Akce Lieovy grupy na varietě

Def A je akce LG na var M

$g \in G \quad A_g : M \to M$

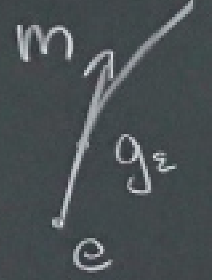
1) $A_{g_1} \circ A_{g_2} = A_{g_1 g_2}$ levá akce $\quad A_g x = g x$

2) $A_{g_1} \circ A_{g_2} = A_{g_2 g_1}$ pravá akce $\quad A_g x = x g$

Př: $L_g \quad R_{g^{-1}} \quad AD_g$ levé akce na G

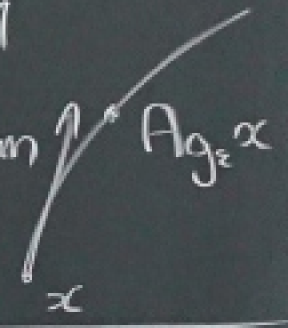
$R_g \quad L_{g^{-1}} \quad AD_{g^{-1}}$ pravé akce na G

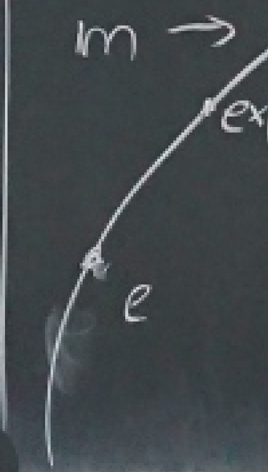
G



$g_\varepsilon \approx e + \varepsilon m$

M





$m \to \exp(\alpha m)$

$\exp((\alpha+\beta)m) = \exp\alpha m \exp\beta m$

Def A_g je akce G na M

a_m generátor akce A_g

$a : \mathfrak{g} \to TM$

$\forall m \in \mathfrak{g} \to a_m \in TM$

$$a_m\big|_x = \frac{D}{d\varepsilon} A_{g_\varepsilon} x \Big|_{\varepsilon=0}$$

$$\frac{Dg_\varepsilon}{d\varepsilon}\Big|_{\varepsilon=0} = m \qquad g_0 = e$$

$\to A_{\exp(\alpha m)} \longleftrightarrow a_m$

Akce Lieovy grupy na varietě

Def A je akce LG na var M

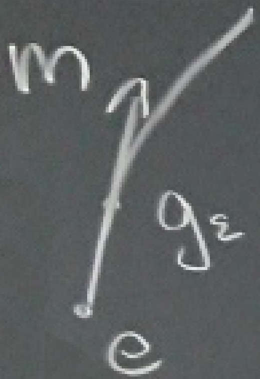
$g \in G \quad A_g : M \to M$

1) $A_{g_1} \circ A_{g_2} = A_{g_1 g_2}$ levá akce $\quad A_g x = g x$

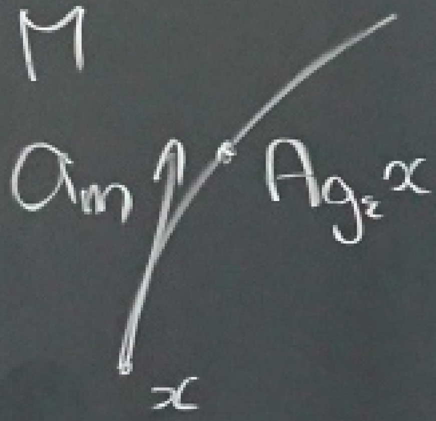
2) $A_{g_1} \circ A_{g_2} = A_{g_2 g_1}$ pravá akce $\quad A_g x = x g$

