{\alpha\beta}{}^{\kappa} \qquad \overset{R}{R}_{\alpha\beta}{}^{\kappa}{}_{\gamma} = 0$$

$$\overset{L}{T}_{\alpha\beta}{}^{\mu} = \overset{L}{T}_{\underline{\kappa\lambda}}{}^{\mu}\, \overset{L}{E}_{\alpha}{}^{\underline{\kappa}}\, \overset{L}{E}_{\beta}{}^{\underline{\lambda}} =$$

$$= \overset{L}{\nabla}_{\overset{L}{E}_{\alpha}} \overset{L}{E}_{\beta}{}^{\mu} - \overset{L}{\nabla}_{\overset{L}{E}_{\beta}} \overset{L}{E}_{\alpha}{}^{\mu} - \left[\overset{L}{E}_{\alpha}, \overset{L}{E}_{\beta}\right]^{\mu}$$

$$= -C_{\underline{\kappa\lambda}}{}^{\mu}\, \overset{L}{E}_{\alpha}{}^{\underline{\kappa}}\, \overset{L}{E}_{\beta}{}^{\underline{\lambda}} = -C_{\alpha\beta}{}^{\mu}$$

$$\overset{L}{\nabla}_{\ell_m} \Omega_n = \overset{L}{\nabla}_{\ell_m} \Omega_n - \underbrace{\overset{L}{\nabla}_{\Omega_n}}_{0} \ell_m - \underbrace{[\ell_m, \Omega_n]}_{0}$$

$$= \ell_m \cdot \overset{L}{T} \cdot \Omega_n = - \ell_m \cdot C \cdot \Omega_n$$

$$\overset{L}{\nabla} \Omega_n = - C \cdot \Omega_n$$

$$\overset{L}{\nabla}_{l_m} \Omega_n = \overset{L}{\nabla}_{l_m} \Omega_n - \underbrace{\overset{L}{\nabla}_{\Omega_n} l_m}_{0} - \underbrace{[l_m, \Omega_n]}_{0}$$

$$= l_m \cdot \overset{L}{T} \cdot \Omega_n = - l_m \cdot C \cdot \Omega_n$$

$$\overset{L}{\nabla} \Omega_n = - C \cdot \Omega_n$$

$$\overset{R}{\nabla} - \overset{L}{\nabla} = C \qquad C_{\underline{\alpha}\kappa}{}^{\gamma}$$

$$0 = \overset{R}{\nabla}_{\underline{\alpha}} \Omega_n^{\kappa} = \underline{\overset{L}{\nabla}_{\underline{\alpha}} \Omega_n^{\kappa}} + A_{\underline{\alpha}\gamma}{}^{\kappa} \Omega_n^{\gamma} = \left(A_{\underline{\alpha}\gamma}{}^{\kappa} - C_{\underline{\alpha}\gamma}{}^{\kappa}\right) \Omega_n^{\gamma}$$

Strukturní rovnice pro $\overset{L}{E}{}^{\alpha}$ $\overset{R}{E}{}^{\alpha}$

$$d\overset{L}{E}{}^{\alpha} + \frac{1}{2} C_{\mu\nu}{}^{\alpha} \overset{L}{E}{}^{\mu} \wedge \overset{L}{E}{}^{\nu} = 0$$

$$d\overset{R}{E}{}^{\alpha} - \frac{1}{2} C_{\mu\nu}{}^{\alpha} \overset{R}{E}{}^{\mu} \wedge \overset{R}{E}{}^{\nu} = 0$$

$$d_{\underline{\kappa}} \overset{L}{E}{}^{\alpha}_{\underline{\nu}} = \underbrace{\overset{L}{\nabla}_{\underline{\mu}} \wedge \overset{L}{E}{}^{\alpha}_{\nu}}_{0} + \overset{L}{T}{}^{\underline{\kappa}}_{\mu\underline{\nu}} \overset{L}{E}{}^{\alpha}_{\kappa} = -C_{\mu\underline{\nu}}{}^{\underline{\kappa}} \overset{L}{E}{}^{\alpha}_{\underline{\kappa}}$$

$$= -C_{\kappa\lambda}{}^{\alpha} \overset{L}{E}{}^{\kappa}_{\underline{\mu}} \overset{L}{E}{}^{\lambda}_{\underline{\nu}} = -\frac{1}{2} C_{\kappa\lambda}{}^{\alpha} \overset{L}{E}{}^{\kappa}_{\underline{\mu}} \wedge \overset{L}{E}{}^{\lambda}_{\underline{\nu}}$$

Strukturní rovnice pro $\overset{L}{E}^{\alpha}$ $\overset{R}{E}^{\alpha}$

$$d\overset{L}{E}^{\alpha} + \frac{1}{2} C_{\mu\nu}{}^{\alpha} \overset{L}{E}^{\mu} \wedge \overset{L}{E}^{\nu} = 0$$

$$d\overset{R}{E}^{\alpha} - \frac{1}{2} C_{\mu\nu}{}^{\alpha} \overset{R}{E}^{\mu} \wedge \overset{R}{E}^{\nu} = 0$$

$$d\overset{L}{E}{}^{\alpha} + \frac{1}{2} C_{\mu\nu}{}^{\alpha} \overset{L}{E}{}^{\mu} \wedge \overset{L}{E}{}^{\nu} = 0$$

$$d_{\underline{\kappa}} \overset{L}{E}{}^{\alpha}_{\underline{\nu}} = \underbrace{\overset{L}{\nabla}_{\underline{\kappa}} \wedge \overset{L}{E}{}^{\alpha}_{\underline{\nu}}}_{0} + \overset{L}{T}{}^{\underline{\kappa}}_{\underline{\kappa\nu}} \overset{L}{E}{}^{\alpha}_{\kappa} = -C_{\underline{\kappa\nu}}{}^{\underline{\kappa}} \overset{L}{E}{}^{\alpha}_{\underline{\kappa}}$$

$$= -C_{\kappa\lambda}{}^{\alpha} \overset{L}{E}{}^{\kappa}_{\underline{\kappa}} \overset{L}{E}{}^{\lambda}_{\underline{\nu}} = -\frac{1}{2} C_{\kappa\lambda}{}^{\alpha} \overset{L}{E}{}^{\kappa}_{\underline{\kappa}} \wedge \overset{L}{E}{}^{\lambda}_{\underline{\nu}}$$

Levi-Civitova metrická derivace na Lieově grupě

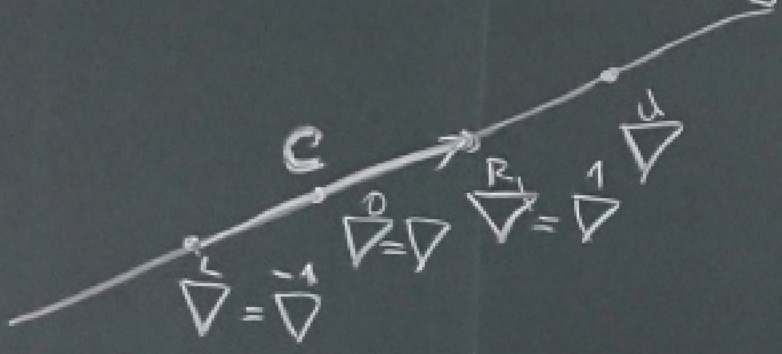
$$\overset{L}{\nabla} \qquad \overset{R}{\nabla}$$

$$\overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2}\mathbf{C} =$$

$$= \overset{R}{\nabla} + \frac{\lambda-1}{2}\mathbf{C}$$

$$\overset{\lambda}{\nabla}_{\xi} a^{\kappa} = \overset{L}{\nabla}_{\xi} a^{\kappa} + \frac{\lambda+1}{2} C_{\mu\nu}{}^{\kappa} a^{\nu}$$

$$= \overset{R}{\nabla}_{\xi} a^{\kappa} + \frac{\lambda-1}{2} C_{\mu\nu}{}^{\kappa} a^{\nu}$$



$$\overset{\lambda}{\nabla}_{\xi} l^{\kappa}_{m} = \frac{\lambda+1}{2} C_{\mu\nu}{}^{\kappa} l^{\nu}_{m}$$

$$\overset{\lambda}{\nabla}_{\xi} r^{\kappa}_{m} = \frac{\lambda-1}{2} C_{\mu\nu}{}^{\kappa} r^{\nu}_{m}$$

$$\overset{\lambda}{\nabla}_{l_m} l_n = \frac{\lambda+1}{2} l_m \cdot c \cdot l_n = \frac{\lambda+1}{2}[l_m, l_n]$$

