Grupa isometrií

$\mathrm{Iso}(M, g)$ $\varphi : M \to M$

$\varphi^* g = g$

$\varphi_1 \circ \varphi_2 = \varphi_1 \varphi_2$

$\mathrm{Iso}(M, g)$ tvoří L.G.

$\mathrm{iso}(M, g)$ příslušná LA

M dim D

$\mathrm{Iso}(M, g)$ má maximálně dim $\frac{D(D+1)}{2}$

$\mathrm{Iso}(M, g)$ M

$\varphi \to \Xi_\varphi$

$\Xi_\varphi x = \varphi x$

$\mathrm{iso}(M, g)$

$S \to \xi_S \in TM$

$\xi_S \equiv S$

$\exp(\tau S) = \varphi_\tau$

$\mathcal{L}_{\xi_S} g = 0$ $\varphi_* g = g$

ξ_S je Killingův vektor

$[\xi_m, \xi_n] = -\xi_{[m,n]}$

báze Kill. vekt. ξ_α

$$[\xi_\alpha, \xi_\beta] = -\underbrace{\xi_{[\xi_\alpha, \xi_\beta]}}_{C_{\alpha\beta}{}^\mu \xi_\mu} = -C_{\alpha\beta}{}^\mu \xi_\mu$$

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Akce symetrie na generátorech symetrie

Killing. vektory R K_μ

$r \in iso(M,g)$ $R_\varphi = \exp(\varphi r) \in Iso(M,g)$

$\xi_r = R$ $\Xi_{R_\varphi} = R_\varphi$

$$[R, K_\mu] = -C_{R\mu}{}^{\nu} K_\nu$$

$V_\varphi = R_{\varphi *} V_0$ $V = V^\mu K_\mu$

konstantní na M

$$\frac{d}{d\varphi} V_\varphi = -\mathcal{L}_R V_\varphi = -[R, V_\varphi] = -[R, K_\mu] V^\mu_\varphi = C_{R\mu}{}^{\nu} V^\mu_\varphi K_\nu$$

$\frac{d}{d\varphi} V^\nu_\varphi = C_{R\mu}{}^{\nu} V^\mu_\varphi$ $V^\nu_\varphi = \exp(C_R)^\nu{}_\mu V^\mu_0$ $C_{R\mu}{}^{\nu} = ad_{R\mu}{}^{\nu}$

Akce symetrie na generátorech symetrie

Killing. vektory R $\quad K_\mu$

$r \in iso(M,g) \qquad \mathcal{R}_\varphi = \exp(\varphi r) \in Iso(M,g)$

$\xi_r = R \qquad\qquad \Xi_{\mathcal{R}_\varphi} = \mathcal{R}_\varphi$

$$[R, K_\mu] = -c_{R\mu}{}^{\nu} K_\nu$$

$V_\varphi = \mathcal{R}_{\varphi *} V_0 \qquad\qquad V = V^\mu K_\mu$

konstantní na M

$$V_\varphi = R_{\varphi *} V_0 \qquad V = V^\mu K_\mu$$

costanti su M

$$\frac{d}{d\varphi} V_\varphi = -\mathcal{L}_R V_\varphi = -[R, V_\varphi] = -[R, K_\mu] V_\varphi^\mu = C_{R\mu}{}^\nu V_\varphi^\mu K_\nu$$

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isometrie v E^2

$R \quad X \quad Y$

$[R, X] = -Y$

$[R, Y] = X$

$[X, Y] = 0$

$C_R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad C_R^2 = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\exp(\varphi C_R) = \cos\varphi \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \sin\varphi \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$V_0 = a_0 X + b_0 Y$

$V_\varphi^\mu = \exp(\varphi C_R)^\mu{}_\nu V_0^\nu = \begin{bmatrix} \cos\varphi\, a_0 - \sin\varphi\, b_0 \\ \sin\varphi\, a_0 + \cos\varphi\, b_0 \end{bmatrix}$

X, Y, Z translace v E^3

$[X, Y] = [Y, Z] = [Z, X] = 0$

$C_X = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \exp(x C_X) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$V_0 = a_0 X + b_0 Y$

$V_x^\mu = \exp(x C_X) \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}$

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