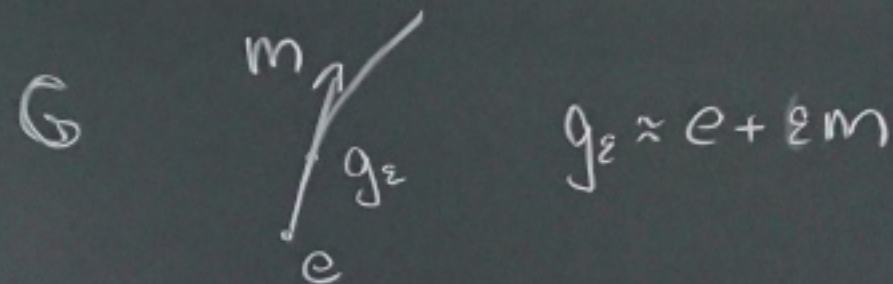


Akce Lieovy grupy na varietě

Def A je akce LG na var M



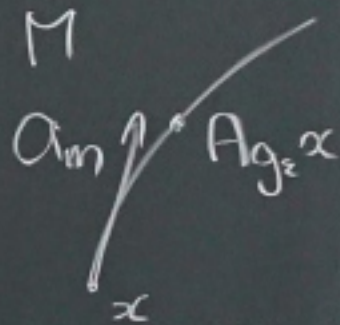
$g \in G \quad A_g : M \rightarrow M$

1) $A_{g_1} \circ A_{g_2} = A_{g_1 g_2}$ levá akce

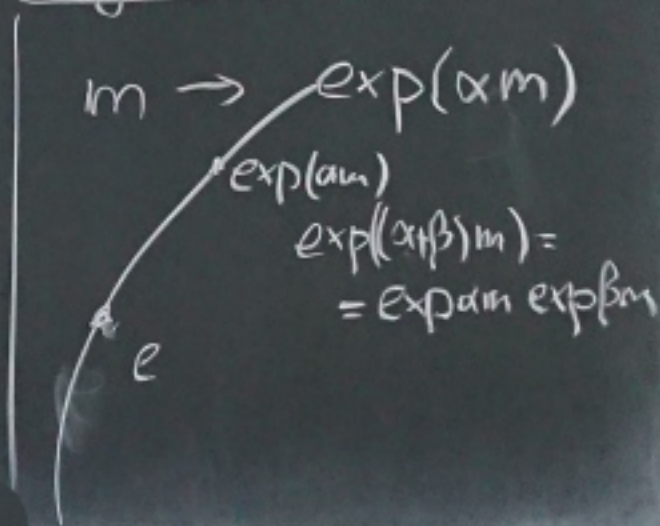
2) $A_{g_1} \circ A_{g_2} = A_{g_2 g_1}$ pravá akce

$A_g x = g x$

$A_g x = x g$



Pr: $L_g \quad R_{g^{-1}} \quad AD_g$ levé akce na G
 $R_g \quad L_{g^{-1}} \quad AD_{g^{-1}}$ pravé akce na G



Def A_g je akce G na M

A_m generátor akce A_g

$a : \mathfrak{g} \rightarrow \mathcal{T}M$

$\forall m \in \mathfrak{g} \rightarrow A_m \in \mathcal{T}M$

$A_m|_x = \frac{D}{dz} A_{g_z} x|_{z=0}$

$\frac{Dg_z}{dz}|_{z=0} = m \quad g_0 = e$

$\rightarrow A_{\exp(\alpha m)} \rightarrow A_m$

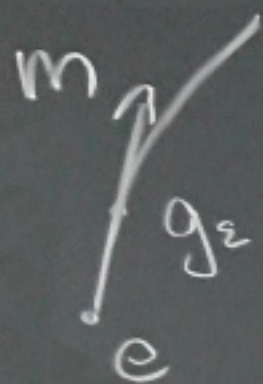
Akce Lieovy grupy na varietě

Def A je akce LG na var M

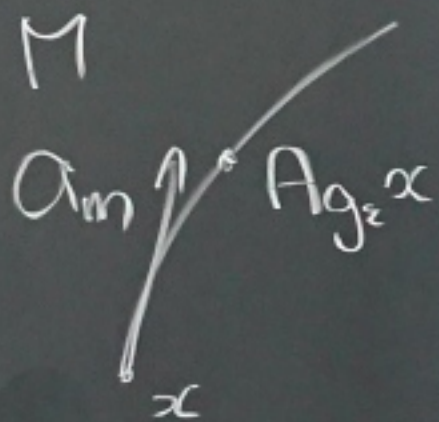
$$g \in G \quad A_g: M \rightarrow M$$

$$\begin{array}{lll} 1) & A_{g_1} \circ A_{g_2} = A_{g_1 g_2} & \text{levá akce} & A_g x = g x \\ 2) & A_{g_1} \circ A_{g_2} = A_{g_2 g_1} & \text{pravá akce} & A_g x = x g \end{array}$$

