Geometrie Lieových grup

$T_e M$ Lieova alg. $[\,,\,]$ $C_{\alpha\beta}{}^{\kappa}$ $k_{\alpha\beta}$ $C_{\alpha\beta\gamma} = C_{[\alpha\beta\gamma]}$

$\overset{L}{E}_\alpha$ $\overset{L}{E}^\alpha$ $\overset{R}{E}_\alpha$ $\overset{R}{E}^\alpha$

1) $\forall A$ levoinv. tenz. pole

$$\overset{L}{\nabla} A = 0$$

$$A = \underbrace{A^{\alpha\dots}_{\beta\dots}}_{\text{konst.}} \overset{L}{E}_\alpha \dots \overset{L}{E}^\beta \dots$$

$$\overset{L}{\nabla}\overset{L}{E}_\alpha = 0 \qquad \overset{L}{\nabla}\overset{L}{E}^\alpha = 0$$

$$\overset{L}{T}_{\alpha\beta}{}^{\kappa} = -C_{\alpha\beta}{}^{\kappa} \qquad \overset{L}{R}_{\alpha\beta}{}^{\kappa}{}_{\gamma} = 0$$

2) $\forall A$ pravoinv. tenz. pole

$$\overset{R}{\nabla} A = 0$$

$$\overset{R}{\nabla}\overset{R}{E}_\alpha = 0$$

$$\overset{R}{T}_{\alpha\beta}{}^{\kappa} = C_{\alpha\beta}{}^{\kappa} \qquad \overset{R}{R}_{\alpha\beta}{}^{\kappa}{}_{\gamma} = 0$$

$$\overset{R}{\nabla} - \overset{L}{\nabla} = C \qquad C_{\alpha\kappa}{}^{\gamma}$$

$$0 = \overset{R}{\nabla}_\alpha r_n^\kappa = \overset{L}{\nabla}_\alpha r_n^\kappa + A_{\alpha\gamma}{}^{\kappa} r_n^\gamma = (A_{\alpha\gamma}{}^{\kappa} - C_{\alpha\gamma}{}^{\kappa}) r_n^\gamma$$

$$\overset{L}{T}_{\alpha\beta}{}^{\kappa} = \overset{L}{T}_{\kappa\lambda}{}^{\rho} \overset{L}{E}^\kappa_\alpha \overset{L}{E}^\lambda_\beta =$$

$$= \overset{L}{\nabla}_{\overset{L}{E}_\alpha}\overset{L}{E}^\kappa_\beta - \overset{L}{\nabla}_{\overset{L}{E}_\beta}\overset{L}{E}^\kappa_\alpha - [\overset{L}{E}_\alpha, \overset{L}{E}_\beta]^\kappa$$

$$= -C_{\kappa\lambda}{}^{\kappa}\overset{L}{E}^\kappa_\alpha\overset{L}{E}^\lambda_\beta = -C_{\alpha\beta}{}^{\kappa}$$

$$\overset{L}{\nabla}_{l_m} r_n = \overset{L}{\nabla}_{l_m} r_n - \underbrace{\overset{L}{\nabla}_{r_n} l_m}_{0} - \underbrace{[l_m, r_n]}_{0}$$

$$= l_m \cdot \overset{L}{T} \cdot r_n = -l_m \cdot C \cdot r_n$$

$$\overset{L}{\nabla} r_n = -C \cdot r_n$$

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$T_e M$ Lieova alg. $[\, , \,]$ $c_{\alpha\beta}{}^{\gamma}$ $k_{\alpha\beta}$ $c_{\alpha\beta\gamma} = c_{[\alpha\beta\gamma]}$

$\overset{L}{E}_{\alpha}$ $\overset{L}{E}{}^{\alpha}$ $\overset{R}{E}_{\alpha}$ $\overset{R}{E}{}^{\alpha}$

