

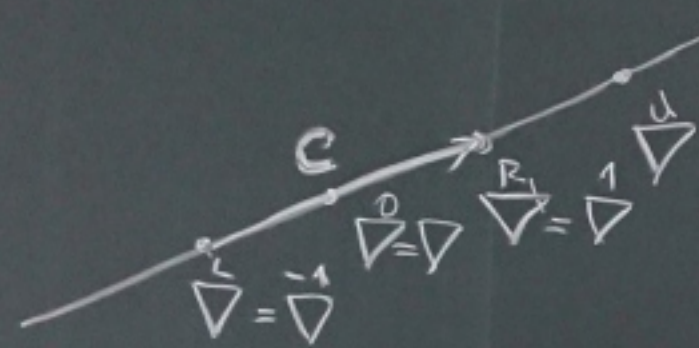
Levi-Civita metrická derivace na Lieově grupě

$$\overset{L}{\nabla} \quad \overset{R}{\nabla}$$

$$\overset{J}{\nabla} = \overset{L}{\nabla} + \frac{J+1}{2} \mathbf{C} =$$

$$= \overset{R}{\nabla} + \frac{J-1}{2} \mathbf{C}$$

$$\begin{aligned} \overset{J}{\nabla}_F a^k &= \overset{L}{\nabla}_F a^k + \frac{J+1}{2} C_{Fk}^k a^k \\ &= \overset{R}{\nabla}_F a^k + \frac{J-1}{2} C_{Fk}^k a^k \end{aligned}$$



$$\overset{J}{\nabla}_F l_m^k = \frac{J+1}{2} C_{Fm}^k l_m^k$$

$$\overset{J}{\nabla}_F \Omega_m^k = \frac{J-1}{2} C_{Fm}^k \Omega_m^k$$

$$\begin{aligned} \overset{J}{\nabla}_{l_m} l_n &= \frac{J+1}{2} l_m \cdot C \cdot l_n = \frac{J+1}{2} [l_m, l_n] \\ &= \frac{J+1}{2} l_{[m,n]} \end{aligned}$$

$$\overset{J}{\nabla}_{\Omega_m} \Omega_n = \frac{1-J}{2} \Omega_{[m,n]}$$

$$\overset{J}{\nabla} c = 0 \quad \overset{J}{\nabla} k = 0$$

$$\begin{aligned} C_{\alpha\beta}^k C_{\beta\gamma}^k &= C_{\alpha\beta}^k C_{\beta\gamma}^k - C_{\alpha\beta}^k C_{\beta\gamma}^k - C_{\alpha\beta}^k C_{\beta\gamma}^k \\ &= C_{\alpha\beta}^k C_{\beta\gamma}^k + C_{\alpha\beta}^k C_{\beta\gamma}^k + C_{\alpha\beta}^k C_{\beta\gamma}^k = 0 \end{aligned}$$

$$[[m,n],p] + [(n,p),m] + [(p,m),n] = 0$$



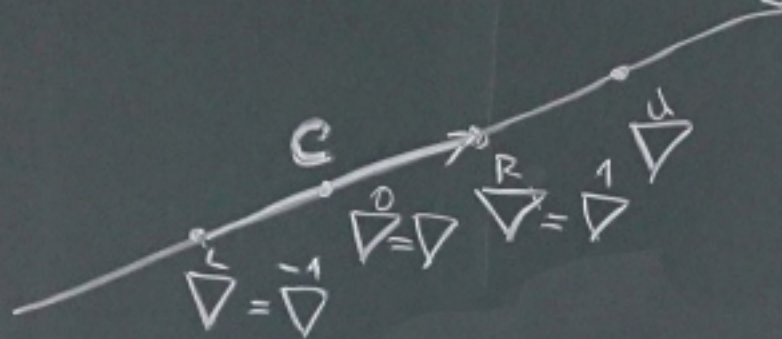
Levi-Civita metrická derivace na Lieově grupě

$$\begin{matrix} L \\ \nabla \end{matrix} \quad \begin{matrix} R \\ \nabla \end{matrix}$$

$$\begin{matrix} J \\ \nabla \end{matrix} = \begin{matrix} L \\ \nabla \end{matrix} + \frac{J+1}{2} \mathbf{C} =$$

$$= \begin{matrix} R \\ \nabla \end{matrix} + \frac{J-1}{2} \mathbf{C}$$

$$\begin{aligned} \begin{matrix} J \\ \nabla_F \end{matrix} a^K &= \begin{matrix} L \\ \nabla_F \end{matrix} a^K + \frac{J+1}{2} C_{FK}^K a^K \\ &= \begin{matrix} R \\ \nabla_F \end{matrix} a^K + \frac{J-1}{2} C_{FK}^K a^K \end{aligned}$$



