Levi-Civitova metrická derivace na Lieově grupě

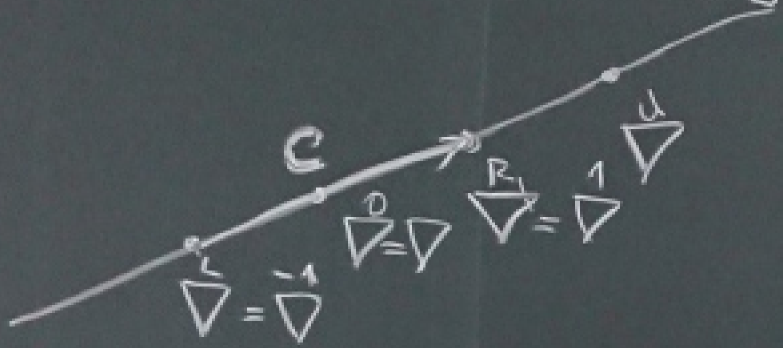
$\overset{L}{\nabla} \qquad \overset{R}{\nabla}$

$$\overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2}\mathbf{C} =$$

$$= \overset{R}{\nabla} + \frac{\lambda-1}{2}\mathbf{C}$$

$$\overset{\lambda}{\nabla}_F a^k = \overset{L}{\nabla}_F a^k + \frac{\lambda+1}{2} C_{\mu\nu}{}^k a^\nu$$

$$= \overset{R}{\nabla}_F a^k + \frac{\lambda-1}{2} C_{\mu\nu}{}^k a^\nu$$



$$\overset{\lambda}{\nabla}_F l^k_m = \frac{\lambda+1}{2} C_{\mu\nu}{}^k l^\nu_m$$

$$\overset{\lambda}{\nabla}_r r^k_m = \frac{\lambda-1}{2} C_{\mu\nu}{}^k r^\nu_m$$

$$\overset{\lambda}{\nabla}_{l_m} l_n = \frac{\lambda+1}{2} l_m \cdot C \cdot l_n = \frac{\lambda+1}{2}[l_m, l_n]$$

$$= \frac{\lambda+1}{2} l_{[m,n]}$$

$$\overset{\lambda}{\nabla}_{r_m} r_n = \frac{1-\lambda}{2} r_{[m,n]}$$

$$\overset{\lambda}{\nabla} C = 0 \qquad \overset{\lambda}{\nabla} k = 0$$

$$\mathbf{C}_{\alpha} C_{\beta\gamma}{}^k = C_{\alpha\lambda}{}^k C_{\beta\gamma}{}^\lambda - C_{\alpha\beta}{}^\lambda C_{\lambda\gamma}{}^k - C_{\alpha\gamma}{}^\lambda C_{\beta\lambda}{}^k$$

$$= C_{\alpha\lambda}{}^k C_{\beta\gamma}{}^\lambda + C_{\gamma\lambda}{}^k C_{\alpha\beta}{}^\lambda + C_{\beta\lambda}{}^k C_{\gamma\alpha}{}^\lambda = 0$$

$$[[m,n],p] + [[n,p],m] + [[p,m],n] = 0$$

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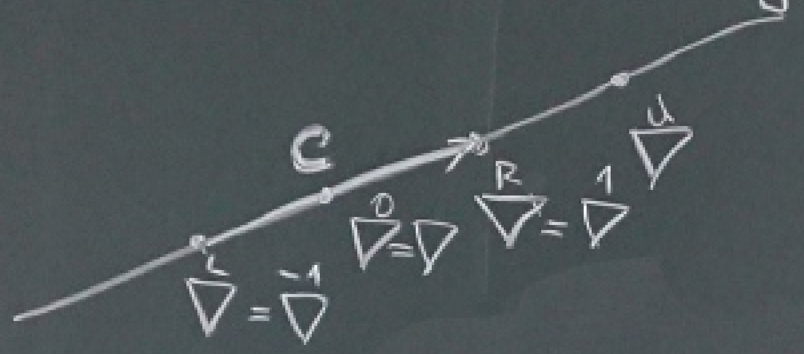
$$\overset{L}{\nabla} \qquad \overset{R}{\nabla}$$



$$\overset{\nu}{\nabla} = \overset{L}{\nabla} + \frac{\nu+1}{2}\,\mathbb{C} =$$

$$= \overset{R}{\nabla} + \frac{\nu-1}{2}\,\mathbb{C}$$

$$\overset{\nu}{\nabla}_{\mu} a^{\kappa} = \overset{L}{\nabla}_{\mu} a^{\kappa} + \frac{\nu+1}{2} C_{\mu\lambda}{}^{\kappa} a^{\lambda}$$