Geometrie Lieových grup

$T_e M$ Lieova alg. $[\,,\,]$ $c_{\alpha\beta}{}^{\kappa}$ $k_{\alpha\beta}$ $c_{\alpha\beta\gamma} = c_{[\alpha\beta\gamma]}$

$$\overset{L}{E}_\alpha \quad \overset{L}{E}{}^\alpha \qquad \overset{R}{E}_\alpha \quad \overset{R}{E}{}^\alpha$$

1) $\forall A$ levoinv. tens. pole

$$\overset{L}{\nabla} A = 0$$

$$A = \underbrace{A^{\alpha\dots}_{\beta\dots}}_{\text{konst.}} \overset{L}{E}_\alpha \dots \overset{L}{E}{}^\beta \dots$$

$$\overset{L}{\nabla} \overset{L}{E}_\alpha = 0 \qquad \overset{L}{\nabla} \overset{L}{E}{}^\alpha = 0$$

$$\overset{L}{T}{}_{\alpha\beta}{}^{\kappa} = -c_{\alpha\beta}{}^{\kappa} \qquad \overset{L}{R}{}_{\alpha\beta}{}^{\kappa}{}_{\gamma} = 0$$

2) $\forall A$ pravoinv. tens. pole

$$\overset{R}{\nabla} A = 0$$

$$\overset{R}{\nabla} \overset{R}{E}_\alpha = 0$$

$$\overset{R}{T}{}_{\alpha\beta}{}^{\kappa} = c_{\alpha\beta}{}^{\kappa} \qquad \overset{R}{R}{}_{\alpha\beta}{}^{\kappa}{}_{\gamma} = 0$$

$$\overset{L}{T}_{\alpha\beta}{}^{\kappa} = \overset{L}{T}_{\underline{\kappa\lambda}}{}^{r}\, \overset{L}{E}{}^{\underline{\kappa}}_{\alpha}\, \overset{L}{E}{}^{\underline{\lambda}}_{\beta} =$$

$$= \overset{L}{\nabla}_{\overset{L}{E}_{\alpha}} \overset{L}{E}{}^{\kappa}_{\beta} - \overset{L}{\nabla}_{\overset{L}{E}_{\beta}} \overset{L}{E}{}^{\kappa}_{\alpha} - \left[\overset{L}{E}_{\alpha}, \overset{L}{E}_{\beta}\right]^{\kappa}$$

$$= - C_{\underline{\kappa\lambda}}{}^{\kappa}\, \overset{L}{E}{}^{\underline{\kappa}}_{\alpha}\, \overset{L}{E}{}^{\underline{\lambda}}_{\beta} = - C_{\alpha\beta}{}^{\kappa}$$

$$\overset{L}{\nabla}_{\ell_m} \Omega_n = \overset{L}{\nabla}_{\ell_m} \Omega_n - \underbrace{\overset{L}{\nabla}_{\Omega_n} \ell_m}_{0} - \underbrace{[\ell_m, \Omega_n]}_{0}$$

$$= \ell_m \cdot \overset{L}{T} \cdot \Omega_n = - \ell_m \cdot C \cdot \Omega_n$$

$$\overset{L}{\nabla} \Omega_n = - C \cdot \Omega_n$$

$$\overset{L}{\nabla}_{\ell_m} \Omega_n = \overset{L}{\nabla}_{\ell_m} \Omega_n - \underbrace{\overset{L}{\nabla}_{\Omega_n} \ell_m}_{0} - \underbrace{[\ell_m, \Omega_n]}_{0}$$

$$= \ell_m \cdot \overset{L}{T} \cdot \Omega_n = - \ell_m \cdot C \cdot \Omega_n$$

$$\overset{L}{\nabla} \Omega_n = - C \cdot \Omega_n$$

$$\overset{R}{\nabla} - \overset{L}{\nabla} = C \qquad C_{\underline{\alpha}\kappa}{}^{\gamma}$$

$$0 = \overset{R}{\nabla}_{\underline{\alpha}} \Omega_n^{\kappa} = \underline{\overset{L}{\nabla}_{\underline{\alpha}} \Omega_n^{\kappa}} + A_{\underline{\alpha}\gamma}{}^{\kappa} \Omega_n^{\gamma} = \left(A_{\underline{\alpha}\gamma}{}^{\kappa} - C_{\underline{\alpha}\gamma}{}^{\kappa}\right) \Omega_n^{\gamma}$$