$$= \frac{\lambda+1}{2} l_{[m,n]}$$

$$\overset{\lambda}{\nabla}_{r_m} r_n = \frac{1-\lambda}{2} r_{[m,n]}$$

$$\overset{\lambda}{\nabla} c = 0 \qquad \overset{\lambda}{\nabla} k = 0$$

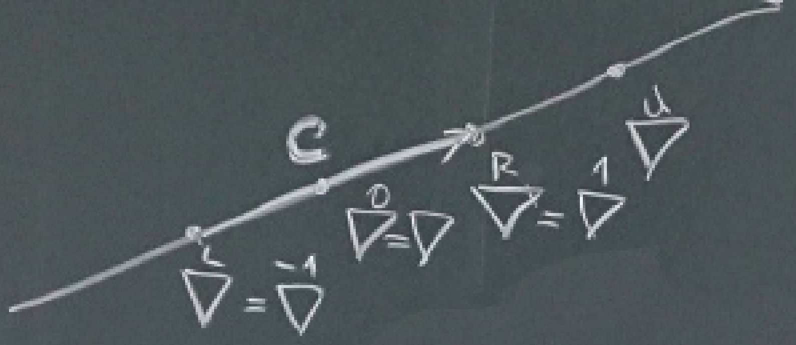
$$C_{\underline{\alpha}} C_{\underline{\beta\gamma}}{}^{\underline{\kappa}} = C_{\underline{\alpha\lambda}}{}^{\underline{\kappa}} C_{\underline{\beta\gamma}}{}^{\underline{\lambda}} - C_{\underline{\alpha\beta}}{}^{\underline{\lambda}} C_{\underline{\lambda\gamma}}{}^{\underline{\kappa}} - C_{\underline{\alpha\gamma}}{}^{\lambda} C_{\underline{\beta\lambda}}{}^{\underline{\kappa}}$$

$$= C_{\underline{\alpha\lambda}}{}^{\underline{\kappa}} C_{\underline{\beta\gamma}}{}^{\underline{\lambda}} + C_{\underline{\gamma\lambda}}{}^{\underline{\kappa}} C_{\underline{\alpha\beta}}{}^{\underline{\lambda}} + C_{\underline{\beta\lambda}}{}^{\underline{\kappa}} C_{\underline{\gamma\alpha}}{}^{\underline{\lambda}} = 0$$

$$[[m,n],p] + [(n,p),m] + [[p,m],n] = 0$$

Levi-Civitova metrická derivace na Lieově grupě

$$\overset{L}{\nabla} \qquad \overset{R}{\nabla}$$



$$\overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2}\mathbf{C} =$$

$$= \overset{R}{\nabla} + \frac{\lambda-1}{2}\mathbf{C}$$

$$\overset{\lambda}{\nabla}_\mu a^\kappa = \overset{L}{\nabla}_\mu a^\kappa + \frac{\lambda+1}{2} C_{\mu\nu}{}^{\kappa} a^\nu$$

$$= \overset{R}{\nabla}_\mu a^\kappa + \frac{\lambda-1}{2} C_{\mu\nu}{}^{\kappa} a^\nu$$

$$\overset{\lambda}{\nabla}_\mu \ell^\kappa_m = \frac{\lambda+1}{2} C_{\mu\nu}{}^{\kappa} \ell^\nu_m$$

$$\overset{\lambda}{\nabla}_\mu \Omega^\kappa_m = \frac{\lambda-1}{2} C_{\mu\nu}{}^{\kappa} \Omega^\nu_m$$

$$\overset{\lambda}{\nabla}_{\ell_m} \ell_n = \frac{\lambda+1}{2} \ell_m \cdot C \cdot \ell_n = \frac{\lambda+1}{2} [\ell_m, \ell_n]$$

$$= \frac{\lambda+1}{2} \ell_{[m,n]}$$

$$\overset{\lambda}{\nabla}_{\Omega_m} \Omega_n = \frac{1-\lambda}{2} \Omega_{[m,n]}$$

$$\overset{\lambda}{\nabla} C = 0 \qquad \overset{\lambda}{\nabla} k = 0$$

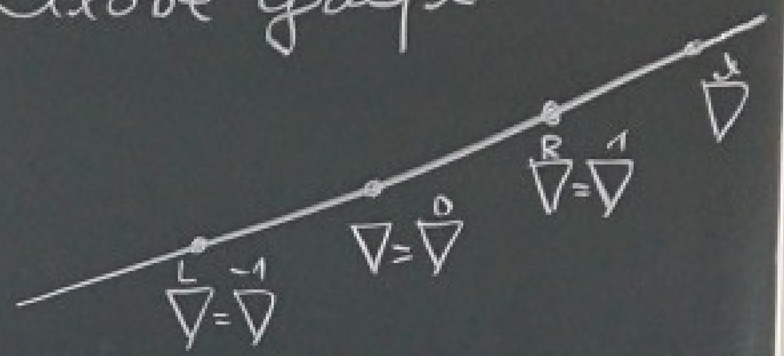
$$C_{\underline{\alpha}} C_{\underline{\beta\gamma}}{}^{\underline{k}} = C_{\underline{\alpha\lambda}}{}^{\underline{k}} C_{\underline{\beta\gamma}}{}^{\underline{\lambda}} - C_{\underline{\alpha\beta}}{}^{\underline{\lambda}} C_{\underline{\lambda\gamma}}{}^{\underline{k}} - C_{\underline{\alpha\gamma}}{}^{\lambda} C_{\underline{\beta\lambda}}{}^{k}$$

$$= C_{\underline{\alpha\lambda}}{}^{\underline{k}} C_{\underline{\beta\gamma}}{}^{\underline{\lambda}} + C_{\underline{\gamma\lambda}}{}^{\underline{k}} C_{\underline{\alpha\beta}}{}^{\underline{\lambda}} + C_{\underline{\beta\lambda}}{}^{\underline{k}} C_{\underline{\gamma\alpha}}{}^{\underline{\lambda}} = 0$$

$$[[m,n],p] + [[n,p],m] + [[p,m],n] = 0$$

Levi-Civitova metrická derivace na Lieově grupě

$$\overset{L}{\nabla},\ \overset{R}{\nabla},\quad \overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2}\mathbb{C} = \overset{R}{\nabla} + \frac{\lambda-1}{2}\mathbb{C}$$



$$\overset{\lambda}{T}_{\mu\nu}{}^{\kappa} = \lambda\, C_{\mu\nu}{}^{\kappa} \qquad \overset{\lambda}{R}_{\mu\nu}{}^{\kappa}{}_{\lambda} = -\frac{1-\lambda^2}{2} C_{\mu\nu}{}^{\alpha} C_{\alpha\lambda}{}^{\kappa}$$

$$\overset{\lambda}{Ric}_{\alpha\beta} = \frac{1-\lambda^2}{2} k_{\alpha\beta}$$

$$\overset{\lambda}{T}_{\mu\nu}{}^{\kappa} = \overset{L}{T}_{\mu\nu}{}^{\kappa} + 2\frac{\lambda+1}{2} C_{[\mu\nu]}{}^{\kappa} = \lambda\, C_{\mu\nu}{}^{\kappa}$$

$$\overset{\lambda}{R}(l_a,l_b)\cdot l_c = \overset{\lambda}{\nabla}_{l_a}\overset{\lambda}{\nabla}_{l_b} l_c - \overset{\lambda}{\nabla}_{l_b}\overset{\lambda}{\nabla}_{l_a} l_c - \overset{\lambda}{\nabla}_{[l_a,l_b]} l_c =$$

$$= \frac{\lambda+1}{2}\left(\overset{\lambda}{\nabla}_{l_a} l_{[b,c]} - \overset{\lambda}{\nabla}_{l_b} l_{[a,c]} - \overset{\lambda}{\nabla}_{l_{[a,b]}} l_c\right) - \frac{1-\lambda}{2}\overset{\lambda}{\nabla}_{l_{[a,b]}} l_c$$

$$= \left(\frac{\lambda+1}{2}\right)^2 l\big[[a,(b,c]) - [b,[a,c]] - [[a,b],c]\big] - \frac{1-\lambda^2}{2} l_{[[a,b],c]}$$

$$= -\frac{1-\lambda^2}{2}\,[[l_a,l_b],l_c]^{\nu} = -\frac{1-\lambda^2}{2} C_{\alpha\beta}{}^{\kappa} C_{\kappa\gamma}{}^{\nu}\, l_a^{\alpha} l_b^{\beta} l_c^{\gamma}$$