$$\nabla_{\mu}^{\lambda} l_m^{\nu} = \frac{\lambda+1}{2} C_{\mu\nu}^{\kappa} l_m^{\nu}$$

$$\nabla_{\mu}^{\lambda} \Omega_m^{\kappa} = \frac{\lambda-1}{2} C_{\mu\nu}^{\kappa} \Omega_m^{\nu}$$

$$\begin{aligned} \nabla_{\mu}^{\lambda} l_m l_n &= \frac{\lambda+1}{2} l_m \cdot C \cdot l_n = \frac{\lambda+1}{2} [l_m, l_n] \\ &= \frac{\lambda+1}{2} l_{[m,n]} \end{aligned}$$

$$\nabla_{\mu}^{\lambda} \Omega_m \Omega_n = \frac{1-\lambda}{2} \Omega_{[m,n]}$$

$$\nabla^{\lambda} c = 0$$

$$\nabla^{\lambda} k = 0$$

$$\begin{aligned} C_{\alpha} C_{\beta\gamma}^{\kappa} &= C_{\alpha\lambda}^{\kappa} C_{\beta\gamma}^{\lambda} - C_{\alpha\beta}^{\lambda} C_{\lambda\gamma}^{\kappa} - C_{\alpha\gamma}^{\lambda} C_{\beta\lambda}^{\kappa} \\ &= C_{\alpha\lambda}^{\kappa} C_{\beta\gamma}^{\lambda} + C_{\alpha\lambda}^{\kappa} C_{\alpha\beta}^{\lambda} + C_{\beta\lambda}^{\kappa} C_{\alpha\gamma}^{\lambda} = 0 \end{aligned}$$

$$[[m, n], p] + [(n, p), m] + [(p, m), n] = 0$$



Levi-Civita metrická derivace na Lieově grupě

$$\overset{L}{\nabla}, \overset{R}{\nabla}, \overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2} C = \overset{R}{\nabla} + \frac{\lambda-1}{2} C$$

$$\overset{\lambda}{T}_{\mu\nu}^{\kappa} = \lambda C_{\mu\nu}^{\kappa} \quad \overset{\lambda}{R}_{\mu\nu}^{\kappa} = -\frac{1-\lambda^2}{2} C_{\mu\nu}^{\kappa} C_{\alpha\beta}^{\kappa}$$

$$\overset{\lambda}{Ric}_{\alpha\beta} = \frac{1-\lambda^2}{2} k_{\alpha\beta}$$

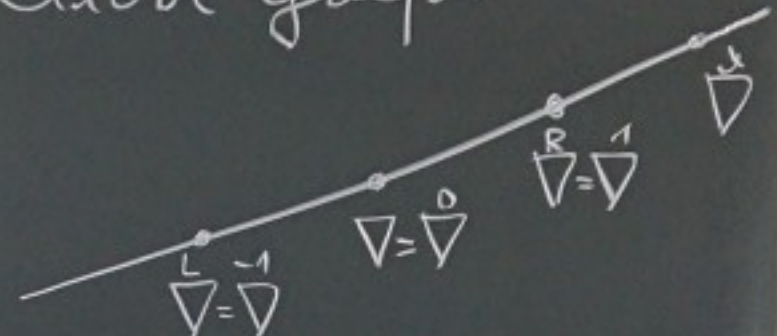
$$\overset{\lambda}{T}_{\mu\nu}^{\kappa} = \overset{L}{T}_{\mu\nu}^{\kappa} + 2 \frac{\lambda+1}{2} C_{[\mu\nu]}^{\kappa} = \lambda C_{\mu\nu}^{\kappa}$$

$$\overset{\lambda}{R}(l_a, l_b) \cdot l_c = \overset{\lambda}{\nabla}_{l_a} \overset{\lambda}{\nabla}_{l_b} l_c - \overset{\lambda}{\nabla}_{l_b} \overset{\lambda}{\nabla}_{l_a} l_c - \overset{\lambda}{\nabla}_{[l_a, l_b]} l_c =$$

$$= \frac{\lambda+1}{2} \left( \overset{\lambda}{\nabla}_{l_a} l_{[b,c]} - \overset{\lambda}{\nabla}_{l_b} l_{[a,c]} - \overset{\lambda}{\nabla}_{l_{[a,b]}} l_c \right) - \frac{1-\lambda^2}{2} \overset{\lambda}{\nabla}_{l_{[a,b]}} l_c$$

