# Fibrované prostory

Def $M$ $\mathbb{P}$ dif. variety

$\mathbb{P}M$ je dif. varieta

$\pi : \mathbb{P}M \to M$ projekce

$\mathbb{P}M$ je lokálně triviální

$\forall x \in M \ \exists U$ okolí $x$

$\pi^{-1}(U) \cong U \times \mathbb{P}$

$\mathbb{P}_x M = \pi^{-1} x \cong \mathbb{P}$ fiber nad $x$

$\mathbb{P}$ stand. fiber

$\mathbb{P}M$ fibr. prostor

# Vektorové bundly

stand. fiber $\mathbb{A}$ - vekt. prostor

$\mathbb{A}M$

tenzorový bundle

$\mathbb{A}^k_l M = (\underbrace{\mathbb{A} \otimes \cdots \otimes \mathbb{A}}_{k} \otimes \underbrace{\mathbb{A}^* \otimes \cdots \otimes \mathbb{A}^*}_{l}) M$

Def řez fibr. prostoru

$\phi : M \to \mathbb{A}M$

$\pi \phi = id$

$x \to \phi(x) \in \mathbb{A}_x M$

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# Trivializace vekt. bundlu

Def $\mathcal{A}M$ vekt. bundle dim $D$

- soubor oblastí $U_{(\alpha)}$ v $M$ pokrývající $M$
- na $U_{(\alpha)}$ dána báze ${}^{(\alpha)}E_A$ $A=1..D$

  ${}^{(\alpha)}E_A^M$ řezy v $\mathcal{A}U_{(\alpha)}$

$\Downarrow$

1) trivializační zobr.

$${}^{(\alpha)}\Phi_x : \mathcal{A}_x M \to \mathbb{R}^D$$

$${}^{(\alpha)}\Phi_x : \psi \to \psi^A = {}^{(\alpha)}E^A \cdot \psi$$

2) inverzní triv. zobr.

$${}^{(\alpha)}\Phi_x^{-1} : \mathbb{R}^D \to \mathcal{A}_x M$$

$$\psi^A \to \psi = \psi^A \, {}^{(\alpha)}E_A$$

3) přechodová zobrazení

$${}^{(\alpha\beta)}\Phi_x : \mathbb{R}^D \to \mathbb{R}^D$$

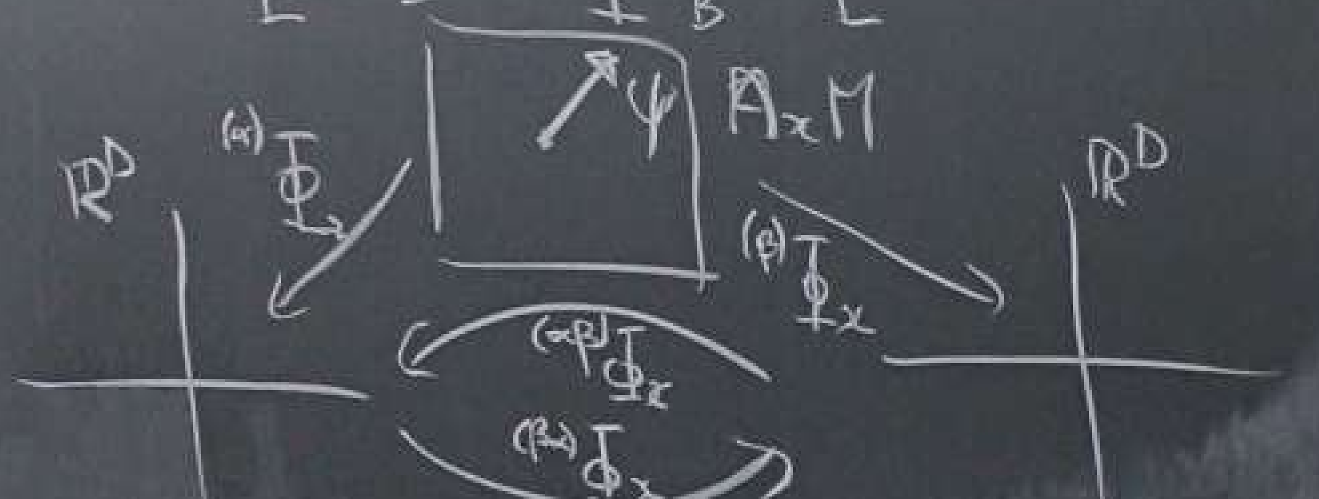
pro $x \in U_{(\alpha)} \cap U_{(\beta)}$