$$\nabla = \overset{0}{\nabla} \qquad \nabla k = 0 \qquad T = 0$$

$$R = -\frac{1}{2} c\cdot c \qquad Ric = \frac{1}{2} k$$

$$Ein + \frac{D-2}{4} k = 0 \qquad \mathcal{R} = \frac{D}{2}$$

orbity l_m r_m jsou geodetiky

$$\nabla_{l_m} l_m = l_{[m,m]} = 0$$

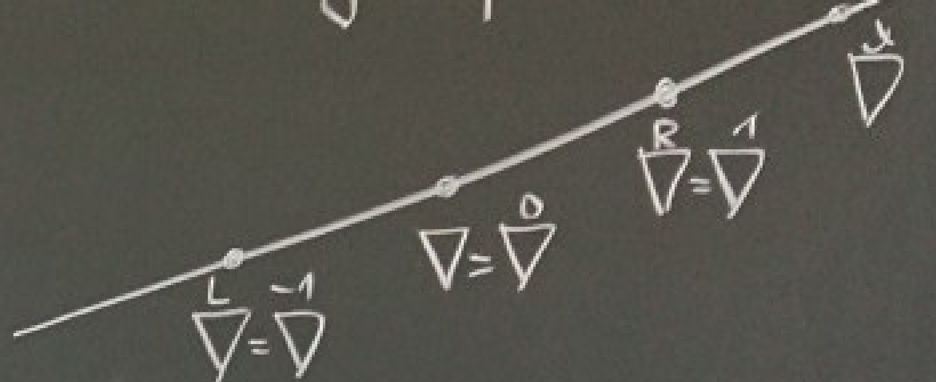
$\exp(\alpha m)$ geodetika

Levi-Civitova metrická derivace na Lieově grupě

$\overset{L}{\nabla}$, $\overset{R}{\nabla}$, $\overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2}\mathbb{C} = \overset{R}{\nabla} + \frac{\lambda-1}{2}\mathbb{C}$

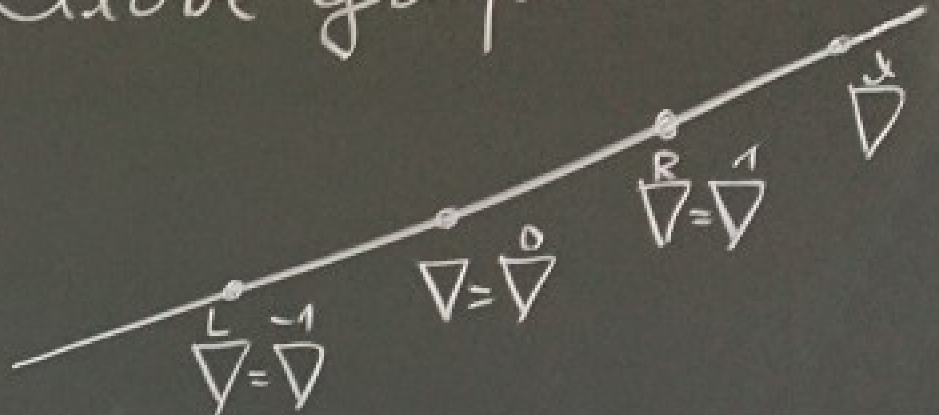
$\overset{\lambda}{T}_{\underline{\mu\nu}}{}^{\underline{\kappa}} = \lambda\, C_{\underline{\mu\nu}}{}^{\underline{\kappa}}$

$\overset{\lambda}{R}_{\underline{\mu\nu}}{}^{\underline{\kappa}}{}_{\underline{\lambda}} = -\frac{1-\lambda^2}{2} C_{\underline{\mu\nu}}{}^{\underline{\alpha}} C_{\underline{\alpha\lambda}}{}^{\underline{\kappa}}$



$\overset{\lambda}{Ric}_{\underline{\alpha\beta}} = \frac{1-\lambda^2}{2} k_{\underline{\alpha\beta}}$

Levi-Civitova metrická derivace na Lieově grupě



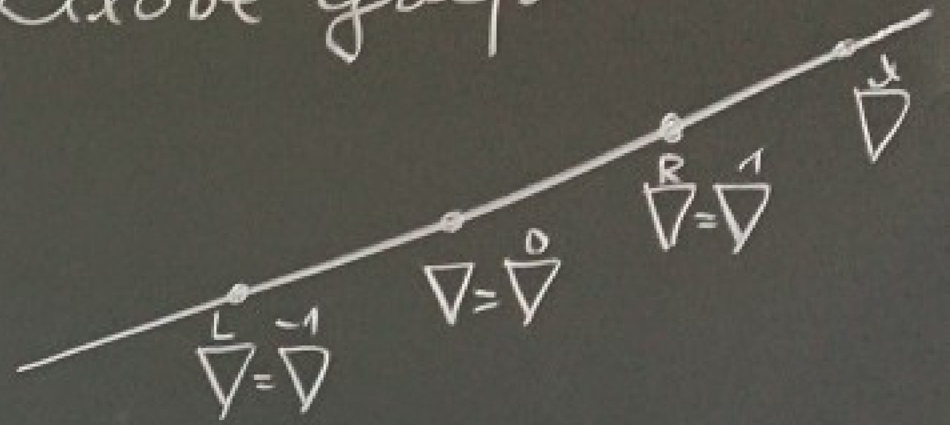
$$\overset{L}{\nabla},\ \overset{R}{\nabla},\quad \overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2}\mathbb{C} = \overset{R}{\nabla} + \frac{\lambda-1}{2}\mathbf{C}$$

$$\overset{\lambda}{T}_{\underline{\mu\nu}}{}^{\underline{\kappa}} = \lambda\, C_{\underline{\mu\nu}}{}^{\underline{\kappa}} \qquad \overset{\lambda}{R}_{\underline{\mu\nu}}{}^{\underline{\kappa}}{}_{\underline{\lambda}} = -\frac{1-\lambda^2}{2} C_{\underline{\mu\nu}}{}^{\underline{\alpha}} C_{\underline{\alpha\lambda}}{}^{\underline{\kappa}}$$

$$\overset{\lambda}{Ric}_{\underline{\alpha\beta}} = \frac{1-\lambda^2}{2} k_{\underline{\alpha\beta}}$$

$$\overset{\lambda}{T}_{\underline{\mu\nu}}{}^{\underline{\kappa}} = \overset{L}{T}_{\underline{\mu\nu}}{}^{\underline{\kappa}} + 2\frac{\lambda+1}{2} C_{[\underline{\mu\nu}]}{}^{\underline{\kappa}} = \lambda\, C_{\underline{\mu\nu}}{}^{\underline{\kappa}}$$

Levi-Civitova metrická derivace na Lieově grupě



$$\overset{L}{\nabla}, \quad \overset{R}{\nabla}, \quad \overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2}\mathbb{C} = \overset{R}{\nabla} + \frac{\lambda-1}{2}\mathbb{C}$$

$$\overset{\lambda}{T}_{\mu\nu}{}^{\kappa} = \lambda\, c_{\mu\nu}{}^{\kappa} \qquad \overset{\lambda}{R}_{\mu\nu}{}^{\kappa}{}_{\lambda} = -\frac{1-\lambda^2}{2} c_{\mu\nu}{}^{\alpha} c_{\alpha\lambda}{}^{\kappa}$$

$$\overset{\lambda}{Ric}_{\alpha\beta} = \frac{1-\lambda^2}{2} k_{\alpha\beta}$$

$$\overset{\lambda}{R}(l_a, l_b)\cdot l_c = \overset{\lambda}{\nabla}_{l_a}\overset{\lambda}{\nabla}_{l_b} l_c - \overset{\lambda}{\nabla}_{l_b}\overset{\lambda}{\nabla}_{l_a} l_c - \overset{\lambda}{\nabla}_{[l_a, l_b]} l_c =$$

$$= \frac{\lambda+1}{2}\left(\overset{\lambda}{\nabla}_{l_a} l_{[b,c]} - \overset{\lambda}{\nabla}_{l_b} l_{[a,c]} - \overset{\lambda}{\nabla}_{l_{[a,b]}} l_c\right) - \frac{1-\lambda}{2}\overset{\lambda}{\nabla}_{l_{[a,b]}} l_c$$

$$= \left(\frac{\lambda+1}{2}\right)^2 l\left[[a,(b,c]]) - [b,[a,c]] - [[a,b],c]\right] - \frac{1-\lambda^2}{2} l_{[[a,b],c]}$$

$$= -\frac{1-\lambda^2}{2}[[l_a, l_b], l_c]^{\nu} = -\frac{1-\lambda^2}{2} c_{\alpha\beta}{}^{\kappa} c_{\kappa\gamma}{}^{\nu} l_a^{\alpha} l_b^{\beta} l_c^{\gamma}$$