$$\Rightarrow \left(\frac{\lambda+1}{2}\right)^2 l([a, [b, c]] - [b, [a, c]] - [[a, b], c]) - \frac{1-\lambda^2}{2} l_{[a,b],c}$$

$$= -\frac{1-\lambda^2}{2} l_{[a,b],c} = -\frac{1-\lambda^2}{2} C_{\alpha\beta}^{\kappa} C_{\mu\nu}^{\lambda} l_a^{\alpha} l_b^{\beta} l_c^{\kappa}$$



$$\overset{0}{\nabla} = \overset{0}{\nabla} \quad \nabla k = 0 \quad T = 0$$

$$R = -\frac{1}{2} C \cdot C \quad Ric = \frac{1}{2} k$$

$$Ein + \frac{D-2}{4} k = 0 \quad R = \frac{D}{2}$$

orbity  $l_m$   $\Omega_m$  jsou geodeticky

$$\overset{\lambda}{\nabla}_{l_m} l_m = l_{[m,m]} = 0$$

$\exp(\alpha m)$  geodetika

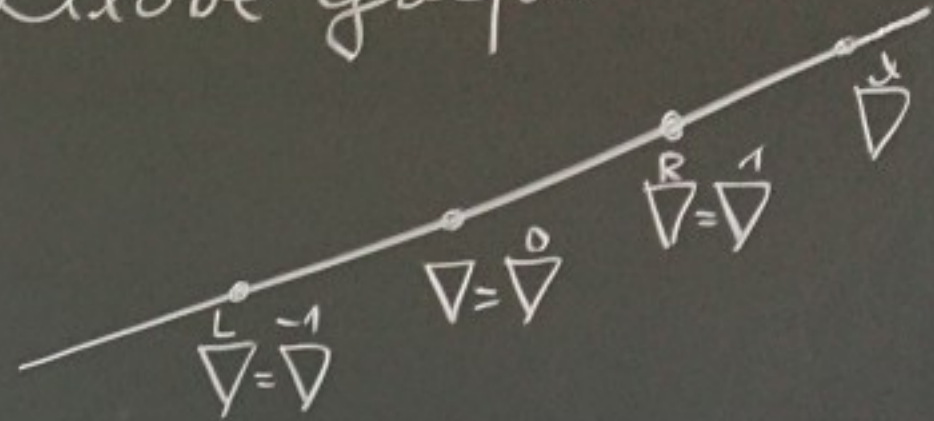
Levi-Civita metrická derivace na Lieově grupě

$$\nabla^L, \nabla^R, \nabla^{\lambda} = \nabla^L + \frac{\lambda+1}{2} \mathbb{C} = \nabla^R + \frac{\lambda-1}{2} \mathbb{C}$$

$$\Gamma_{\mu\nu}^{\lambda} = \lambda C_{\mu\nu}^{\lambda}$$

$$R_{\mu\nu}^{\lambda\alpha} = -\frac{1-\lambda^2}{2} C_{\mu\nu}^{\alpha\beta} C_{\beta\alpha}^{\lambda}$$