$${}^{(\alpha\beta)}\Phi_x = {}^{(\alpha)}\Phi_x \circ {}^{(\beta)}\Phi_x^{-1}$$

$$\psi^A \to {}^{(\alpha\beta)}\Phi_x{}^A{}_B \, \psi^B$$

$${}^{(\beta)}E_B = {}^{(\alpha)}E_A \, {}^{(\alpha\beta)}\Phi^A{}_B$$

$${}^{(\alpha)}E^A = {}^{(\alpha\beta)}\Phi^A{}_B \, {}^{(\beta)}E^B$$



# Trivializace vekt. bundlu

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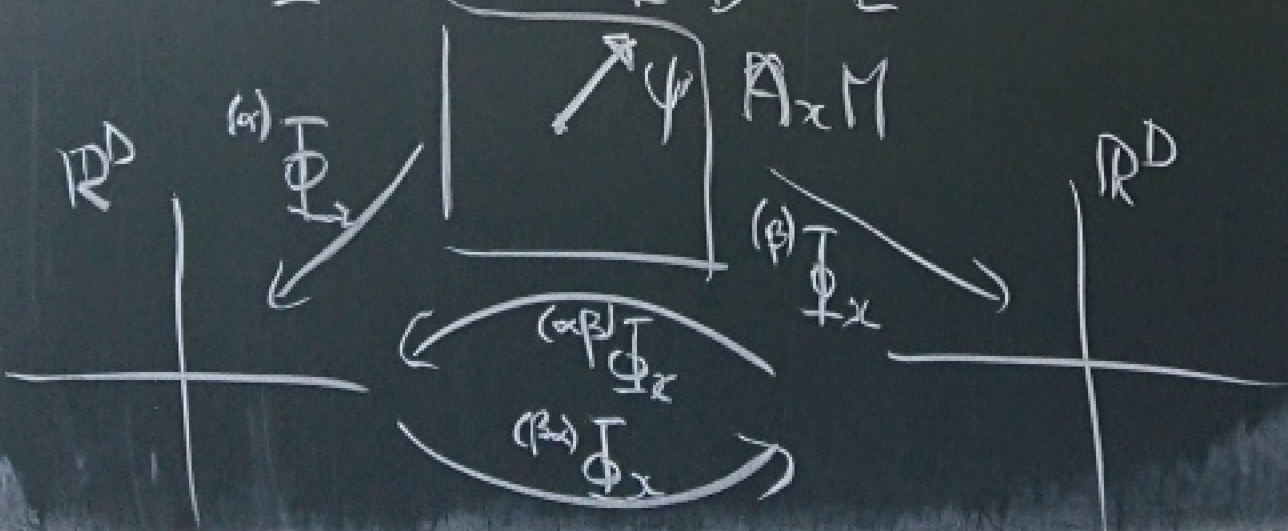
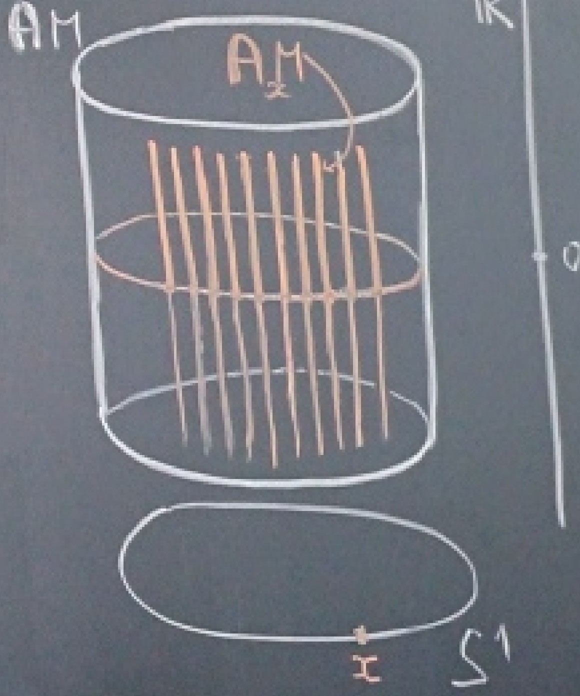