$$= \overset{R}{\nabla}_{\mu} a^{\kappa} + \frac{\nu-1}{2} C_{\mu\lambda}{}^{\kappa} a^{\lambda}$$

$$\overset{\lambda}{\nabla}_{\mu} \ell^{\kappa}_{m} = \frac{\lambda+1}{2} C_{\mu\nu}{}^{\kappa} \ell^{\nu}_{m}$$

$$\overset{\lambda}{\nabla}_{\mu} \Omega^{\kappa}_{m} = \frac{\lambda-1}{2} C_{\mu\nu}{}^{\kappa} \Omega^{\nu}_{m}$$

$$\overset{\lambda}{\nabla}_{\ell_m} \ell_n = \frac{\lambda+1}{2} \ell_m \cdot C \cdot \ell_n = \frac{\lambda+1}{2} [\ell_m, \ell_n]$$

$$= \frac{\lambda+1}{2} \ell_{[m,n]}$$

$$\overset{\lambda}{\nabla}_{\Omega_m} \Omega_n = \frac{1-\lambda}{2} \Omega_{[m,n]}$$

$$\overset{\lambda}{\nabla} C = 0 \qquad \overset{\lambda}{\nabla} k = 0$$

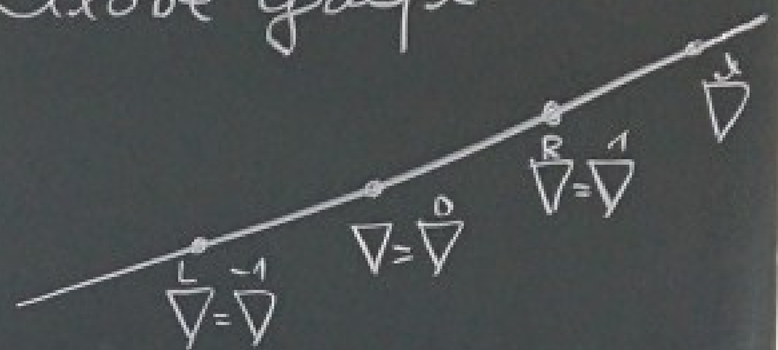
$$C_{\underline{\alpha}} C_{\underline{\beta\gamma}}{}^{\underline{\kappa}} = C_{\underline{\alpha\lambda}}{}^{\underline{\kappa}} C_{\underline{\beta\gamma}}{}^{\underline{\lambda}} - C_{\underline{\alpha\beta}}{}^{\underline{\lambda}} C_{\underline{\lambda\gamma}}{}^{\underline{\kappa}} - C_{\underline{\alpha\gamma}}{}^{\lambda} C_{\underline{\beta\lambda}}{}^{\underline{\kappa}}$$

$$= C_{\underline{\alpha\lambda}}{}^{\underline{\kappa}} C_{\underline{\beta\gamma}}{}^{\underline{\lambda}} + C_{\underline{\gamma\lambda}}{}^{\underline{\kappa}} C_{\underline{\alpha\beta}}{}^{\underline{\lambda}} + C_{\underline{\beta\lambda}}{}^{\underline{\kappa}} C_{\underline{\gamma\alpha}}{}^{\underline{\lambda}} = 0$$

$$[[m,n],p] + [[n,p],m] + [[p,m],n] = 0$$

Levi-Civitova metrická derivace na Lieově grupě

$$\overset{L}{\nabla},\ \overset{R}{\nabla},\quad \overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2}\mathbb{C} = \overset{R}{\nabla} + \frac{\lambda-1}{2}\mathbb{C}$$



$$\overset{\lambda}{T}_{\mu\nu}{}^{\kappa} = \lambda\, C_{\mu\nu}{}^{\kappa} \qquad \overset{\lambda}{R}_{\mu\nu}{}^{\kappa}{}_{\lambda} = -\frac{1-\lambda^2}{2}\, C_{\mu\nu}{}^{\alpha} C_{\alpha\lambda}{}^{\kappa}$$

$$\overset{\lambda}{Ric}_{\alpha\beta} = \frac{1-\lambda^2}{2}\, k_{\alpha\beta}$$

$$\overset{\lambda}{T}_{\mu\nu}{}^{\kappa} = \overset{L}{T}_{\mu\nu}{}^{\kappa} + 2\,\frac{\lambda+1}{2}\, C_{[\mu\nu]}{}^{\kappa} = \lambda\, C_{\mu\nu}{}^{\kappa}$$

