

# Reprezentace L.G. a L.A. na vektorovém prost. $V$

Def  $T: G \rightarrow \text{Lin } V \quad g \rightarrow Tg \in V_1$

$$Tg_2 = Tg \cdot Tg_2 \quad Tg_{\mathbb{B}}^{\mathbb{A}} = Tg_{\mathbb{M}}^{\mathbb{A}} \cdot Tg_{\mathbb{R}}^{\mathbb{B}}$$

Def  $t: \mathfrak{g} \rightarrow \text{Lin } V \quad m \rightarrow t_m \in V_1$

$$t_{[m,n]} = [t_m, t_n] = t_m t_n - t_n t_m$$

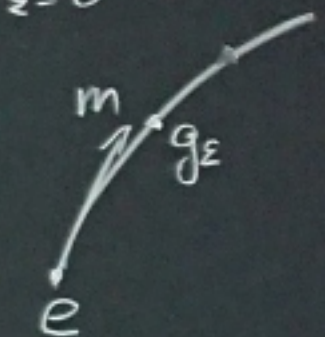
$$t_m^{\mathbb{A}}_{\mathbb{B}} = m^{\alpha} t_{\alpha}^{\mathbb{A}}_{\mathbb{B}}$$

$$C_{\mu\nu}^{\alpha} t_{\alpha}^{\mathbb{K}}_{\mathbb{L}} = t_{\mu}^{\mathbb{K}}_{\mathbb{M}} t_{\nu}^{\mathbb{M}}_{\mathbb{L}} - t_{\nu}^{\mathbb{K}}_{\mathbb{M}} t_{\mu}^{\mathbb{M}}_{\mathbb{L}}$$

$$e + \varepsilon m \quad T_{e+\varepsilon m} = \delta + \varepsilon t_m$$

Def  $T$  rep. LG na  $V$   
 $\downarrow$   $t$  generátor této rep.

$$t_m = \left. \frac{d}{d\varepsilon} Tg_{\varepsilon} \right|_{\varepsilon=0}$$

$$T_{\exp m} = \exp t_m$$


$$T_{\exp 0} = \delta$$

$$\left. \frac{d}{d\varepsilon} T_{\exp(\varepsilon m)} \right|_{\varepsilon=\varepsilon_0} = \left. \frac{d}{d\varepsilon} T_{\exp(\varepsilon m)} \right|_{\varepsilon=0} T_{\exp \varepsilon_0 m}$$

$$= t_m \cdot T_{\exp(\varepsilon_0 m)}$$

$$\Rightarrow T_{\exp \varepsilon m} = \exp(\varepsilon t_m)$$

# Reprezentace L.G. a L.A. na vektorovém prost. $V$

Def  $T: G \rightarrow \text{Lin } V \quad g \rightarrow T_g \in V_1^1$   
 $T_{g_2} = T_{g_1} \cdot T_{g_2} \quad T_{g_2}^A_B = T_{g_1}^A_M \cdot T_{g_2}^M_B$

Def  $t: \mathfrak{g} \rightarrow \text{Lin } V \quad m \rightarrow t_m \in V_1^1$   
 $t_{[m,n]} = [t_m, t_n] = t_m t_n - t_n t_m$

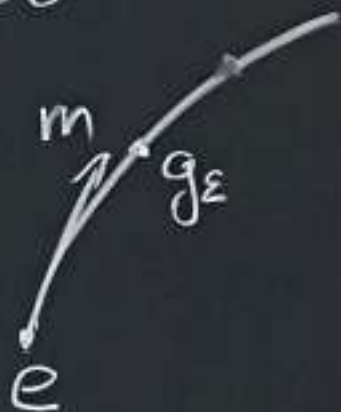
$t_m^A_B = m^\alpha t_\alpha^A_B$   
 $C_{K^\alpha} t_{K^\alpha}^K_L = t_{K^\alpha}^K_M t_{K^\beta}^M_L - t_{K^\beta}^M_L t_{K^\alpha}^K_M$

$$e + \varepsilon m \quad T_{e + \varepsilon m} = \delta + \varepsilon t_m$$

D-f

$\downarrow$   $T$  rep. LG  $m \in V$   
 $t$  generator této rep.