Př: Lineární bundle nad $S^1$

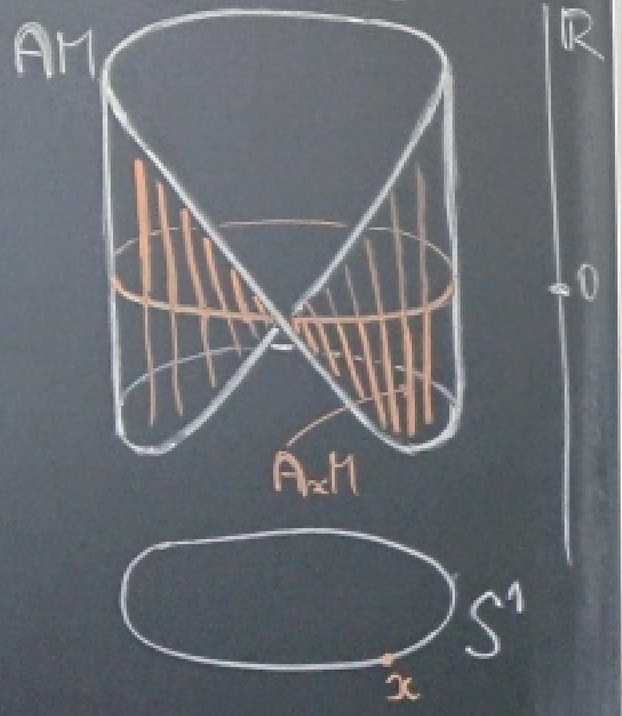
$M = S^1 \qquad A = \mathbb{R}$

triviální bundle



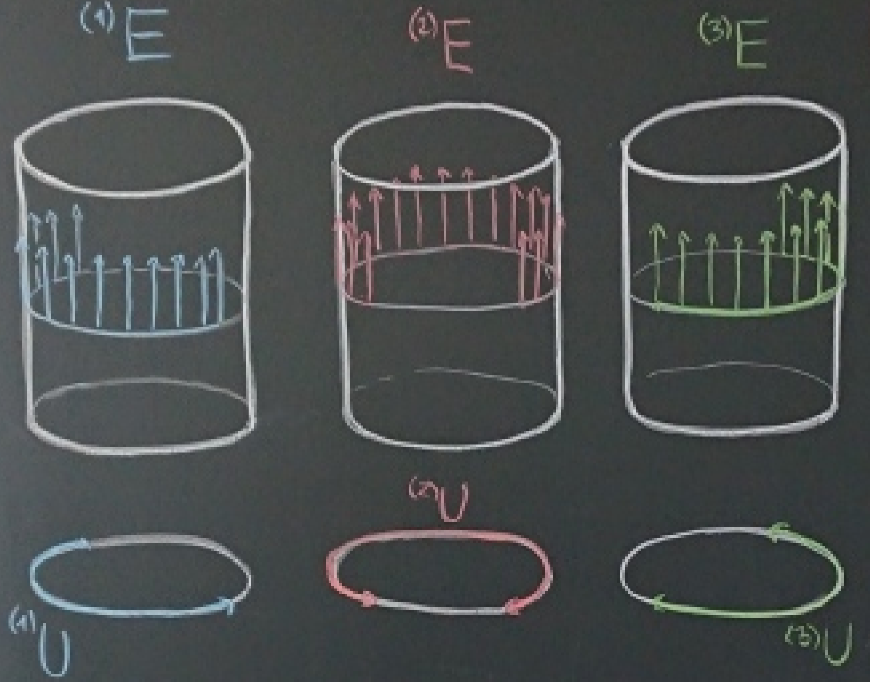

$AM \cong S^1 \times \mathbb{R}$

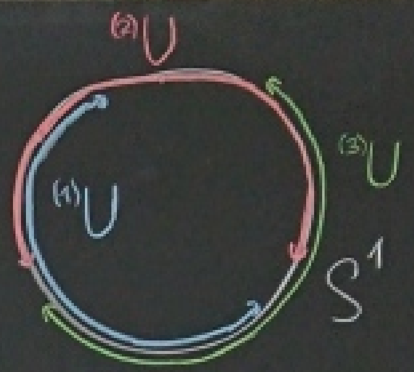
twistovaný bundle




$AM \not\cong S^1 \times \mathbb{R}$

triviální bundle



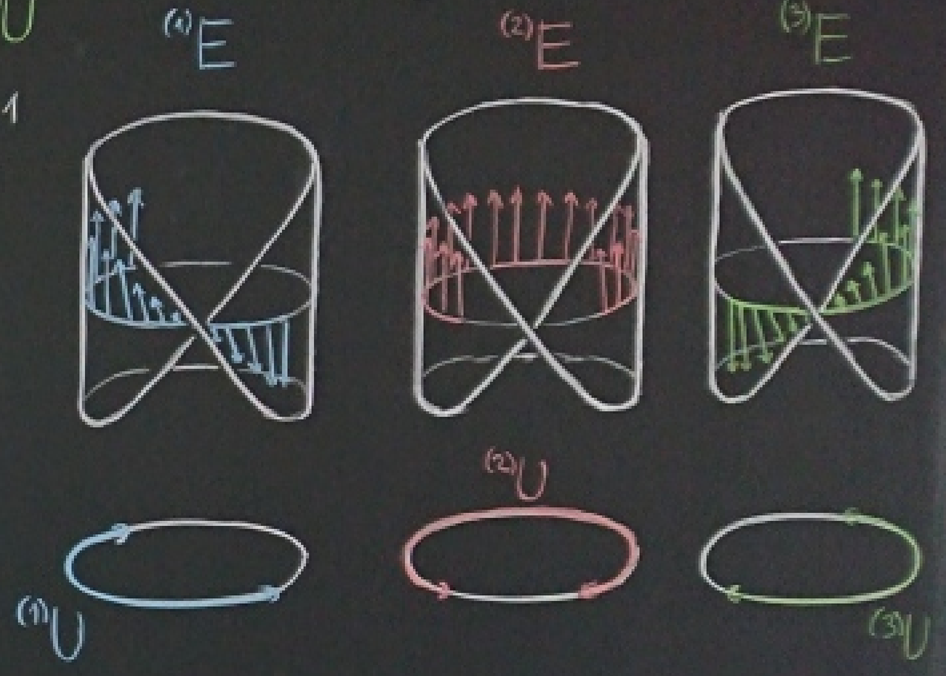




$${}^{(12)}\Phi = [1]$$

$${}^{(23)}\Phi = [1]$$

$${}^{(31)}\Phi = [1]$$

twistovaný bundle




$${}^{(12)}\Phi = [1]$$

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# Př: Lineární bundle nad $S^1$