$$\nabla = \overset{0}{\nabla} \qquad \nabla k = 0 \qquad T = 0$$

$$R = -\frac{1}{2} c \cdot c \qquad \mathrm{Ric} = \frac{1}{2} k$$

$$\mathrm{Ein} + \frac{D-2}{4} k = 0 \qquad \mathbb{R} = \frac{D}{2}$$

orbity l_m Ωm jsou geodetiky

$$\nabla_{l_m} l_m = l_{[m,m]} = 0$$

$\exp(\alpha m)$ geodetika

$$\overset{\lambda}{\nabla}_{l_m} = \mathcal{L}_{l_m} + \mathrm{Ad}_{\frac{\lambda-1}{2} m}$$

$$\mathrm{Ad}_a \longleftrightarrow -a^k c_{k\mu}{}^{\nu} = -\mathrm{ad}_a{}^{\nu}{}_{\mu}$$

$$T_eG \qquad -l_a^k c_{k\mu}{}^{\nu}$$

$$\overset{\lambda}{\nabla}_{r_m} = \mathcal{L}_{r_m} + \mathrm{Ad}_{\frac{\lambda+1}{2} m}$$

konkrétně

$$\overset{R}{\nabla}_{l_m} = \mathcal{L}_{l_m} \qquad \overset{L}{\nabla}_{r_m} = \mathcal{L}_{r_m}$$

$$\mathcal{L}_{l_m} = \overset{\lambda}{\nabla}_{l_m} + \Lambda \qquad \Lambda = -\overset{\lambda}{\nabla} l_m - l_m \cdot \overset{\lambda}{T} = -\frac{\lambda+1}{2} c \cdot l_m - \lambda\, l_m \cdot c$$

$$= -\frac{\lambda-1}{2} l_m \cdot c$$

$$\overset{\lambda}{\nabla}_{l_m} = \mathcal{L}_{l_m} + \mathrm{Ad}_{\frac{\lambda-1}{2}m}$$

$$\overset{\lambda}{\nabla}_{r_m} = \mathcal{L}_{r_m} + \mathrm{Ad}_{\frac{\lambda+1}{2}m}$$

konkrétně

$$\overset{R}{\nabla}_{l_m} = \mathcal{L}_{l_m} \qquad \overset{L}{\nabla}_{r_m} = \mathcal{L}_{r_m}$$

$$\mathrm{Ad}_a \longleftrightarrow -a^{\underline{k}} c_{\underline{k}\underline{\mu}}{}^{\underline{\nu}} = -\mathrm{ad}_a{}^{\underline{\nu}}{}_{\underline{\mu}}$$

$$T_eG \qquad -l_a^{\underline{k}} c_{\underline{k}\underline{\mu}}{}^{\underline{\nu}}$$

$$\overset{\lambda}{\nabla}_{l_m} = \mathcal{L}_{l_m} + \mathrm{Ad}_{\frac{\lambda-1}{2}m}$$

$$\begin{array}{ll} \mathrm{Ad}_a \longleftrightarrow & -a^{\underline{k}} c_{\underline{k}\underline{r}}{}^{\underline{r}} = -\mathrm{ad}_a{}^{\underline{r}}{}_{\underline{k}} \\ \quad T_eG & -l_a^{\underline{k}} c_{\underline{k}\underline{r}}{}^{\underline{r}} \end{array}$$

$$\mathcal{L}_{l_m} = \overset{\lambda}{\nabla}_{l_m} + \Lambda$$

$$\begin{aligned} \Lambda = -\overset{\lambda}{\nabla} l_m - l_m \cdot \overset{\lambda}{T} &= -\frac{\lambda+1}{2} c \cdot l_m - \lambda\, l_m \cdot c \\ &= -\frac{\lambda-1}{2} l_m \cdot c \end{aligned}$$

Akce Lieovy grupy na varietě

Def A je akce LG na var M

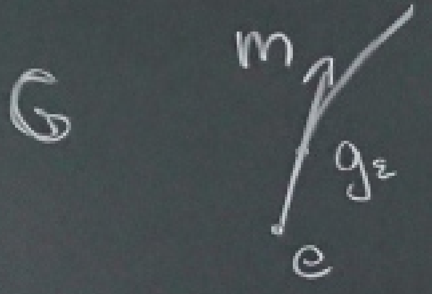
$g \in G \quad A_g : M \to M$

1) $A_{g_1} \circ A_{g_2} = A_{g_1 g_2}$ levá akce $\quad A_g x = g x$

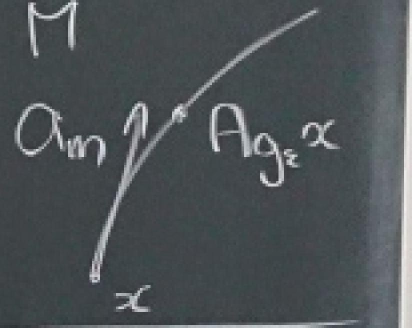
2) $A_{g_1} \circ A_{g_2} = A_{g_2 g_1}$ pravá akce $\quad A_g x = x g$

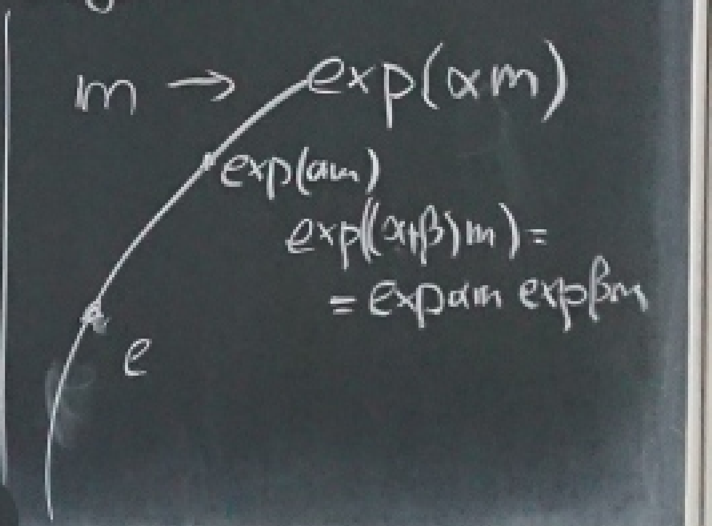
Př: $L_g \quad R_{g^{-1}} \quad AD_g$ levé akce na G

$R_g \quad L_{g^{-1}} \quad AD_{g^{-1}}$ pravé akce na G



$g_\varepsilon \approx e + \varepsilon m$





$\exp((\alpha+\beta)m) =$
$= \exp \alpha m \exp \beta m$

Def A_g je akce G na M

a_m generátor akce A_g

$a : \mathfrak{g} \to TM$

$\forall m \in \mathfrak{g} \to a_m \in TM$

$$a_m\big|_x = \frac{D}{d\varepsilon} A_{g_\varepsilon} x \Big|_{\varepsilon=0}$$

$$\frac{D g_\varepsilon}{d\varepsilon}\Big|_{\varepsilon=0} = m \qquad g_0 = e$$

$$\to A_{\exp(\alpha m)} \longleftrightarrow a_m$$

Akce Lieovy grupy na varietě

Def A je akce LG na var M

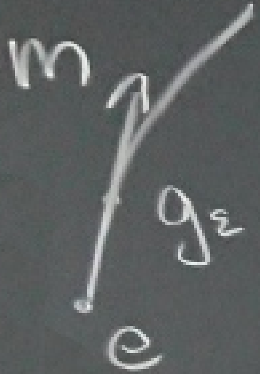
$g \in G$ $\quad A_g : M \to M$

1) $A_{g_1} \circ A_{g_2} = A_{g_1 g_2}$ levá akce $\quad A_g x = g x$

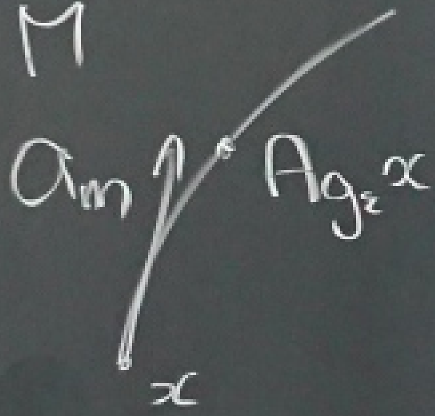
2) $A_{g_1} \circ A_{g_2} = A_{g_2 g_1}$ pravá akce $\quad A_g x = x g$