$$\overset{\lambda}{R}(l_a,l_b)\cdot l_c = \overset{\lambda}{\nabla}_{l_a}\overset{\lambda}{\nabla}_{l_b} l_c - \overset{\lambda}{\nabla}_{l_b}\overset{\lambda}{\nabla}_{l_a} l_c - \overset{\lambda}{\nabla}_{[l_a,l_b]} l_c =$$
$$= \frac{\lambda+1}{2}\left(\overset{\lambda}{\nabla}_{l_a} l_{[b,c]} - \overset{\lambda}{\nabla}_{l_b} l_{[a,c]} - \overset{\lambda}{\nabla}_{l_{[a,b]}} l_c\right) - \frac{1-\lambda}{2}\overset{\lambda}{\nabla}_{l_{[a,b]}} l_c$$
$$= \left(\frac{\lambda+1}{2}\right)^2 l\big[[a,[b,c]] - [b,[a,c]] - [[a,b],c]\big] - \frac{1-\lambda^2}{2}\, l_{[[a,b],c]}$$
$$= -\frac{1-\lambda^2}{2}\,[[l_a,l_b],l_c]^{\gamma} = -\frac{1-\lambda^2}{2}\, C_{\alpha\beta}{}^{\mu} C_{\mu\gamma}{}^{\nu}\, l_a^{\alpha} l_b^{\beta} l_c^{\gamma}$$

$$\nabla = \overset{0}{\nabla} \qquad \nabla k = 0 \qquad T = 0$$

$$R = -\frac{1}{2}\, C\cdot C \qquad Ric = \frac{1}{2}\, k$$

$$Ein + \frac{D-2}{4}\, k = 0 \qquad \mathcal{R} = \frac{D}{2}$$

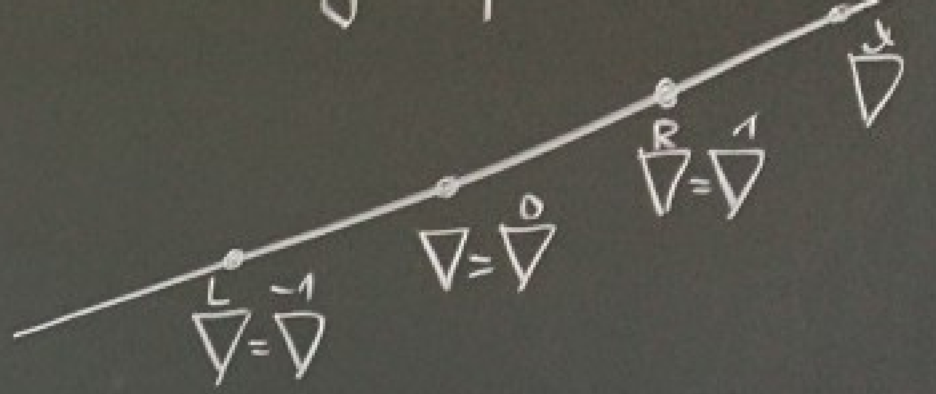
orbity l_m r_m jsou geodetiky

$$\nabla_{l_m} l_m = l_{[m,m]} = 0$$

$\exp(\alpha m)$ geodetika

Levi-Civitova metrická derivace na Lieově grupě

$$\overset{L}{\nabla}, \quad \overset{R}{\nabla}, \quad \overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2}\,\mathbb{C} = \overset{R}{\nabla} + \frac{\lambda-1}{2}\,\mathbb{C}$$

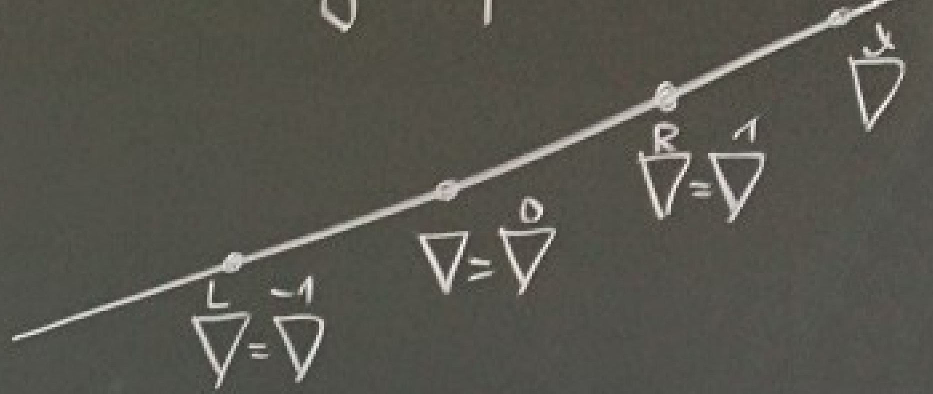


$$\overset{\lambda}{T}_{\underline{\mu\nu}}{}^{\underline{\kappa}} = \lambda\, C_{\underline{\mu\nu}}{}^{\underline{\kappa}} \qquad \overset{\lambda}{R}_{\underline{\mu\nu}}{}^{\underline{\kappa}}{}_{\underline{\lambda}} = -\frac{1-\lambda^2}{2}\, C_{\underline{\mu\nu}}{}^{\underline{\alpha}}\, C_{\underline{\alpha\lambda}}{}^{\underline{\kappa}}$$