$$M = S^1 \qquad A = \mathbb{R}$$

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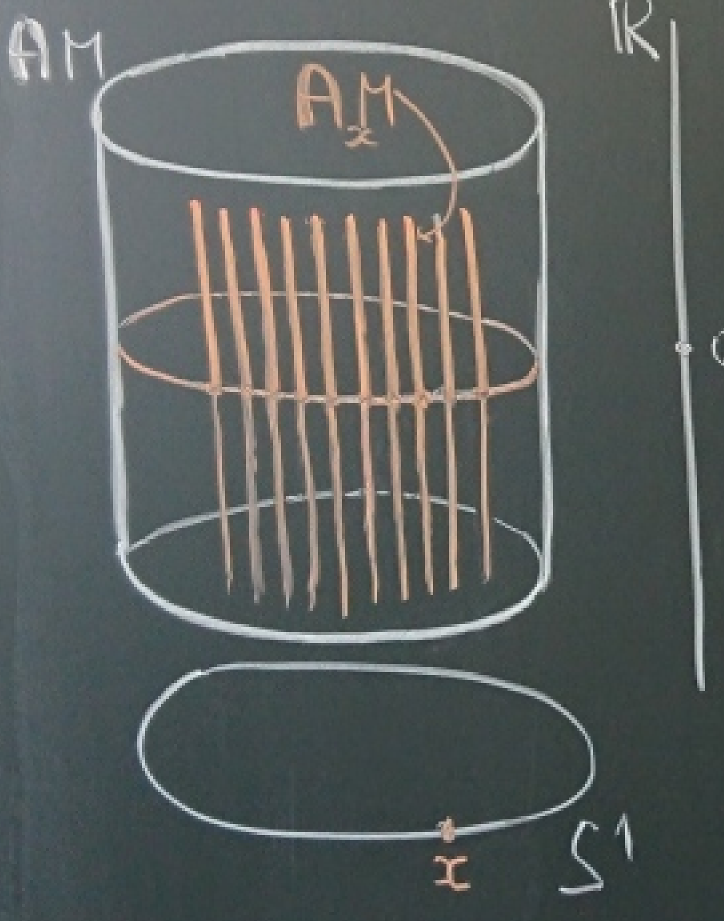
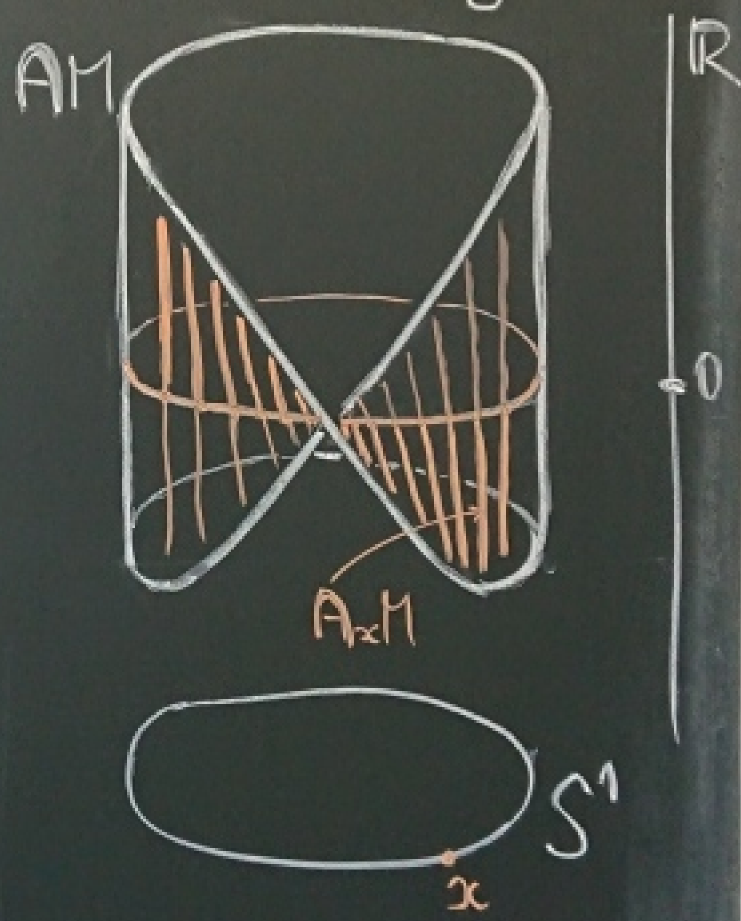


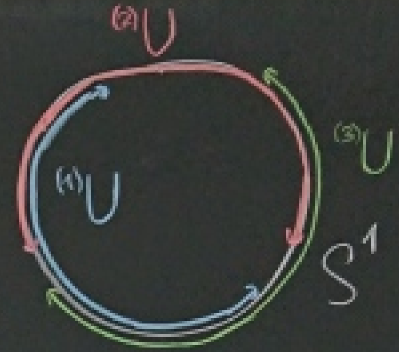

$$AM \cong S^1 \times \mathbb{R}$$

twistovaný bundle




$$AM \ncong S^1 \times \mathbb{R}$$

triviální bundle

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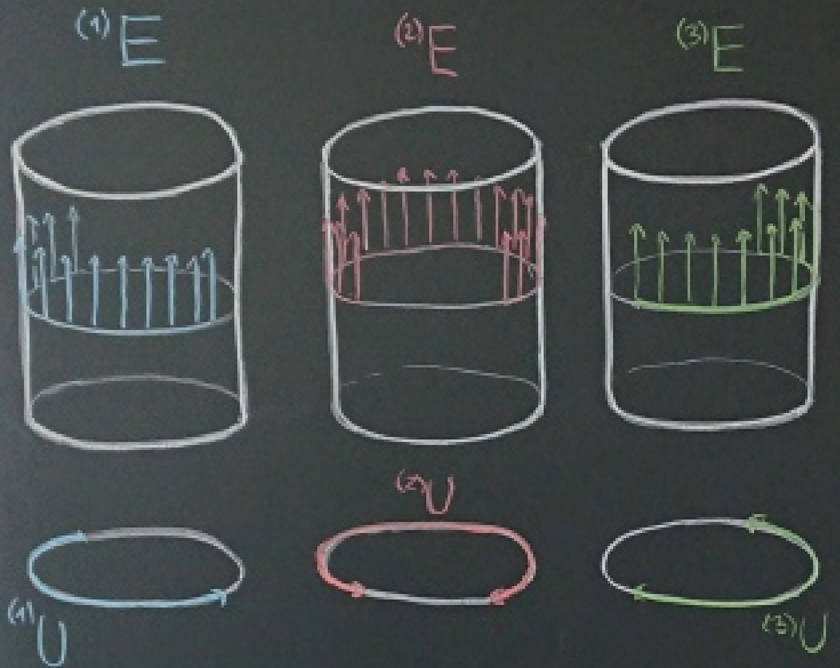