Strukturní rovnice pro $\overset{L}{E}{}^{\alpha}$ $\overset{R}{E}{}^{\alpha}$

$$d\overset{L}{E}{}^{\alpha} + \frac{1}{2} C_{\mu\nu}{}^{\alpha} \overset{L}{E}{}^{\mu} \wedge \overset{L}{E}{}^{\nu} = 0$$

$$d\overset{R}{E}{}^{\alpha} - \frac{1}{2} C_{\mu\nu}{}^{\alpha} \overset{R}{E}{}^{\mu} \wedge \overset{R}{E}{}^{\nu} = 0$$

$$d_{\underline{\kappa}} \overset{L}{E}{}^{\alpha}_{\underline{\nu}} = \underbrace{\overset{L}{\nabla}_{\underline{\kappa}} \wedge \overset{L}{E}{}^{\alpha}_{\underline{\nu}}}_{0} + \overset{L}{T}{}^{\kappa}_{\underline{\mu\nu}} \overset{L}{E}{}^{\alpha}_{\kappa} = -C_{\underline{\mu\nu}}{}^{\kappa} \overset{L}{E}{}^{\alpha}_{\underline{\kappa}}$$

$$= -C_{\kappa\lambda}{}^{\alpha} \overset{L}{E}{}^{\kappa}_{\underline{\mu}} \overset{L}{E}{}^{\lambda}_{\underline{\nu}} = -\frac{1}{2} C_{\kappa\lambda}{}^{\alpha} \overset{L}{E}{}^{\kappa}_{\underline{\mu}} \wedge \overset{L}{E}{}^{\lambda}_{\underline{\nu}}$$

Strukturní rovnice pro $\overset{L}{E}{}^{\alpha}$ $\overset{R}{E}{}^{\alpha}$

$$d\overset{L}{E}{}^{\alpha} + \frac{1}{2} C_{\mu\nu}{}^{\alpha} \overset{L}{E}{}^{\mu} \wedge \overset{L}{E}{}^{\nu} = 0$$

$$d\overset{R}{E}{}^{\alpha} - \frac{1}{2} C_{\mu\nu}{}^{\alpha} \overset{R}{E}{}^{\mu} \wedge \overset{R}{E}{}^{\nu} = 0$$

$$d\overset{L}{E}{}^{\alpha} + \frac{1}{2} C_{\mu\nu}{}^{\alpha}\, \overset{L}{E}{}^{\mu} \wedge \overset{L}{E}{}^{\nu} = 0$$

$$d_{\underline{\kappa}} \overset{L}{E}{}^{\alpha}_{\underline{\nu}} = \underbrace{\overset{L}{\nabla}_{\underline{\kappa}} \wedge \overset{L}{E}{}^{\alpha}_{\underline{\nu}}}_{0} + \overset{L}{T}{}^{\underline{\kappa}}_{\mu\underline{\nu}}\, \overset{L}{E}{}^{\alpha}_{\kappa} = -C_{\mu\underline{\nu}}{}^{\underline{\kappa}}\, \overset{L}{E}{}^{\alpha}_{\underline{\kappa}}$$

$$= -C_{\kappa\lambda}{}^{\alpha}\, \overset{L}{E}{}^{\kappa}_{\underline{\kappa}}\, \overset{L}{E}{}^{\lambda}_{\underline{\nu}} = -\frac{1}{2} C_{\kappa\lambda}{}^{\alpha}\, \overset{L}{E}{}^{\kappa}_{\underline{\kappa}} \wedge \overset{L}{E}{}^{\lambda}_{\underline{\nu}}$$