$$\overset{\lambda}{Ric}_{\underline{\alpha\beta}} = \frac{1-\lambda^2}{2}\, k_{\underline{\alpha\beta}}$$

Levi-Civitova metrická derivace na Lieově grupě

$$\overset{L}{\nabla}, \quad \overset{R}{\nabla}, \quad \overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2}\,\mathbb{C} = \overset{R}{\nabla} + \frac{\lambda-1}{2}\,C$$



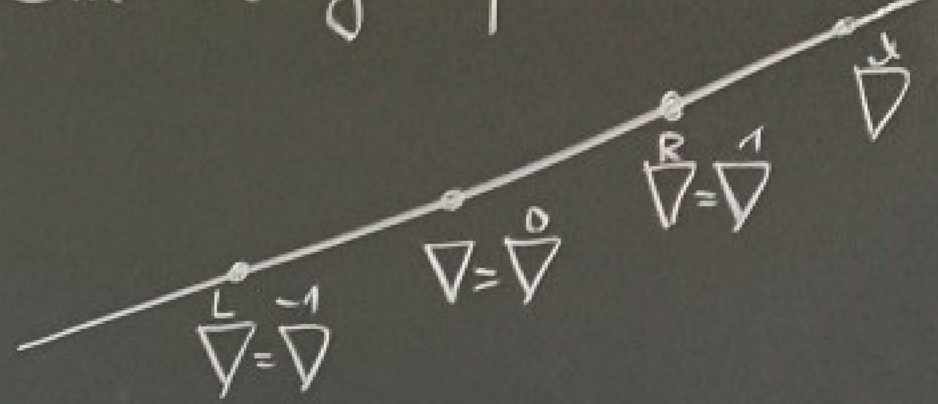
$$\overset{\lambda}{T}_{\mu\nu}{}^{\kappa} = \lambda\, C_{\mu\nu}{}^{\kappa} \qquad \overset{\lambda}{R}_{\mu\nu}{}^{\kappa}{}_{\lambda} = -\frac{1-\lambda^2}{2}\, C_{\mu\nu}{}^{\alpha}\, C_{\alpha\lambda}{}^{\kappa}$$

$$\overset{\lambda}{Ric}_{\alpha\beta} = \frac{1-\lambda^2}{2}\, k_{\alpha\beta}$$

$$\overset{\lambda}{T}_{\mu\nu}{}^{\kappa} = \overset{L}{T}_{\mu\nu}{}^{\kappa} + 2\,\frac{\lambda+1}{2}\, C_{[\mu\nu]}{}^{\kappa} = \lambda\, C_{\mu\nu}{}^{\kappa}$$

Levi-Civitova metrická derivace na Lieově grupě

$$\overset{L}{\nabla},\ \overset{R}{\nabla},\quad \overset{\lambda}{\nabla} = \overset{L}{\nabla} + \frac{\lambda+1}{2}\,\mathbb{C} = \overset{R}{\nabla} + \frac{\lambda-1}{2}\,\mathbb{C}$$



$$\overset{\lambda}{T}_{\mu\nu}{}^{\kappa} = \lambda\, c_{\mu\nu}{}^{\kappa} \qquad \overset{\lambda}{R}_{\mu\nu\lambda}{}^{\kappa} = -\frac{1-\lambda^2}{2}\, c_{\mu\nu}{}^{\alpha} c_{\alpha\lambda}{}^{\kappa}$$

$$\overset{\lambda}{Ric}_{\alpha\beta} = \frac{1-\lambda^2}{2}\, k_{\alpha\beta}$$

$$\overset{\lambda}{R}(l_a, l_b)\cdot l_c = \overset{\lambda}{\nabla}_{l_a} \overset{\lambda}{\nabla}_{l_b} l_c - \overset{\lambda}{\nabla}_{l_b} \overset{\lambda}{\nabla}_{l_a} l_c - \overset{\lambda}{\nabla}_{[l_a, l_b]} l_c =$$