$${}^{(12)}\Phi = [1]$$
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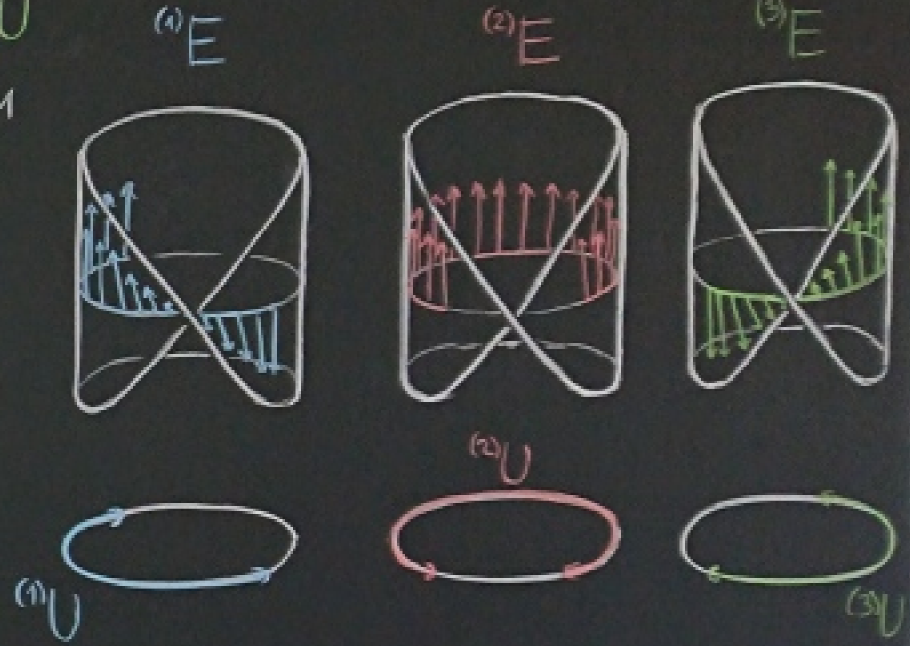


$${}^{(12)}\Phi = [1]$$
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# Kovariantní derivace na vektorovém bundlu

$\phi, \psi \in \mathrm{Sect}\, AM \quad f \in \mathcal{F}M$

$AM \qquad \xi, \varsigma \in TM$

$D_\xi$ kovar. der. na $AM$ ve směru $\xi \in TM$

$$D_\xi : \mathrm{Sect}\, AM \to \mathrm{Sect}\, AM$$

$D_\xi \phi|_x$ je dáno znalostí $\phi$ na libov. okolí $U$ bodu $x \in M$

$$D_\xi(\phi + \psi) = D_\xi \phi + D_\xi \psi$$

$$D_\xi(f\phi) = f D_\xi \phi + (\xi \cdot df)\, \phi$$

$$D_{\xi + f\varsigma} \phi = D_\xi \phi + f D_\varsigma \phi$$

## Kovariantní diferenc.

$$D_\xi \phi = \xi \cdot D\phi$$

$$D_\xi \phi^A = \xi^m D_m \phi^A$$

$$D_m \phi^A$$

rozšíření na tens. bundle

$$D(\phi \otimes \psi) = (D\phi) \otimes \psi + \phi \otimes (D\psi)$$

$$D\, C\phi = C\, D\phi$$

$$Df = df$$

# Kovariantní derivace na vektorovém bundlu

$AM$ $\quad \phi, \psi \in \mathrm{Sect}\, AM \quad f \in \mathcal{F}M$

$\xi, \zeta \in \mathcal{T}M$

$D_\xi$ kovar. der. na $AM$ ve směru $\xi \in \mathcal{T}M$

$$D_\xi : \mathrm{Sect}\, AM \to \mathrm{Sect}\, AM$$

$D_\xi \phi\big|_x$ je dáno znalostí $\phi$ na libov. okolí $U$ bodu $x \in M$

$$D_\xi(\phi + \psi) = D_\xi \phi + D_\xi \psi$$

$$D_\xi(f\phi) = f D_\xi \phi + (\xi \cdot df)\, \phi$$

$$D_{\xi + f\zeta} \phi = D_\xi \phi + f D_\zeta \phi$$

Kovariantní diference

$$D_{\xi}\phi = \xi\cdot D\phi$$

$$D_{\xi}\phi^{\underline{A}} = \xi^{\underline{m}} D_{\underline{m}}\phi^{\underline{A}}$$

$$D_{\underline{m}}\phi^{\underline{A}}$$

rozšíření na tens. bundle

$$D(\phi \otimes \psi) = (D\phi) \otimes \psi + \phi \otimes (D\psi)$$

$$D C \phi = C D \phi$$

$$D f = df$$

vztah kov. derivací

$D, \tilde D$ kov. der. na $\mathbf{A}M$

$$\tilde D\phi - D\phi = \mathbf{A}\cdot\phi$$

$$\tilde D_m \phi^{\underline{A}} - D_m \phi^{\underline{A}} = \mathbf{A}_m{}^{\underline{A}}{}_{\underline{B}}\, \phi^{\underline{B}}$$

$$\tilde D T - D T = \mathbf{A} T$$

$$\mathbf{A}_m T^{\underline{A}\underline{B}\dots}_{\underline{C}\underline{D}\dots} = \mathbf{A}_m{}^{\underline{A}}{}_{\underline{M}}\, T^{\underline{M}\underline{B}\dots}_{\underline{C}\underline{D}\dots} + \mathbf{A}_m{}^{\underline{B}}{}_{\underline{M}}\, T^{\underline{A}\underline{M}\dots}_{\underline{C}\underline{D}\dots} + \dots - \mathbf{A}_m{}^{\underline{M}}{}_{\underline{C}}\, T^{\underline{A}\underline{B}\dots}_{\underline{M}\underline{D}\dots} - \mathbf{A}_m{}^{\underline{M}}{}_{\underline{D}}\, T^{\underline{A}\underline{B}\dots}_{\underline{C}\underline{M}\dots} - \dots$$