Př: $L_g \quad R_{g^{-1}} \quad AD_g$ levé akce na G

$R_g \quad L_{g^{-1}} \quad AD_{g^{-1}}$ pravé akce na G



$$g_\varepsilon \approx e + \varepsilon m$$



Def A_g je akce G na M

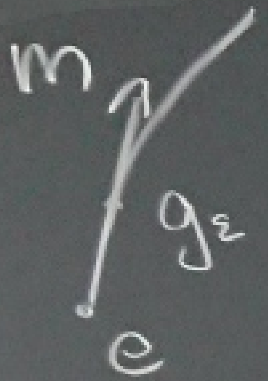
a_m generátor akce A_g

$$a : \mathfrak{g} \to TM$$

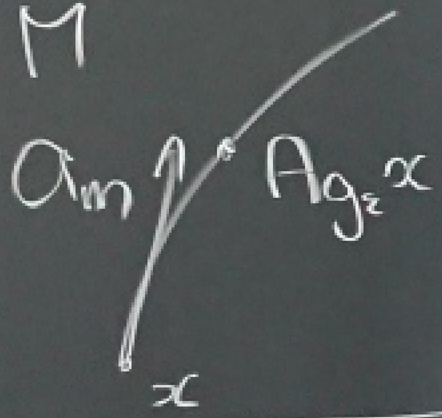
$$\forall m \in \mathfrak{g} \to a_m \in TM$$

$$a_m\Big|_x = \frac{D}{d\varepsilon} A_{g_\varepsilon} x \Big|_{\varepsilon=0}$$

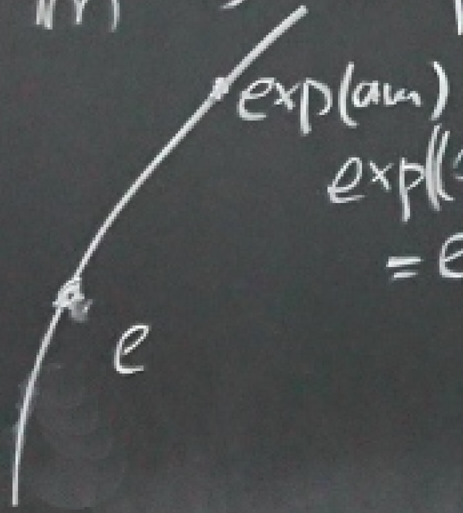
$$\frac{D g_\varepsilon}{d\varepsilon}\Big|_{\varepsilon=0} = m \qquad g_0 = e$$



$g_\varepsilon \approx e + \varepsilon m$



$m \rightarrow \exp(\alpha m)$



$\exp((\alpha+\beta)m) =$
$= \exp \alpha m \, \exp \beta m$

Def A_g je akce G na M

a_m generátor akce A_g

$$a : \mathfrak{g} \rightarrow TM$$

$$\forall m \in \mathfrak{g} \rightarrow a_m \in TM$$

$$a_m \Big|_x = \frac{D}{d\varepsilon} A_{g_\varepsilon} x \Big|_{\varepsilon=0}$$

$$\frac{D g_\varepsilon}{d\varepsilon}\Big|_{\varepsilon=0} = m \qquad g_0 = e$$

$$\rightarrow A_{\exp(\alpha m)} \longleftrightarrow a_m$$

$$a_{m+\alpha n} = a_m + \alpha a_n$$

Př l-akce L_g $\quad r_m \quad L_{\exp(\alpha m)}$

r-akce R_g $\quad l_m$

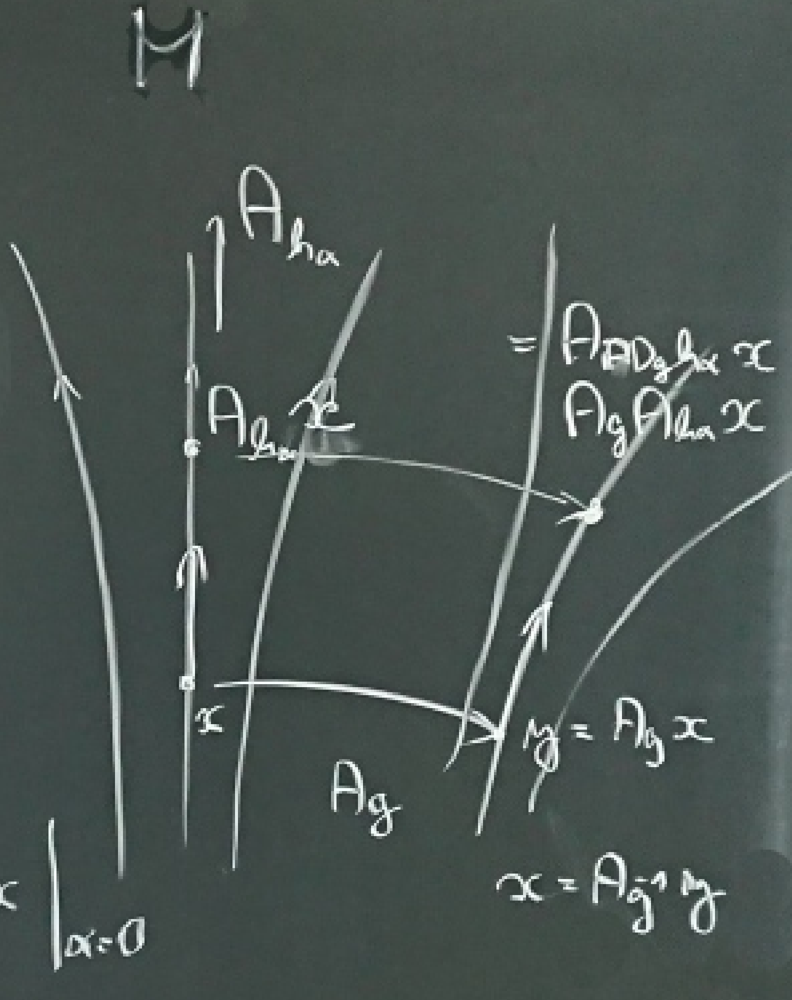
Lemma

1, $A_{g*} a_m = a_{\mathrm{Ad}_g m}$ – A_g levá akce

2, $A_{g^{-1}*} a_m = a_{\mathrm{Ad}_g m}$ – A_g pravá akce

dů: $A_{g*} a_m|_x = A_{g*} \frac{D}{d\alpha} A_{h_\alpha} x\Big|_{\alpha=0} = \frac{D}{d\alpha} A_g A_{h_\alpha} x\Big|_{\alpha=0}$

$$= \frac{D}{d\alpha} A_{AD_g h_\alpha} \underbrace{A_g x}_{y}\Big|_{\alpha=0} = a_{\mathrm{Ad}_g m}\Big|_{A_g x}$$



Věta

A_g levá akce $[a_m, a_n] = - a_{[m,n}$

A_g pravá akce $[a_m, a_n] = a_{[m,n]}$

$$[a_m, a_n] = \mathcal{L}_{a_m} a_n = -\frac{d}{d\alpha} A_{g_\alpha *} a_n$$

$$= -\frac{d}{d\alpha} a_{\mathrm{Ad}_{g_\alpha} n}\Big|_{\alpha=0} = - a_{\underbrace{\mathrm{ad}_m n}_{[m,n]}} =$$