$$\text{Pr:} \quad \begin{array}{llll} L_g & R_{g^{-1}} & AD_g & \text{levé akce na } G \\ R_g & L_{g^{-1}} & AD_{g^{-1}} & \text{pravé akce na } G \end{array}$$



$$g_\epsilon \approx e + \epsilon m$$



Def A_g je akce G na M

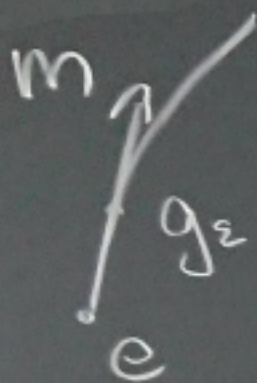
A_m generátor akce A_g

$$a : \mathfrak{g} \rightarrow TM$$

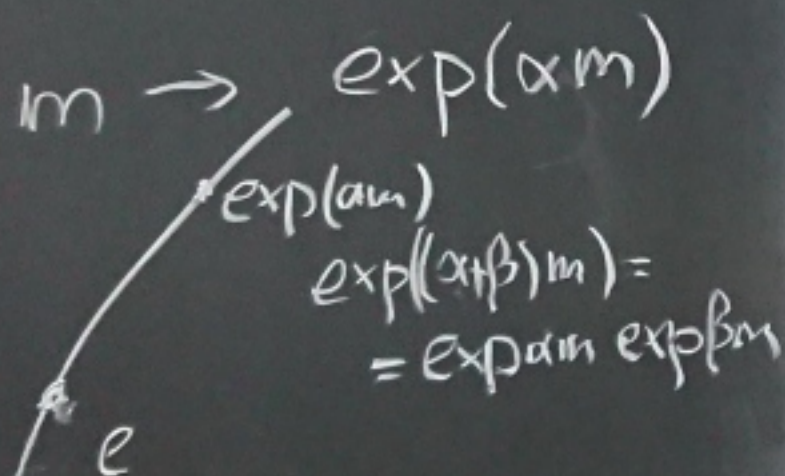
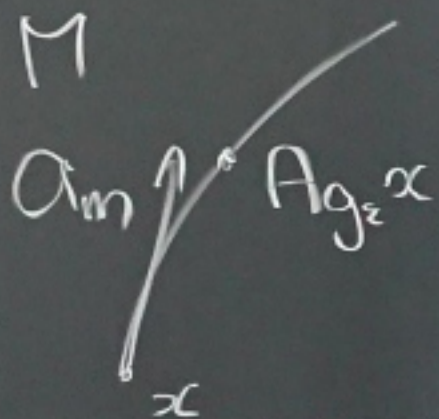
$$\forall m \in \mathfrak{g} \rightarrow A_m \in TM$$