Př. $\mathbf{A}_m M^{\underline{A}}{}_{\underline{B}} = \mathbf{A}_m{}^{\underline{A}}{}_{\underline{N}}\, M^{\underline{N}}{}_{\underline{B}} - \mathbf{A}_m{}^{\underline{N}}{}_{\underline{B}}\, M^{\underline{A}}{}_{\underline{N}}$

$$\mathbf{A}_m M = [\mathbf{A}_m, M]$$

trivializace

$E_{\underline{A}}$ báze v $\mathbf{A}M$

$$\partial E_{\underline{A}} = 0 \qquad \partial E^{\underline{A}} = 0$$

$$\phi = \phi^{\underline{A}} E_{\underline{A}}$$

$$\partial\phi = (d\phi^{\underline{A}}) \otimes E_{\underline{A}}$$

$$\partial_m \phi^{\underline{A}} = (\partial\phi)^{\underline{A}}_m = \phi^{\underline{A}}{}_{,m}$$

$$D = \partial + \mathbf{A} \qquad \mathbf{A}_m{}^{\underline{A}}{}_{\underline{B}}$$

$\mathbb{M}$ pseudoderivace $\mathbf{A}^k_l M$

$$\mathbb{M} : \mathrm{Sect}\, \mathbf{A}^k_l M \to \mathrm{Sect}\, \mathbf{A}^k_l M$$

$$\mathbb{M}(\phi + r\psi) = \mathbb{M}\phi + r\,\mathbb{M}\psi$$

$$\mathbb{M}(\phi\psi) = (\mathbb{M}\phi)\psi + \phi(\mathbb{M}\psi)$$

$$\mathbb{M}\, C\phi = C\,\mathbb{M}\phi$$

$$\mathbb{M} f = 0$$

$$\mathbb{M} T^{\underline{A}\dots}_{\underline{B}\dots} = M^{\underline{A}}{}_{\underline{N}}\, T^{\underline{N}\dots}_{\underline{B}\dots} + \dots - M^{\underline{N}}{}_{\underline{B}}\, T^{\underline{A}\dots}_{\underline{N}\dots} - \dots$$

$$\mathbb{M}\phi^{\underline{A}} = M^{\underline{A}}{}_{\underline{N}}\, \phi^{\underline{N}}$$

vztah kov. derivací

$D, \tilde{D}$ kov. der. na AM

$$\tilde{D}\phi - D\phi = A\cdot\phi$$

$$\tilde{D}_{\underline{m}}\phi^{\underline{A}} - D_{\underline{m}}\phi^{\underline{A}} = A_{\underline{m}}{}^{\underline{A}}{}_{\underline{B}}\,\phi^{\underline{B}}$$

$$\tilde{D}T - DT = \mathbb{A}\,T$$

$$\mathbb{A}_{\underline{m}} T^{\underline{A}\underline{B}\ldots}_{\underline{C}\underline{D}\ldots} = A_{\underline{m}}{}^{\underline{A}}{}_{\underline{M}}\, T^{\underline{M}\underline{B}\ldots}_{\underline{C}\underline{D}\ldots} + A_{\underline{m}}{}^{\underline{B}}{}_{\underline{M}}\, T^{\underline{A}\underline{M}\ldots}_{\underline{C}\underline{D}\ldots} + \ldots$$
$$- A_{\underline{m}}{}^{\underline{M}}{}_{\underline{C}}\, T^{\underline{A}\underline{B}\ldots}_{\underline{M}\underline{D}\ldots} - A_{\underline{m}}{}^{\underline{M}}{}_{\underline{D}}\, T^{\underline{A}\underline{B}\ldots}_{\underline{C}\underline{M}\ldots} - \ldots$$

Př: $\mathbb{A}_{\underline{m}} M^{\underline{A}}{}_{\underline{B}} = A_{\underline{m}}{}^{\underline{A}}{}_{\underline{N}}\, M^{\underline{N}}{}_{\underline{B}} - A_{\underline{m}}{}^{\underline{N}}{}_{\underline{B}}\, M^{\underline{A}}{}_{\underline{N}}$

$$\mathbb{A}_{\underline{m}} M = [A_{\underline{m}}, M]$$

trivializace

$E_A$ báze v $A^M$

$\partial E_A = 0 \qquad \partial E^A = 0$

$\phi = \phi^A E_A$

$\partial \phi = (d\phi^A) \otimes E_A$

$\partial_m \phi^A = (\partial \phi)^A_m = \phi^A_{\ ,m}$

$D = \partial + \mathbb{A} \qquad \mathbb{A}_{\underline{m}}{}^{\underline{A}}{}_{\underline{B}}$