$$m = \frac{Dg_\alpha}{d\alpha}\Big|_{\alpha=0}$$

$$= - a_{[m,n]}$$

$$a_{m+\alpha n} = a_m + \alpha a_n$$

Př l-akce L_g $\quad$ R_m $\quad$ $L_{\exp(\alpha m)}$

p-akce R_g $\quad$ l_m

Lemma

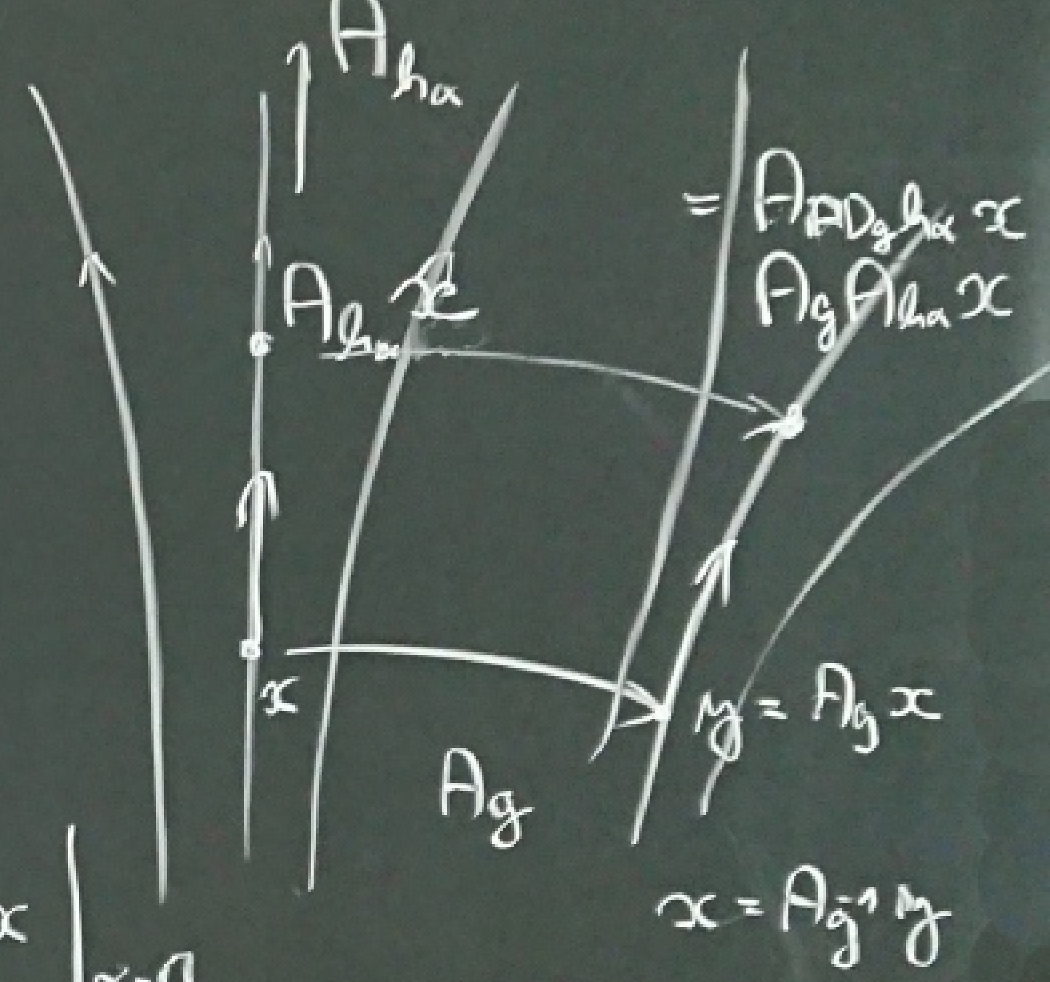
1, $A_{g*}\, a_m = a_{Ad_g m}$ – A_g levá akce

2, $A_{g^{-1}*}\, a_m = a_{Ad_g m}$ – A_g pravá akce

Lema

1, $A_{g*}\, a_m = a_{Ad_g m}$ – A_g levá akce

2, $A_{g^{-1}*}\, a_m = a_{Ad_g m}$ – A_g pravá akce



dů 1 $A_{g*}\, a_m\big|_x = A_{g*} \frac{D}{d\alpha} A_{h_\alpha} x\Big|_{\alpha=0} = \frac{D}{d\alpha} A_g A_{h_\alpha} x\Big|_{\alpha=0}$

$$= \frac{D}{d\alpha} A_{AD_g h_\alpha} \underbrace{A_g x}_{y}\Big|_{\alpha=0} = a_{Ad_g m}\Big|_{A_g x}$$

Věta

A_g levá akce $\quad [a_m, a_n] = -\, a_{[m,n]}$

A_g pravá akce $\quad [a_m, a_n] = a_{[m,n]}$

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$$= -\frac{d}{d\alpha} a_{\mathrm{Ad}_{g_\alpha} n} \Big|_{\alpha=0} = -a_{\underbrace{\mathrm{ad}_m n}_{[m,n]}} =$$

$$m = \frac{Dg_\alpha}{d\alpha}\Big|_{\alpha=0}$$

$$= -a_{[m,n]}$$

Reprezentace L.G. a L.A. na vektorovém prost. V

Def $\quad T \quad G \to \operatorname{Lin} V \qquad g \to T_g \in V^1_1$

$$T_{gh} = T_g \cdot T_h \qquad T_{gh}{}^{\underline{A}}{}_{\underline{B}} = T_g{}^{\underline{A}}{}_{\underline{M}}\, T_h{}^{\underline{M}}{}_{\underline{B}}$$

Def $\quad t \quad \mathfrak{g} \to \operatorname{Lin} V \qquad m \to t_m \in V^1_1$

$$t_{[m,n]} = [t_m, t_n] = t_m \cdot t_n - t_n \cdot t_m$$

$$t_m{}^{\underline{A}}{}_{\underline{B}} = m^{\alpha}\, t_{\underline{\alpha}}{}^{\underline{A}}{}_{\underline{B}}$$

$$C_{\underline{\mu\nu}}{}^{\underline{\alpha}}\, t_{\underline{\alpha}}{}^{\underline{K}}{}_{\underline{L}} = t_{\underline{\mu}}{}^{\underline{K}}{}_{\underline{N}}\, t_{\underline{\nu}}{}^{\underline{N}}{}_{\underline{L}} - t_{\underline{\nu}}{}^{\underline{K}}{}_{\underline{N}}\, t_{\underline{\mu}}{}^{\underline{N}}{}_{\underline{L}}$$

$$e + \varepsilon m \qquad T_{e+\varepsilon m} = \delta + \varepsilon t_m$$

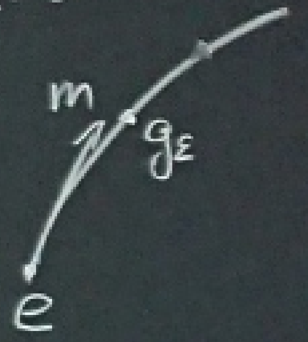
Def

T repr. LG na V

$\downarrow$

t generátor této repr.

$$t_m = \frac{d}{d\varepsilon} T_{g_\varepsilon} \Big|_{\varepsilon = 0}$$



$$T_{\exp m} = \exp t_m$$

$$T_{\exp 0} = \delta$$

$$\frac{d}{d\varepsilon} T_{\exp(\varepsilon m)} \Big|_{\varepsilon = \varepsilon_0} = \frac{d}{d\varepsilon} T_{\exp(\varepsilon m)} \Big|_{\varepsilon = 0} T_{\exp \varepsilon_0 m}$$

$$= t_m \cdot T_{\exp(\varepsilon_0 m)}$$

$$\Rightarrow T_{\exp \varepsilon m} = \exp(\varepsilon t_m)$$

Reprezentace L.G. a L.A. na vektorovém prost. V

Def $\quad T \quad G \rightarrow \operatorname{Lin} V \qquad g \rightarrow T_g \in V^1_1$

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Def $\quad t \quad \mathfrak{g} \rightarrow \operatorname{Lin} V \qquad m \rightarrow t_m \in V^1_1$

$$t_{[m,n]} = [t_m, t_n] = t_m \cdot t_n - t_n \cdot t_m$$

$$t_m{}^{\underline{A}}{}_{\underline{B}} = m^{\alpha}\, t_{\alpha}{}^{\underline{A}}{}_{\underline{B}}$$

$$C_{\kappa\gamma}{}^{\alpha}\, t_{\alpha}{}^{\underline{K}}{}_{\underline{L}} = t_{\kappa}{}^{\underline{K}}{}_{\underline{M}}\, t_{\gamma}{}^{\underline{M}}{}_{\underline{L}} - t_{\gamma}{}^{\underline{K}}{}_{\underline{M}}\, t_{\kappa}{}^{\underline{M}}{}_{\underline{L}}$$

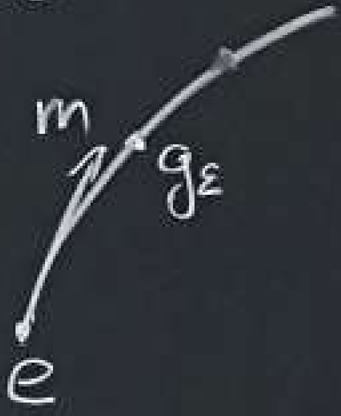
$e+\varepsilon m \qquad T_{e+\varepsilon m} = \delta + \varepsilon t_m$

Def

T repr. LG na V

↓ t generátor této repr.

$$t_m = \frac{d}{d\varepsilon} T_{g_\varepsilon} \Big|_{\varepsilon=0}$$



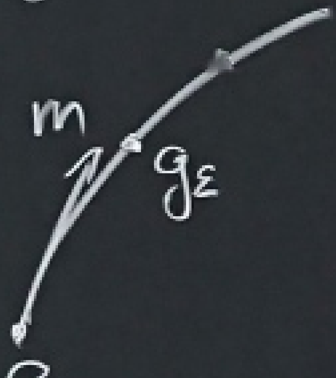
Def

$\downarrow$ T repr. LG na V

t generátor tejto repr.