Př: $L_g \quad R_{g^{-1}} \quad AD_g$ levé akce na G

$R_g \quad L_{g^{-1}} \quad AD_{g^{-1}}$ pravé akce na G



$$g_\varepsilon \approx e + \varepsilon m$$



Def A_g je akce G na M

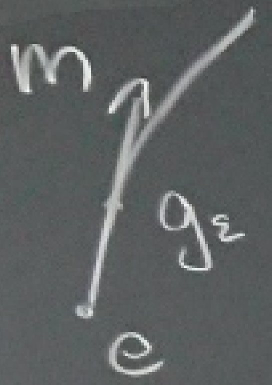
a_m generátor akce A_g

$$a : \mathfrak{g} \to TM$$

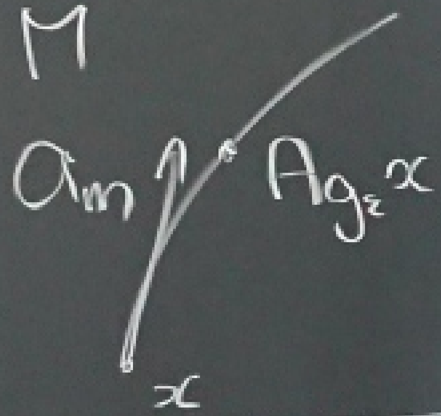
$$\forall m \in \mathfrak{g} \to a_m \in TM$$

$$a_m\big|_x = \frac{D}{d\varepsilon} A_{g_\varepsilon} x \Big|_{\varepsilon=0}$$

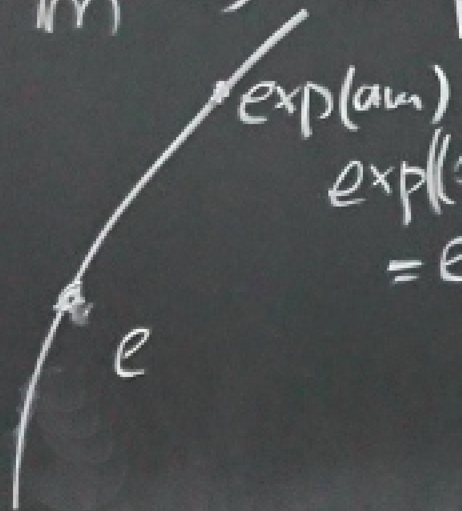
$$\frac{D g_\varepsilon}{d\varepsilon}\Big|_{\varepsilon=0} = m \qquad g_0 = e$$



$$g_\varepsilon \approx e + \varepsilon m$$



$$m \rightarrow \exp(\alpha m)$$



$$\exp((\alpha+\beta)m) = \\ = \exp\alpha m \, \exp\beta m$$

Def A_g je akce G na M

a_m generátor akce A_g

$$a : \mathfrak{g} \rightarrow TM$$

$$\forall m \in \mathfrak{g} \rightarrow a_m \in TM$$

$$a_m\Big|_x = \frac{D}{d\varepsilon} A_{g_\varepsilon} x \Big|_{\varepsilon=0}$$