$\mathbb{M}$ pseudoderivace $\mathbb{A}^{\varepsilon}_{e} M$

$$\mathbb{M} : \mathrm{Sect}\, \mathbb{A}^{\varepsilon}_{e} M \longrightarrow \mathrm{Sect}\, \mathbb{A}^{\varepsilon}_{e} M$$

$$\mathbb{M}(\phi + r\psi) = \mathbb{M}\phi + r\,\mathbb{M}\psi$$

$$\mathbb{M}(\phi\psi) = (\mathbb{M}\phi)\psi + \phi(\mathbb{M}\psi)$$

$$\mathbb{M}\, G\phi = G\, \mathbb{M}\phi$$

$$\mathbb{M} f = 0$$

$$\mathbb{M} T^{\underline{A}\dots}_{\underline{B}\dots} = M^{\underline{A}}{}_{\underline{N}} T^{\underline{N}\dots}_{\underline{B}\dots} + \dots - M^{\underline{N}}{}_{\underline{B}} T^{\underline{A}\dots}_{\underline{N}\dots} - \dots$$

$$\mathbb{M}\phi^{\underline{A}} = M^{\underline{A}}{}_{\underline{N}}\phi^{\underline{N}}$$

Kovariantní vnější der. na vekt. bundle

$\Lambda^p_{\otimes} A^k_l M$

$\phi_1 \wedge \phi_2 \wedge \ldots = \binom{p}{p_1\, p_2 \ldots} \mathcal{A}(\phi_1 \otimes \phi_2 \otimes \ldots)$

$p_1\, {}^{k_1}_{l_1} \quad p_2\, {}^{k_2}_{l_2}$

$\phi^{M}_{\underline{m}} \wedge \psi^{N}_{\underline{n}} = 2\, \phi^{M}_{[\underline{m}} \psi^{N}_{\underline{n}]}$

$= \phi^{M}_{\underline{m}} \otimes \psi^{N}_{\underline{n}} - \phi^{M}_{\underline{n}} \otimes \psi^{N}_{\underline{m}}$

${}^D d$ kov. vnější der. na $\Lambda^p A^k_l M$ $\qquad ({}^D d\phi)_{a_0 a_1 \ldots a_p} = (p+1)\, D_{[a_0} \phi_{a_1 \ldots a_p]}$

${}^D d : \mathrm{Sect}\, \Lambda^p A^k_l M \to \Lambda^{p+1} A^k_l M$

${}^D d(\phi + \psi) = {}^D d\phi + {}^D d\psi$

${}^D d(\phi \wedge \psi) = ({}^D d\phi) \wedge \psi + (-1)^p\, \phi \wedge {}^D d\psi$

${}^D d\phi = D\phi$ pro $\phi \in \mathrm{Sect}\, A^k_l M$ $\quad p = 0$

${}^D d\phi = d\phi$ pro $\phi \in \mathrm{Sect}\, \Lambda^p M$ $\quad k, l = 0$

$$\phi = \sum_{a_1 < \ldots < a_p} \phi_{a_1 \ldots a_p}\, dx^{a_1} \wedge \ldots \wedge dx^{a_p} \qquad \phi_{a_1 \ldots a_p} \in \mathrm{Sect}\, A^k_l M$$

$${}^D d\phi = \sum_{a_1 < \ldots < a_p} {}^D d\phi_{a_1 \ldots a_p} \wedge dx^{a_1} \wedge \ldots \wedge dx^{a_p} = (p+1) \sum_{a_0 < a_1 < \ldots < a_p} D_{[a_0} \phi_{a_1 \ldots a_p]}\, dx^{a_0} \wedge dx^{a_1} \wedge \ldots \wedge dx^{a_p}$$

# Kovariantní vnější der. na vekt. bundle

$$\Lambda^{p}_{\otimes} \mathcal{A}^{k}_{l} M$$

$$\underset{\substack{p_1 \\ l_1}}{\overset{k_1}{\phi_1}} \wedge \underset{\substack{p_2 \\ l_2}}{\overset{k_2}{\phi_2}} \wedge \dots = \binom{p}{p_1\, p_2 \dots} \mathcal{A}\left(\phi_1 \otimes \phi_2 \otimes \dots\right)$$

$$\phi^{\underline{M}}_{\underline{m}} \wedge \psi^{\underline{N}}_{\underline{n}} = 2\, \phi^{\underline{M}}_{[\underline{m}}\, \psi^{\underline{N}}_{\underline{n}]}$$

$$= \phi^{\underline{M}}_{\underline{m}} \otimes \psi^{\underline{N}}_{\underline{n}} - \phi^{\underline{M}}_{\underline{n}} \otimes \psi^{\underline{N}}_{\underline{m}}$$

${}^D d$ kov. vnější der. na $\Lambda^p A^q_l M$

$${}^D d : \mathrm{Sect}\,\Lambda^p A^q_l M \longrightarrow \Lambda^{p+1} A^q_l M$$

$${}^D d(\phi + \psi) = {}^D d\phi + {}^D d\psi$$

$${}^D d(\underset{p}{\phi} \wedge \psi) = ({}^D d\phi) \wedge \psi + (-1)^p \phi \wedge {}^D d\psi$$

${}^D d\phi = D\phi$ pro $\phi \in \mathrm{Sect}\, A^q_l M$ $\quad p=0$

${}^D d\phi = d\phi$ pro $\phi \in \mathrm{Sect}\,\Lambda^p M$ $\quad q,l=0$

${}^{D}d$ kov. vnější der na $\Lambda^p A^{k}_{l} M$ $\qquad ({}^{D}d\phi)_{a_0 a_1 \dots a_p} = (p+1)\, D_{[a_0}\phi_{a_1\dots a_p]}$

$${}^{D}d : \mathcal{S}ect\, \Lambda^p A^{k}_{l} M \to \Lambda^{p+1} A^{k}_{l} M$$

$${}^{D}d(\phi+\psi) = {}^{D}d\phi + {}^{D}d\psi$$

$${}^{D}d(\phi \wedge \psi) = ({}^{D}d\phi)\wedge\psi + (-1)^p\, \phi \wedge {}^{D}d\psi$$