$$t_m = \left. \frac{d}{d\varepsilon} T g_\varepsilon \right|_{\varepsilon=0}$$



Def

$T$  rep. LG  $\mathfrak{m} \subset V$   
 $t$  generator  $t$  is rep.

$$t_m = \left. \frac{d}{d\varepsilon} T g_\varepsilon \right|_{\varepsilon=0}$$

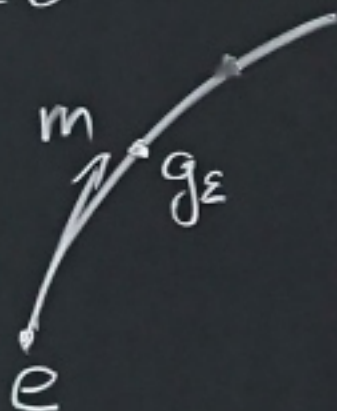
$$T \exp m = \exp t_m$$

$$T \exp 0 = \delta$$

$$\left. \frac{d}{d\varepsilon} T \exp(\varepsilon m) \right|_{\varepsilon=\varepsilon_0} = \left. \frac{d}{d\varepsilon} T \exp(\varepsilon m) \right|_{\varepsilon=0} T \exp \varepsilon_0 m$$

$$= t_m \cdot T \exp(\varepsilon m)$$

$$\Rightarrow T \exp \varepsilon m = \exp(\varepsilon t_m)$$



$$[a, b] = \frac{D}{d\tau} \left( \exp(\sqrt{\tau} a) \exp(\sqrt{\tau} b) \exp(-\sqrt{\tau} a) \exp(-\sqrt{\tau} b) \right) \Big|_{\tau=0}$$

$$\frac{d}{d\tau} T \exp(\sqrt{\tau} a) \exp(\sqrt{\tau} b) \exp(-\sqrt{\tau} a) \exp(-\sqrt{\tau} b) \Big|_{\tau=0} = t_{[a, b]} \stackrel{?}{=} [t_a, t_b]$$