$$\frac{Dg_\varepsilon}{d\varepsilon}\Big|_{\varepsilon=0} = m \qquad g_0 = e$$

$$\rightarrow A_{\exp(\alpha m)} \longleftrightarrow a_m$$

$a_{m+\alpha n} = a_m + \alpha a_n$

Př l-akce L_g $\quad r_m \quad L_{\exp(\alpha m)}$

r-akce R_g $\quad l_m$

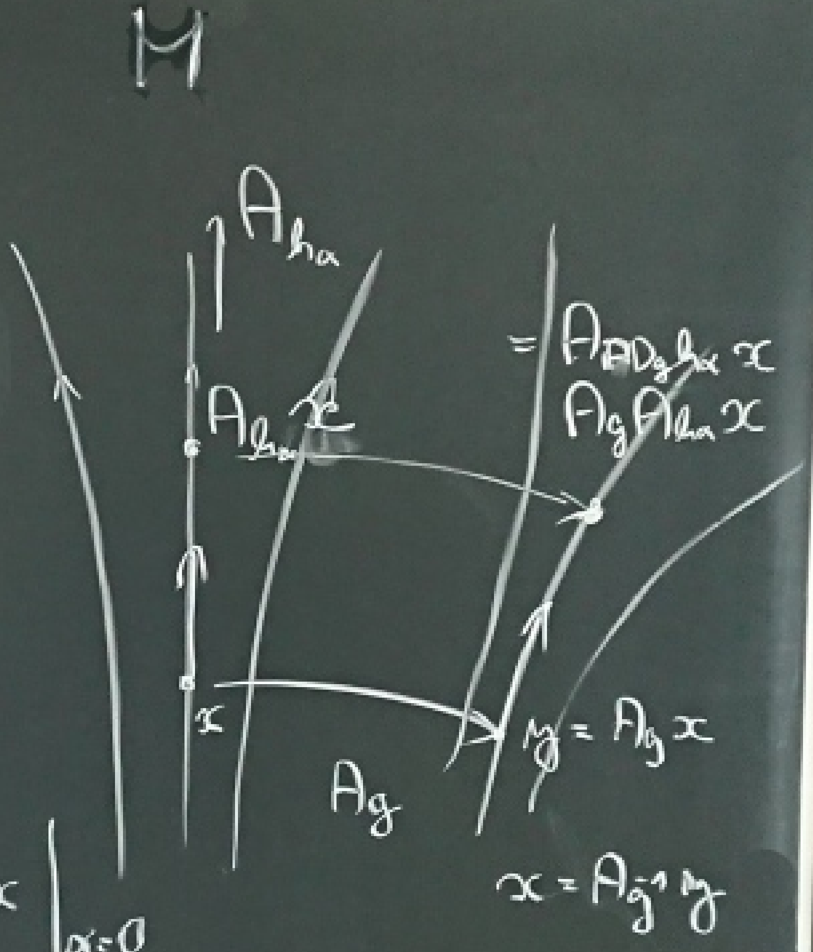
Lema

1, $A_{g*} a_m = a_{Ad_g m}$ − A_g levá akce

2, $A_{g^{-1}*} a_m = a_{Ad_g m}$ − A_g pravá akce

důk $A_{g*} a_m|_x = A_{g*} \frac{D}{d\alpha} A_{h_\alpha} x\Big|_{\alpha=0} = \frac{D}{d\alpha} A_g A_{h_\alpha} x\Big|_{\alpha=0}$

$= \frac{D}{d\alpha} A_{AD_g h_\alpha} \underbrace{A_g x}_{y}\Big|_{\alpha=0} = a_{Ad_g m}\Big|_{A_g x}$



Věta

A_g levá akce $[a_m, a_n] = -a_{[m,n]}$

A_g pravá akce $[a_m, a_n] = a_{[m,n]}$

$[a_m, a_n] = \mathcal{L}_{a_m} a_n = -\frac{d}{d\alpha} A_{g_\alpha *} a_n$

$= -\frac{d}{d\alpha} a_{Ad_{g_\alpha} n}\Big|_{\alpha=0} = -a_{\underbrace{ad_m n}_{[m,n]}} =$