$$A_m|_x = \left. \frac{D}{d\epsilon} A_{g_\epsilon} x \right|_{\epsilon=0}$$

$$\left. \frac{Dg_\epsilon}{d\epsilon} \right|_{\epsilon=0} = m \quad g_0 = e$$



$$g_\epsilon \approx e + \epsilon m$$



Def A_g je atice G na M

A_m generator atice A_g

$$a : \mathfrak{g} \rightarrow TM$$

$$\forall m \in \mathfrak{g} \rightarrow A_m \in TM$$

$$A_m|_x = \left. \frac{D}{d\epsilon} A_{g_\epsilon} x \right|_{\epsilon=0}$$

$$\left. \frac{Dg_\epsilon}{dz} \right|_{\epsilon=0} = m \quad g_0 = e$$

$$\rightarrow A_{\exp(\alpha m)} \iff A_m$$

$$a_{m+\alpha n} = a_m + \alpha a_n$$

PF l-akce L_g Ω_m $L_{\exp(\alpha m)}$
 p-akce R_g Ω_m

Leme

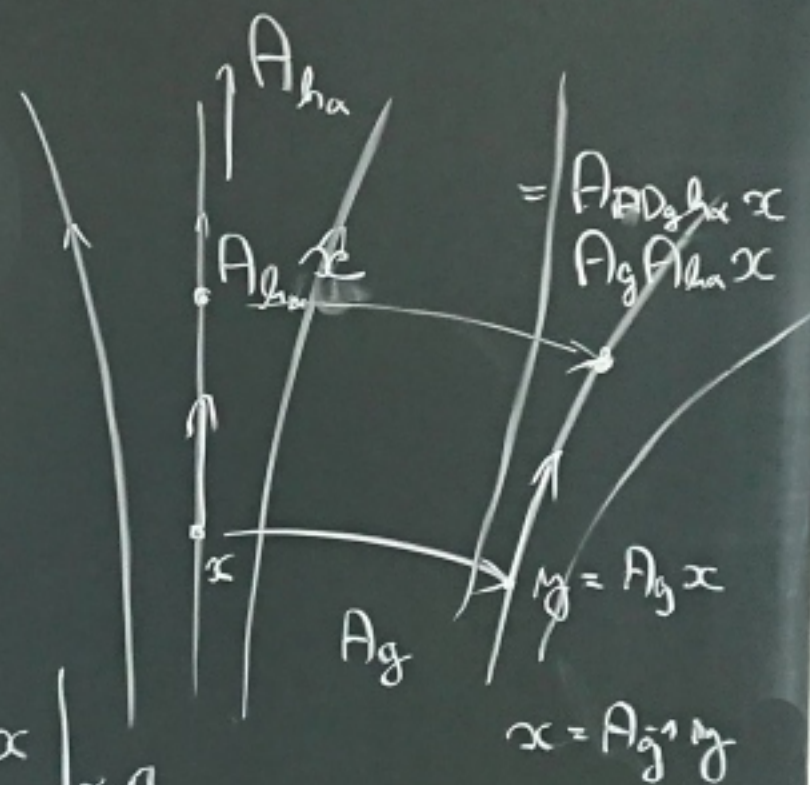
1) $A_g * a_m = a_{Ad_g m}$ - A_g levá akce

2) $A_{g^{-1}} * a_m = a_{Ad_g m}$ - A_g pravá akce

$$d_{\alpha} A_g * a_m |_{\alpha=0} = A_g * \frac{D}{d\alpha} A_{h\alpha} x |_{\alpha=0} = \frac{D}{d\alpha} A_g A_{h\alpha} x |_{\alpha=0}$$

$$= \frac{D}{d\alpha} A_{AD_g h\alpha} \underbrace{A_g x}_y |_{\alpha=0} = a_{Ad_g m} |_{A_g x}$$

M



Víte

A_g levá akce $[a_m, a_n] = -a_{[m, n]}$

A_g pravá akce $[a_m, a_n] = a_{[m, n]}$

$$[a_m, a_n] = \mathcal{L}_{a_m} a_n = -\frac{d}{d\alpha} A_{g\alpha} * a_n$$

$$= -\frac{d}{d\alpha} a_{Ad_{g\alpha} n} |_{\alpha=0} = -a_{\underbrace{ad_m n}_{[m, n]}}$$

$$m = \frac{Dg_\alpha}{d\alpha} |_{\alpha=0}$$

$$= -a_{[m, n]}$$

$$a_{m+\alpha n} = a_m + \alpha a_n$$

PF

l-akce	L_g	Ω_m	$L_{\exp(\alpha m)}$
r-akce	R_g	I_m	

Lemma

- 1, $A_g * a_m = a_{Ad_g m}$ — A_g levá akce
- 2, $A_{g^{-1}} * a_m = a_{Ad_g m}$ — A_g pravá akce