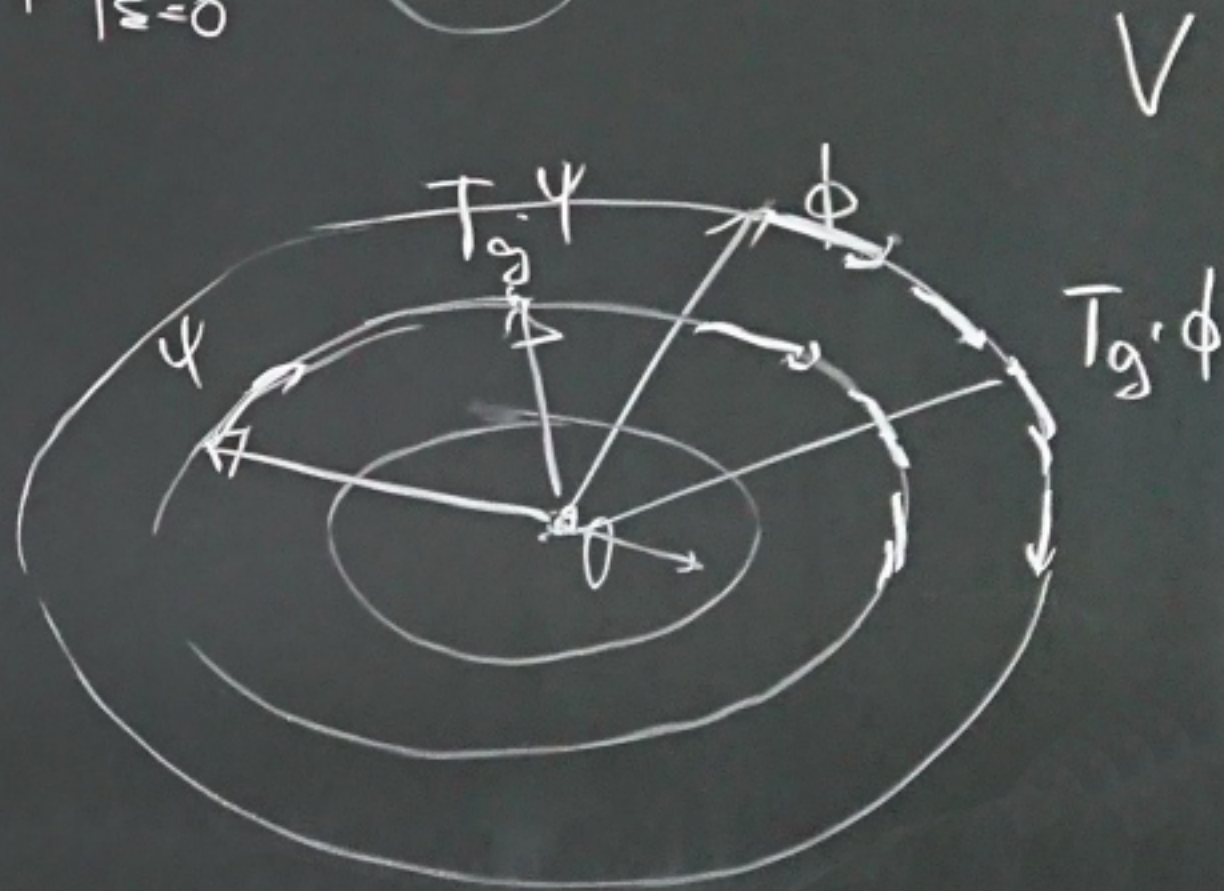
$$\begin{aligned} T \exp(\sqrt{\tau} a) &= \exp(t \sqrt{\tau} a) = \delta + \sqrt{\tau} t_a + \frac{1}{2} \tau t_a^2 + \dots \\ &= \left( \delta + \sqrt{\tau} t_a + \frac{1}{2} \tau t_a^2 + \dots \right) \left( \delta + \sqrt{\tau} t_b + \frac{1}{2} \tau t_b^2 + \dots \right) \left( \delta - \sqrt{\tau} t_a + \frac{1}{2} \tau t_a^2 + \dots \right) \left( \delta - \sqrt{\tau} t_b + \frac{1}{2} \tau t_b^2 + \dots \right) = \\ &= \delta + \sqrt{\tau} (t_a + t_b - t_a - t_b) \\ &\quad + \tau \left( \cancel{t_a^2} + \cancel{t_b^2} + t_a \cdot t_b - \cancel{t_a} t_b - t_b \cancel{t_a} + \cancel{t_a} t_b \right) = \tau [t_a, t_b] \end{aligned}$$

$$t_{[a, b]} = [t_a, t_b]$$

$T$  repr. LG. ma  $V$

$$g \quad T_g \quad \phi \rightarrow T_g \cdot \phi \quad \phi^A = T_g^A_B \phi^B$$

$$m \rightarrow t_m \quad t_m|_\phi = \frac{D}{d\varepsilon} T_{g_\varepsilon} \cdot \phi|_{\varepsilon=0} = \textcircled{t_m} \cdot \phi$$



$T$  репр. LG на  $V$

$$g \quad T_g \quad \phi \rightarrow T_g \cdot \phi \quad \phi^A = T_g^A_B \phi^B$$

$$m \rightarrow t_m \quad t_m|_\phi = \left. \frac{d}{d\varepsilon} T_{g_\varepsilon} \cdot \phi \right|_{\varepsilon=0} = t_m \cdot \phi$$

$T$  репр. LG. на  $V$

$$g \quad T_g \quad \phi \rightarrow T_g \cdot \phi \quad \phi^A = T_g^A{}_B \phi^B$$

$$m \rightarrow t_m \quad t_m|_\phi = \frac{D}{d\varepsilon} T_{g_\varepsilon} \cdot \phi|_{\varepsilon=0} = \textcircled{t_m} \cdot \phi$$