$m = \frac{Dg_\alpha}{d\alpha}\Big|_{\alpha=0}$

$= -a_{[m,n]}$

$$a_{m+\alpha n} = a_m + \alpha a_n$$

Př l-akce L_g $\qquad R_m \qquad L_{\exp(\alpha m)}$

p-akce R_g $\qquad \ell_m$

Lemma

1) $A_{g*}\, a_m = a_{Ad_g m}$ – A_g levá akce

2) $A_{g^{-1}*}\, a_m = a_{Ad_g m}$ – A_g pravá akce

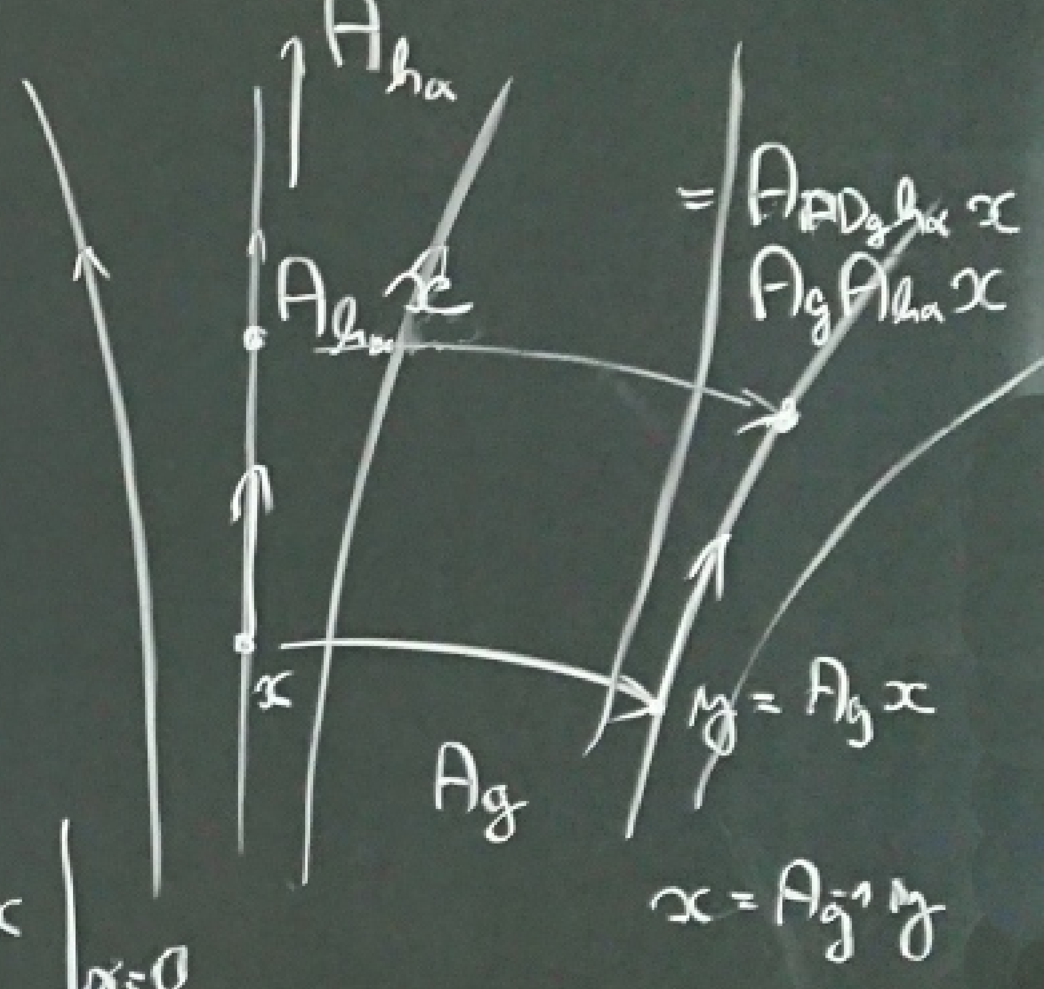
Lema

1, $A_{g*}\, a_m = a_{Ad_g m}$ — A_g levá akce

2, $A_{g^{-1}*}\, a_m = a_{Ad_g m}$ — A_g pravá akce

dů: $A_{g*}\, a_m\big|_x = A_{g*} \frac{D}{d\alpha} A_{h_\alpha} x \Big|_{\alpha=0} = \frac{D}{d\alpha} A_g A_{h_\alpha} x \Big|_{\alpha=0}$

$$= \frac{D}{d\alpha} A_{AD_g h_\alpha} \underbrace{A_g x}_{y} \Big|_{\alpha=0} = a_{Ad_g m} \Big|_{A_g x}$$



Věta

A_g levá akce $\quad [a_m, a_n] = -a_{[m,n]}$

A_g pravá akce $\quad [a_m, a_n] = a_{[m,n]}$

Věta

A_g levá akce $\quad [a_m, a_n] = -a_{[m,n]}$

A_g pravá akce $\quad [a_m, a_n] = a_{[m,n]}$

$$[a_m, a_n] = \mathcal{L}_{a_m} a_n = -\frac{d}{d\alpha} A_{g_\alpha}^* a_n \Big|_{\alpha=0}$$

$$= -\frac{d}{d\alpha} a_{\mathrm{Ad}_{g_\alpha} n} \Big|_{\alpha=0} = -a_{\underbrace{\mathrm{ad}_m n}_{[m,n]}} =$$

$$m = \frac{Dg_\alpha}{d\alpha}\Big|_{\alpha=0}$$

$$= -a_{[m,n]}$$