$$\text{Ric}_{\alpha\beta} = \frac{1-\lambda^2}{2} K_{\alpha\beta}$$



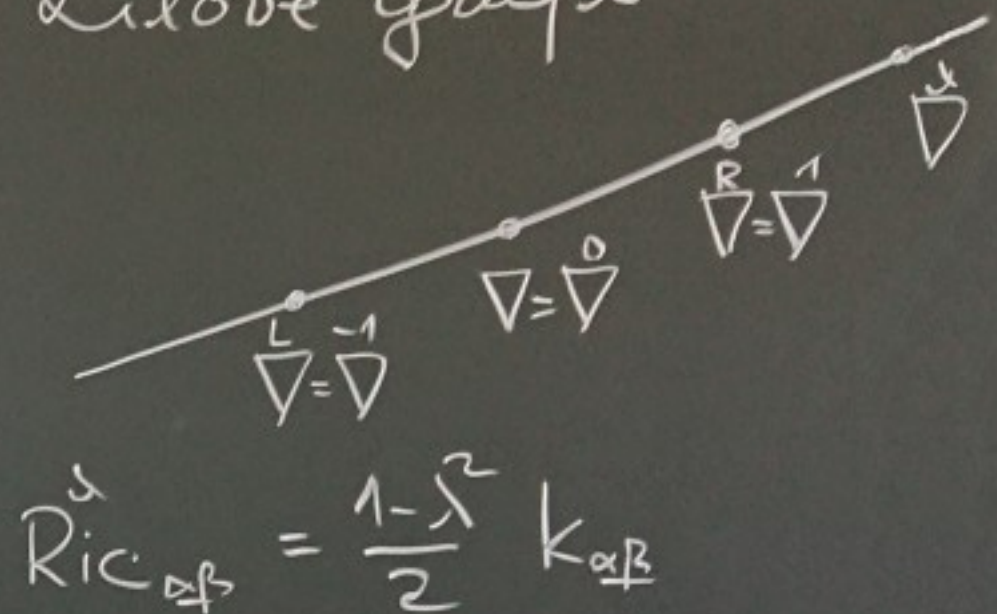


Levi-Civita metrická derivace na Lieově grupě

$$\overset{L}{\nabla}, \overset{R}{\nabla}, \overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2} \mathbf{C} = \overset{R}{\nabla} + \frac{\lambda-1}{2} \mathbf{C}$$

$$\overset{\lambda}{T}_{\mu\nu}^{\mu\nu} = \lambda C_{\mu\nu}^{\mu\nu} \quad R_{\mu\nu}^{\mu\nu} = -\frac{1-\lambda^2}{2} C_{\mu\nu}^{\mu\nu} C_{\mu\nu}^{\mu\nu}$$

$$\overset{\lambda}{T}_{\mu\nu}^{\mu\nu} = \overset{L}{T}_{\mu\nu}^{\mu\nu} + 2\frac{\lambda+1}{2} C_{[\mu\nu]}^{\mu\nu} = \lambda C_{\mu\nu}^{\mu\nu}$$



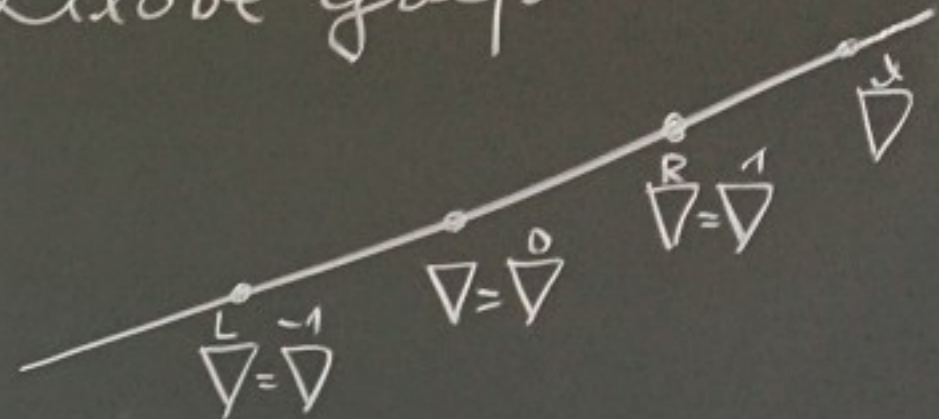
$$Ric_{\alpha\beta}^{\alpha\beta} = \frac{1-\lambda^2}{2} K_{\alpha\beta}$$

Levi-Civita metrická derivace na Lieově grupě

$$\overset{L}{\nabla}, \overset{R}{\nabla}, \overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2} \mathbf{C} = \overset{R}{\nabla} + \frac{\lambda-1}{2} \mathbf{C}$$

$$\overset{\lambda}{T}_{\mu\nu}^{\kappa} = \lambda C_{\mu\nu}^{\kappa} \quad \overset{\lambda}{R}_{\mu\nu}^{\kappa} = -\frac{1-\lambda^2}{2} C_{\mu\nu}^{\alpha} C_{\alpha\lambda}^{\kappa}$$

$$\overset{\lambda}{\text{Ric}}_{\alpha\beta} = \frac{1-\lambda^2}{2} K_{\alpha\beta}$$



$$\overset{\lambda}{R}(l_a, l_b) \cdot l_c = \overset{\lambda}{\nabla}_{l_a} \overset{\lambda}{\nabla}_{l_b} l_c - \overset{\lambda}{\nabla}_{l_b} \overset{\lambda}{\nabla}_{l_a} l_c - \overset{\lambda}{\nabla}_{[l_a, l_b]} l_c =$$

$$= \frac{\lambda+1}{2} \left( \overset{\lambda}{\nabla}_{l_a} l_{[b,c]} - \overset{\lambda}{\nabla}_{l_b} l_{[a,c]} - \overset{\lambda}{\nabla}_{l_{[a,b]}} l_c \right) - \frac{1-\lambda}{2} \overset{\lambda}{\nabla}_{l_{[a,b]}} l_c$$