$${}^{D}d\phi = D\phi \qquad \text{pro } \phi \in \mathcal{S}ect\, A^{k}_{l} M \quad p=0$$

$${}^{D}d\phi = d\phi \qquad \text{pro } \phi \in \mathcal{S}ect\, \Lambda^p M \quad k,l=0$$

$$\phi = \sum_{a_1<\dots<a_p} \phi_{a_1\, a_p}\, dx^{a_1}\wedge\dots\wedge dx^{a_p} \qquad \phi_{a_1\dots a_p} \in \mathcal{S}ect\, A^{k}_{l} M$$

$${}^{D}d\phi = \sum_{a_1< \, <a_p} {}^{D}d\phi_{a_1\dots a_p} \wedge dx^{a_1}\wedge\dots\wedge dx^{a_p} = (p+1)\sum_{a_0<a_1<\dots<a_p} D_{[a_0}\phi_{a_1\dots a_p]}\, dx^{a_0}\wedge dx^{a_1}\wedge\dots\wedge dx^{a_p}$$

$D$ na $\mathbb{A}M$

$\nabla$ na $\mathbb{T}M$ $\quad T^{\underline{a}}_{\underline{mn}}$

$D$ rozšíření $D$ na $\mathbb{T}M$

$$D(\underset{\mathbb{T}M}{A}\otimes\underset{\mathbb{A}M}{\phi}) = \underbrace{(DA)}_{\nabla A}\otimes\phi + A\otimes\underbrace{(D\phi)}_{D\phi}$$

$${}^{D}\!d_{\underline{a}_0}\phi^{\underline{A}}_{\underline{a}_1\ldots\underline{a}_p} = D_{\underline{a}_0}\wedge\phi^{\underline{A}}_{\underline{a}_1\ldots\underline{a}_p} + T^{\underline{n}}_{\underline{a}_0\underline{a}_1}\wedge\phi^{\underline{A}}_{\underline{n}\,\underline{a}_2\ldots\underline{a}_p}$$

$$= (p+1)\,D_{[\underline{a}_0}\phi^{\underline{A}}_{\underline{a}_1\ldots\underline{a}_p]} + \binom{p+1}{2}T^{\underline{n}}_{[\underline{a}_0\underline{a}_1}\phi_{|\underline{n}|\underline{a}_2\ldots\underline{a}_p]}$$

${}^{D}\!d$ kov. vnější der. na $\Lambda^p\mathbb{A}^q_l M$ $\qquad ({}^{D}\!d\phi)_{a_0a_1\ldots a_p} = (p+1)\,D_{[a_0}\phi_{a_1\ldots a_p]}$

$${}^{D}\!d : \mathrm{Sect}\,\Lambda^p\mathbb{A}^q_l M \to \Lambda^{p+1}\mathbb{A}^q_l M$$

$${}^{D}\!d(\phi+\psi) = {}^{D}\!d\phi + {}^{D}\!d\psi$$

$${}^{D}\!d(\underset{p}{\phi}\wedge\psi) = ({}^{D}\!d\phi)\wedge\psi + (-1)^p\,\phi\wedge{}^{D}\!d\psi$$

${}^{D}\!d\phi = D\phi \quad$ pro $\phi\in\mathrm{Sect}\,\mathbb{A}^q_l M \quad p=0$

${}^{D}\!d\phi = d\phi \quad$ pro $\phi\in\mathrm{Sect}\,\Lambda^p M \quad q,l=0$

$$\phi = \sum_{a_1<\ldots<a_p}\phi_{a_1\ldots a_p}\,dx^{a_1}\wedge\ldots\wedge dx^{a_p} \qquad \phi_{a_1\ldots a_p}\in\mathrm{Sect}\,\mathbb{A}^q_l M$$

$${}^{D}\!d\phi = \sum_{a_1<\ldots<a_p}{}^{D}\!d\phi_{a_1\ldots a_p}\wedge dx^{a_1}\wedge\ldots\wedge dx^{a_p} = (p+1)\sum_{a_0<a_1<\ldots<a_p}D_{[a_0}\phi_{a_1\ldots a_p]}\,dx^{a_0}\wedge dx^{a_1}\wedge\ldots\wedge dx^{a_p}$$

$D$ na $\mathbb{A}M$

$\nabla$ na $\mathbb{T}M$ $\qquad T^{\underline{a}}{}_{\underline{mn}}$

$D$ rozšíření $D$ na $\mathbb{T}M$

$$D\big(\underset{\substack{\uparrow \\ \mathbb{T}M}}{A} \otimes \underset{\substack{\uparrow \\ \mathbb{A}M}}{\phi}\big) = \underbrace{(DA)}_{\nabla A} \otimes \phi + A \otimes \underbrace{(D\phi)}_{D\phi}$$

$${}^{D}d_{\underline{a}_0}\phi^{\underline{A}}{}_{\underline{a}_1\dots\underline{a}_p} = D_{\underline{a}_0} \wedge \phi^{\underline{A}}{}_{\underline{a}_1\dots\underline{a}_p} + T^{\underline{n}}{}_{\underline{a}_0\underline{a}_1} \wedge \phi^{\underline{A}}{}_{|\underline{n}|\underline{a}_2\dots\underline{a}_p}$$

$$= (p+1)\, D_{[\underline{a}_0}\phi^{\underline{A}}{}_{\underline{a}_1\dots\underline{a}_p]} + \binom{p+1}{2} T^{\underline{n}}{}_{[\underline{a}_0\underline{a}_1}\phi_{|\underline{n}|\underline{a}_2\dots\underline{a}_p]}$$