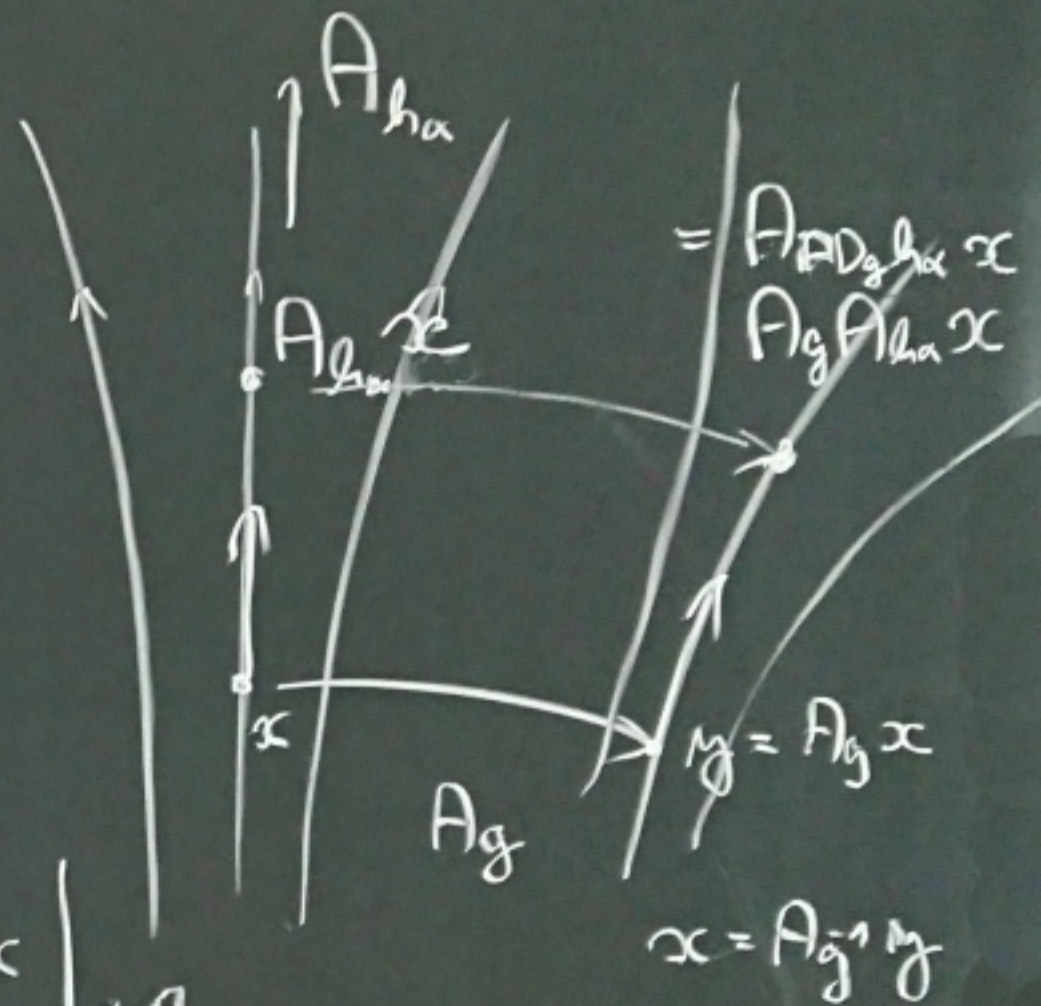
Lens

$$1) A_{g^*} a_m = a_{Adg^m} - A_g \text{ leva' alca}$$

$$2) A_{g^{-1}*} a_m = a_{Adg^m} - A_g \text{ para' alca}$$

$$\text{diz } A_{g^*} a_m|_x = A_{g^*} \frac{D}{d\alpha} A_{h\alpha} x \Big|_{\alpha=0} = \frac{D}{d\alpha} A_g A_{h\alpha} x \Big|_{\alpha=0}$$

$$= \frac{D}{d\alpha} A_{ADg^h\alpha} \underbrace{A_g x}_y \Big|_{\alpha=0} = a_{Adg^m} |_{A_g x}$$



Víte

A_3 levá strana $[a_m, a_n] = -a_{[m, n]}$

A_3 pravá strana $[a_m, a_n] = a_{[m, n]}$

Niite

$$A_g \text{ levá atce } [a_m, a_n] = -a_{[m,n]}$$

$$A_g \text{ pravé atce } [a_m, a_n] = a_{[m,n]}$$

$$[a_m, a_n] = \int a_m a_n = - \frac{d}{d\alpha} A_{g_\alpha} a_n \Big|_{\alpha=0}$$

$$= - \frac{d}{d\alpha} a_{Ad_{g_\alpha} n} \Big|_{\alpha=0} = - a_{\underbrace{ad_m n}_{[m,n]}}$$
$$m = \frac{Dg_\alpha}{d\alpha} \Big|_{\alpha=0}$$

$$= -a_{[m,n]}$$