$$\rightarrow \left( \frac{\lambda+1}{2} \right)^2 l \left( [a, (b, c)] - [b, (a, c)] - [(a, b), c] \right) - \frac{1-\lambda^2}{2} l_{[(a, b), c]}$$

$$= -\frac{1-\lambda^2}{2} l_{[(a, b), c]} = -\frac{1-\lambda^2}{2} C_{\alpha\beta}^{\kappa} C_{\mu\kappa}^{\lambda} l_a^{\alpha} l_b^{\beta} l_c^{\lambda}$$



$$\nabla = \overset{0}{\nabla} \quad \nabla k = 0 \quad T = 0$$

$$R = -\frac{1}{2} c \cdot c \quad Ric = \frac{1}{2} k$$

$$Ein + \frac{D-2}{4} k = 0 \quad R = \frac{D}{2}$$

orbity  $l_m$   $\Omega_m$  joon geoditika

$$\nabla_{l_m} l_m = l_{[m,m]} = 0$$

$\exp(\alpha m)$  geodetika



$$\overset{\lambda}{\nabla}_{l_m} = \mathcal{L}_{l_m} + \text{Ad}_{\frac{\lambda-1}{2} m}$$

$$\overset{\lambda}{\nabla}_{R_m} = \mathcal{L}_{R_m} + \text{Ad}_{\frac{\lambda+1}{2} m}$$

$$\text{Ada} \leftrightarrow \begin{matrix} -\alpha^k C_{k\neq}^{\vee} \\ \text{TeG} \end{matrix} = -\text{ada}^{\vee} = -l_{\alpha}^k C_{k\neq}^{\vee}$$

kontinuität

$$\overset{\mathbb{R}}{\nabla}_{l_m} = \mathcal{L}_{l_m} \quad \overset{\mathbb{L}}{\nabla}_{R_m} = \mathcal{L}_{R_m}$$

$$\mathcal{L}_{l_m} = \overset{\lambda}{\nabla}_{l_m} + \Lambda$$

$$\Lambda = -\overset{\lambda}{\nabla}_{l_m} - l_m \overset{\lambda}{T} = -\frac{\lambda+1}{2} c \cdot l_m - \lambda l_m \cdot c = -\frac{\lambda-1}{2} l_m \cdot c$$

$$\nabla_{\ell_m}^{\downarrow} = \mathcal{L}_{\ell_m} + \text{Ad}_{\frac{\downarrow}{2}} \ell_m$$

$$\nabla_{\Omega_m}^{\downarrow} = \mathcal{L}_{\Omega_m} + \text{Ad}_{\frac{\downarrow}{2}} \Omega_m$$

$$\text{Ad}_a \leftrightarrow \begin{matrix} -\alpha^k C_{k\neq}^{\neq} = -\text{ad}_a^{\neq} \\ \text{TeG} \quad -\ell_a^k C_{k\neq}^{\neq} \end{matrix}$$

Kontinuität  $\bar{e}$

$$\nabla_{\ell_m}^{\mathbb{R}} = \mathcal{L}_{\ell_m}$$

$$\nabla_{\Omega_m}^{\mathbb{L}} = \mathcal{L}_{\Omega_m}$$



$$\overset{\lambda}{\nabla}_{\mathfrak{g}_m} = \mathcal{L}_{\mathfrak{g}_m} + \text{Ad}_{\frac{\lambda-1}{2}} \mathfrak{g}_m$$

$$\text{Ad}_a \leftrightarrow \begin{array}{l} -\alpha^k C_{k\neq}^{\vee} \\ \text{TeG} \end{array} = -\text{ad}_a^{\vee} \\ -\mathfrak{L}_a^k C_{k\neq}^{\vee}$$

$$\mathcal{L}_{\mathfrak{g}_m} = \overset{\lambda}{\nabla}_{\mathfrak{g}_m} + \Lambda$$

$$\begin{aligned} \Lambda &= -\overset{\lambda}{\nabla}_{\mathfrak{g}_m} - \mathfrak{g}_m \overset{\lambda}{\nabla} = -\frac{\lambda+1}{2} \mathfrak{g}_m \cdot \mathfrak{c} - \lambda \mathfrak{g}_m \cdot \mathfrak{c} \\ &= -\frac{\lambda-1}{2} \mathfrak{g}_m \cdot \mathfrak{c} \end{aligned}$$