$$= \frac{\lambda+1}{2}\left(\overset{\lambda}{\nabla}_{l_a} l_{[b,c]} - \overset{\lambda}{\nabla}_{l_b} l_{[a,c]} - \overset{\lambda}{\nabla}_{l_{[a,b]}} l_c \right) - \frac{1-\lambda}{2} \overset{\lambda}{\nabla}_{l_{[a,b]}} l_c$$

$$= \left(\frac{\lambda+1}{2}\right)^2 l\Big[[a,[b,c]] - [b,[a,c]] - [[a,b],c] \Big] - \frac{1-\lambda^2}{2}\, l_{[[a,b],c]}$$

$$= -\frac{1-\lambda^2}{2} [[l_a, l_b], l_c]^{\nu} = -\frac{1-\lambda^2}{2}\, c_{\alpha\beta}{}^{\kappa} c_{\kappa\gamma}{}^{\nu}\, l_a^{\alpha} l_b^{\beta} l_c^{\gamma}$$

$$\nabla = \overset{0}{\nabla} \qquad \nabla k = 0 \qquad T = 0$$

$$R = -\frac{1}{2} c \cdot c \qquad \mathrm{Ric} = \frac{1}{2} k$$

$$\mathrm{Ein} + \frac{D-2}{4} k = 0 \qquad \mathcal{R} = \frac{D}{2}$$

orbity ℓ_m Ω_m jako geodetyki

$$\nabla_{\ell_m} \ell_m = \ell_{[m,m]} = 0$$

$\exp(\alpha m)$ geodetyka

$$\overset{\lambda}{\nabla}_{\ell_m} = \mathcal{L}_{\ell_m} + \mathrm{Ad}_{\frac{\lambda-1}{2}m}$$

$$\overset{\lambda}{\nabla}_{r_m} = \mathcal{L}_{r_m} + \mathrm{Ad}_{\frac{\lambda+1}{2}m}$$

$$\mathrm{Ad}_a \longleftrightarrow -a^k c_{k\mu}{}^{\nu} = -\mathrm{ad}_a{}^{\nu}{}_{\mu}$$

$$T_eG \qquad -\ell_a^k c_{k\mu}{}^{\nu}$$

konkrétně

$$\overset{R}{\nabla}_{\ell_m} = \mathcal{L}_{\ell_m} \qquad \overset{L}{\nabla}_{r_m} = \mathcal{L}_{r_m}$$

$$\mathcal{L}_{\ell_m} = \overset{\lambda}{\nabla}_{\ell_m} + \Lambda \qquad \Lambda = -\overset{\lambda}{\nabla}\ell_m - \ell_m \cdot \overset{\lambda}{T} = -\frac{\lambda+1}{2}c \cdot \ell_m - \lambda\, \ell_m \cdot c$$

$$= -\frac{\lambda-1}{2}\ell_m \cdot c$$

$$\overset{\lambda}{\nabla}_{l_m} = \mathcal{L}_{l_m} + \mathrm{Ad}_{\frac{\lambda-1}{2}m}$$

$$\overset{\lambda}{\nabla}_{r_m} = \mathcal{L}_{r_m} + \mathrm{Ad}_{\frac{\lambda+1}{2}m}$$

konkrétně

$$\overset{R}{\nabla}_{l_m} = \mathcal{L}_{l_m} \qquad \overset{L}{\nabla}_{r_m} = \mathcal{L}_{r_m}$$

$$\mathrm{Ad}_a \longleftrightarrow -a^{\kappa} c_{\kappa\mu}{}^{\nu} = -\mathrm{ad}_a{}^{\nu}{}_{\mu}$$

$$T_eG \qquad -l_a^{\kappa} c_{\kappa\mu}{}^{\nu}$$

$$\overset{\lambda}{\nabla}_{l_m} = \mathcal{L}_{l_m} + \mathrm{Ad}_{\frac{\lambda-1}{2}m}$$

$$\mathrm{Ad}_a \longleftrightarrow -a^{\underline{k}} c_{\underline{k}\underline{t}}{}^{\underline{v}} = -\mathrm{ad}_a{}^{\underline{v}}{}_{\underline{t}}$$

$$T_eG \qquad -l_a^{\underline{k}} c_{\underline{k}\underline{t}}{}^{\underline{v}}$$

$$\mathcal{L}_{l_m} = \overset{\lambda}{\nabla}_{l_m} + \Lambda$$

$$\Lambda = -\overset{\lambda}{\nabla} l_m - l_m \cdot \overset{\lambda}{T} = -\frac{\lambda+1}{2} c \cdot l_m - \lambda\, l_m \cdot c$$

$$= -\frac{\lambda-1}{2} l_m \cdot c$$