$$t_m = \frac{d}{d\varepsilon} T_{g_\varepsilon}\Big|_{\varepsilon=0}$$

$$T_{\exp m} = \exp t_m$$



$$T_{\exp 0} = \delta$$

$$\frac{d}{d\varepsilon} T_{\exp(\varepsilon m)}\Big|_{\varepsilon=\varepsilon_0} = \frac{d}{d\varepsilon} T_{\exp(\varepsilon m)}\Big|_{\varepsilon=0} T_{\exp \varepsilon_0 m}$$

$$= t_m \cdot T_{\exp(\varepsilon_0 m)}$$

$$\Rightarrow T_{\exp \varepsilon m} = \exp(\varepsilon t_m)$$

$$[a,b] = \frac{D}{d\tau}\left[\exp(\sqrt{\tau}\,a)\exp(\sqrt{\tau}\,b)\exp(-\sqrt{\tau}\,a)\exp(-\sqrt{\tau}\,b)\right]\Big|_{\tau=0}$$

$$\frac{d}{d\tau}\, T_{\exp(\sqrt{\tau}a)\exp(\sqrt{\tau}b)\exp(-\sqrt{\tau}a)\exp(-\sqrt{\tau}b)}\Big|_{\tau=0} = t_{[a,b]} \overset{?}{=} [t_a, t_b]$$

$$T_{\exp(\sqrt{\tau}a)} = \exp(t_{\sqrt{\tau}a}) = \delta + \sqrt{\tau}\,t_a + \frac{1}{2}\tau\, t_a^2 + \dots$$

$$(\delta + \sqrt{\tau}\,t_a + \tfrac{1}{2}\tau t_a^2 + \dots)(\delta + \sqrt{\tau}\,t_b + \tfrac{1}{2}\tau t_b^2 + \dots)(\delta - \sqrt{\tau}\,t_a + \tfrac{1}{2}\tau t_a^2 + \dots)(\delta - \sqrt{\tau}\,t_b + \tfrac{1}{2}\tau t_b^2 + \dots) =$$

$$= \delta + \sqrt{\tau}\,(\cancel{t_a + t_b - t_a - t_b})$$

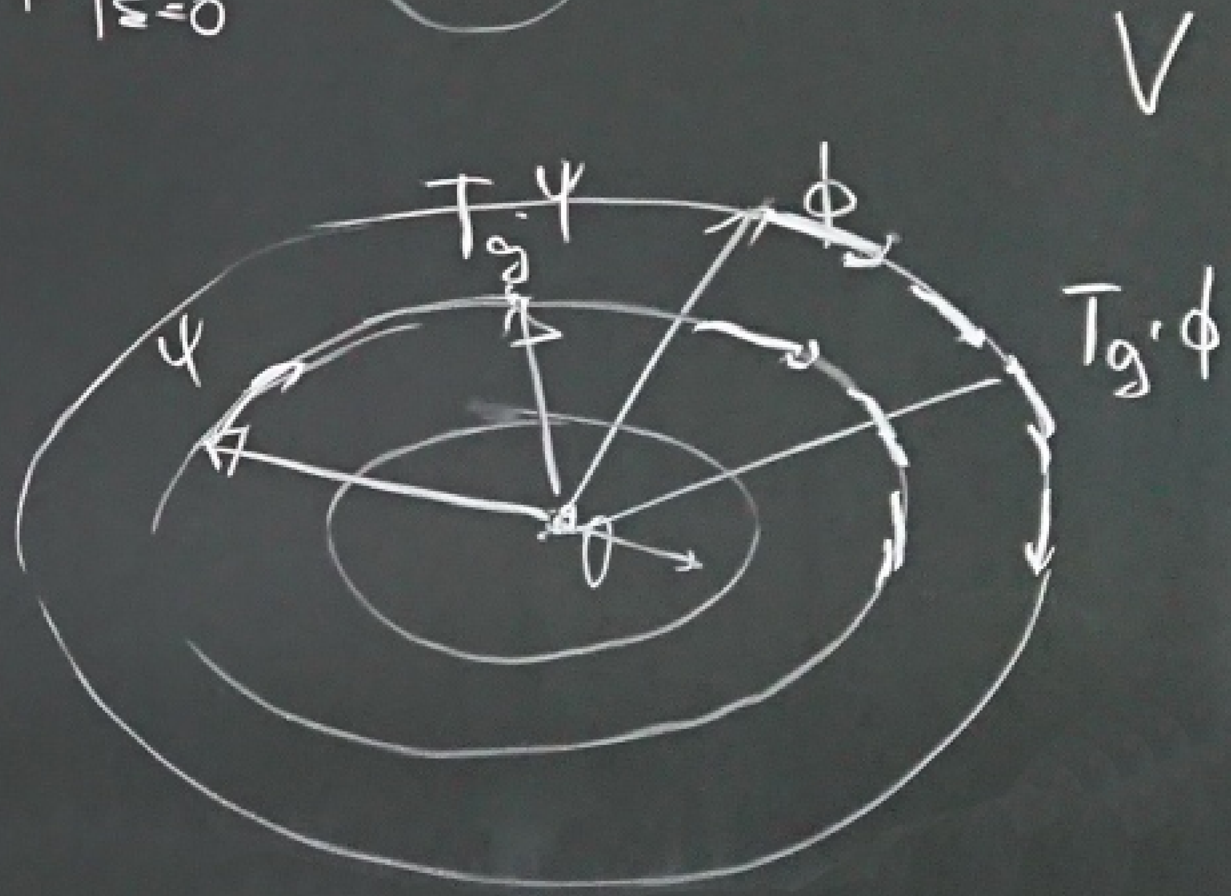
$$+ \tau(\cancel{t_a^2} + \cancel{t_b^2} + t_a \cdot t_b - \cancel{t_a t_b} - t_b \cdot t_a + \cancel{t_a t_b}) = \tau[t_a, t_b]$$

$$t_{[a,b]} = [t_a, t_b]$$

T репр. L.G. на V

$g \quad T_g \qquad \phi \to T_g \cdot \phi \qquad \phi^{\underline{A}} = T_g{}^{\underline{A}}{}_{\underline{B}} \phi^{\underline{B}}$

$m \to t_m \qquad t_m\big|_\phi = \frac{D}{d\varepsilon} T_{g_\varepsilon} \cdot \phi \Big|_{\varepsilon=0} = \boxed{t_m} \cdot \phi$



T repr. L.G. na V

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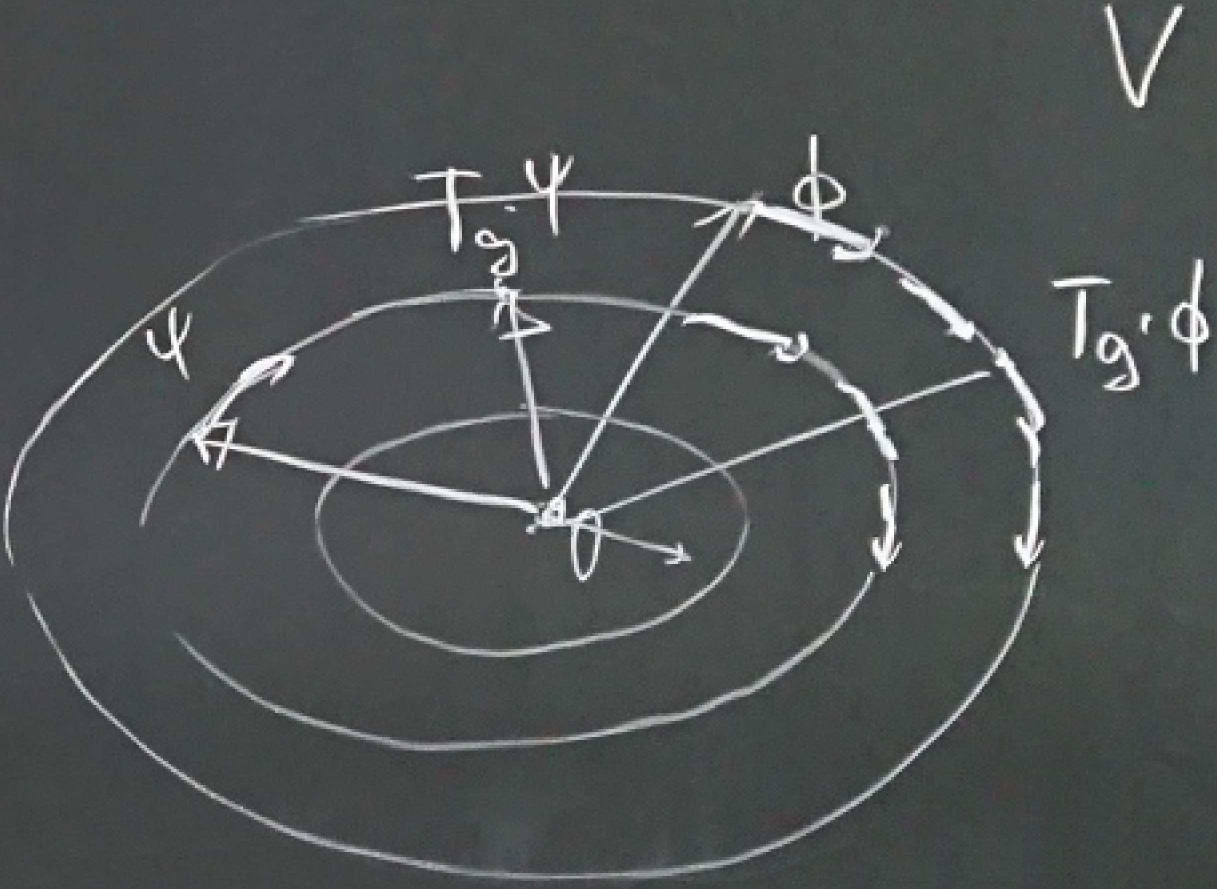
T репр. L.G. на V

$g \quad T_g \qquad \phi \to T_g \cdot \phi \qquad \phi^{\underline{A}} = T_g{}^{\underline{A}}{}_{\underline{B}}\, \phi^{\underline{B}}$

$m \to \mathbf{t}_m \qquad \mathbf{t}_m\big|_\phi = \frac{D}{d\varepsilon} T_{g_\varepsilon} \cdot \phi \Big|_{\varepsilon=0} = \boxed{t_m} \cdot \phi$


V
$T_g \cdot \psi$
ϕ
$T_g \cdot \phi$
ψ
0

Grupa isometrií

$\mathrm{Iso}(M,g)$ $\varphi: M \to M$

$$\varphi^{*} g = g$$

$$\varphi_1 \circ \varphi_2 = \varphi_1 \varphi_2$$

$\mathrm{Iso}(M,g)$ tvoří L.G.

$\mathrm{iso}(M,g)$ příslušná LA

M dim D

$\mathrm{Iso}(M,g)$ má maximální dim $\frac{D(D+1)}{2}$

$\mathrm{Iso}(M,g)$ M

$$\varphi \to \Xi_\varphi$$

$$\Xi_\varphi x = \varphi x$$

$\mathrm{iso}(M,g)$

$$S \to \xi_S \in TM$$

$$\xi_S \equiv S$$

$$\exp(\tau S) = \varphi_\tau$$

$$\mathcal{L}_{\xi_S} g = 0 \quad \varphi_{*} g = g$$

ξ_S je Killingův vektor

$$[\xi_m, \xi_n] = -\xi_{[m,n]}$$

báze Kill. vekt. ξ_α

$$[\xi_\alpha, \xi_\beta] = -\xi_{\underbrace{[\xi_\alpha, \xi_\beta]}_{C_{\alpha\beta}{}^{\mu}\xi_\mu}} = -C_{\alpha\beta}{}^{\mu}\,\xi_\mu$$

Grupa isometrií

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$\mathrm{Iso}(M,g)$ $\quad M$

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$$\xi_s \equiv S$$

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Grupa isometrií

$\mathrm{Iso}(M,g)$ $\varphi : M \to M$

$$\varphi^* g = g$$

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$\mathrm{Iso}(M,g)$ tvoří L.G.

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$\mathrm{Iso}(M,g)$ M

$$\varphi \to \Xi_\varphi$$

$$\Xi_\varphi x = \varphi x$$

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$$S \to \xi_S \in TM$$

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bázis Kill. vekt. ξ_α

$$[\xi_\alpha, \xi_\beta] = -\xi_{\underbrace{[\xi_\alpha, \xi_\beta]}_{C_{\alpha\beta}{}^{\mu}\xi_\mu}} = -C_{\alpha\beta}{}^{\mu}\,\xi_\mu$$

Akce symetrie na generátorech symetrie

Killing. vektory R K_μ

$\mathfrak{r} \in \mathrm{iso}(M,g)$ $\mathcal{R}_\varphi = \exp(\varphi \mathfrak{r}) \in \mathrm{Iso}(M,g)$

$\xi_\mathfrak{r} = R$ $\Xi_{\mathcal{R}_\varphi} = \mathcal{R}_\varphi$

$$[R, K_\mu] = -C_{R\mu}{}^\nu K_\nu$$

$V_\varphi = \mathcal{R}_{\varphi *} V_0$ $V = \underset{\uparrow}{V^\mu} K_\mu$ konstantní na M

$$\frac{d}{d\varphi} V_\varphi = -\mathcal{L}_R V_\varphi = -[R, V_\varphi] = -[R, K_\mu] V^\mu_\varphi = C_{R\mu}{}^\nu V^\mu_\varphi K_\nu$$

$\frac{d}{d\varphi} V^\nu_\varphi = C_{R\mu}{}^\nu V^\mu_\varphi$ $V^\nu_\varphi = \exp(C_R)^\nu{}_\mu V^\mu_0$ $C_{R\mu}{}^\nu = \mathrm{ad}_{\mathfrak{r}\mu}{}^\nu$

Akce symetrie na generátorech symetrie

Killing. vektory R K_μ

$r \in \mathrm{iso}(M,g)$ $\mathcal{R}_\varphi = \exp(\varphi r) \in \mathrm{Iso}(M,g)$

$\xi_r = R$ $\Xi_{\mathcal{R}_\varphi} \equiv \mathcal{R}_\varphi$

$$[R, K_\mu] = - C_{R\mu}{}^{\nu} K_\nu$$

$V_\varphi = \mathcal{R}_{\varphi *} V_0$ $V = \underset{\nwarrow \text{konstantní na } M}{V^\mu} K_\mu$

$V_\varphi = \mathcal{R}_{\varphi *} V_0 \qquad V = V^\mu K_\mu$

$\uparrow$ costanti su M

$$\frac{d}{d\varphi} V_\varphi = -\mathcal{L}_R V_\varphi = -[R, V_\varphi] = -[R, K_\mu] V^\mu_\varphi = C_{R\mu}{}^\nu V^\mu_\varphi K_\nu$$

$$\frac{d}{d\varphi} V^\nu_\varphi = C_{R\mu}{}^\nu V^\mu_\varphi \qquad V^\nu_\varphi = \exp(C_R)^\nu{}_\mu V^\mu_0 \qquad C_{R\mu}{}^\nu = \mathrm{ad}_{R\mu}{}^\nu$$

isometrie v E^2

$R \quad X \quad Y$

$$[R, X] = -Y$$

$$[R, Y] = X$$

$$[X, Y] = 0$$

$$C_R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad C_R^2 = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\exp(\varphi C_R) = \cos\varphi \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \sin\varphi \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$V_0 = a_0 X + b_0 Y$$

$$V_\varphi^\mu = \exp(\varphi C_R)^\mu{}_\nu V_0^\nu = \begin{bmatrix} \cos\varphi\, a_0 - \sin\varphi\, b_0 \\ \sin\varphi\, a_0 + \cos\varphi\, b_0 \end{bmatrix}$$

X, Y, Z translace v E^3

$$[X, Y] = [Y, Z] = [Z, X] = 0$$

$$C_X = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \exp(x C_X) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$V_0 = a_0 X + b_0 Y$$

$$V_x^\mu = \exp(x C_x) \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}$$

X, Y, Z translace v $\mathbb{E}^3$

$$[X, Y] = [Y, Z] = [Z, X] = 0$$

$$C_x = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \exp(xC_x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$V_0 = a_0 X + b_0 Y$$

$$V_x^n = \exp(xC_x) \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}$$

isometrie in E^2

$$R \quad X \quad Y$$

$$[R, X] = -Y$$

$$[R, Y] = X$$

$$[X, Y] = 0$$

$$C_R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad C_R^2 = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\exp(\varphi C_R) = \cos\varphi \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \sin\varphi \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$V_0 = a_0 X + b_0 Y$$

$$V_\varphi^\mu = \exp(\varphi C_R)^\mu{}_\nu V_0^\nu = \begin{bmatrix} \cos\varphi\, a_0 - \sin\varphi\, b_0 \\ \sin\varphi\, a_0 + \cos\varphi\, b_0 \end{bmatrix}$$