Kovariantní derivace na vektorovém bundlu

$\phi, \psi \in \mathrm{Sect}\,AM \quad f \in \mathcal{F}M$

$AM \qquad \xi, \zeta \in TM$

D_ξ kovar. der. na AM ve směru $\xi \in TM$

$$D_\xi : \mathrm{Sect}\,AM \to \mathrm{Sect}\,AM$$

$D_\xi \phi|_x$ je dáno znalostí ϕ na libov. okolí U bodu $x \in M$

$$D_\xi(\phi + \psi) = D_\xi \phi + D_\xi \psi$$

$$D_\xi(f\phi) = f D_\xi \phi + (\xi \cdot df)\, \phi$$

$$D_{\xi + f\zeta} \phi = D_\xi \phi + f D_\zeta \phi$$

Kovariantní diference

$$D_\xi \phi = \xi \cdot D\phi$$

$$D_\xi \phi^A = \xi^m D_m \phi^A$$

$$D_m \phi^A$$

rozšíření na tenz. bundle

$$D(\phi \otimes \psi) = (D\phi) \otimes \psi + \phi \otimes (D\psi)$$

$$D\, G\phi = G\, D\phi$$

$$Df = df$$

Kovariantní derivace na vektorovém bundlu

$\phi, \psi \in \mathrm{Sect}\, AM \quad f \in \mathcal{F}M$

$AM \quad \xi, \zeta \in \mathcal{T}M$

D_ξ kovar. der. na AM ve směru $\xi \in \mathcal{T}M$

$$D_\xi : \mathrm{Sect}\, AM \to \mathrm{Sect}\, AM$$

$D_\xi \phi\big|_x$ je dáno znalostí ϕ na libov. okolí U bodu $x \in M$

$$D_\xi(\phi + \psi) = D_\xi \phi + D_\xi \psi$$

$$D_\xi(f\phi) = f D_\xi \phi + (\xi \cdot df)\,\phi$$

$$D_{\xi + f\zeta}\phi = D_\xi \phi + f D_\zeta \phi$$

Kovariantní diference.

$$D_\xi \phi = \xi \cdot D \phi$$

$$D_\xi \phi^{\underline{A}} = \xi^{\underline{m}} D_{\underline{m}} \phi^{\underline{A}}$$

$$D_{\underline{m}} \phi^{\underline{A}}$$

rozšíření na tens. bundle

$$D(\phi \otimes \psi) = (D\phi) \otimes \psi + \phi \otimes (D\psi)$$

$$D\, C\phi = C\, D\phi$$

$$Df = df$$

vztah kov. derivací

$D, \tilde{D}$ kov. der. na AM

$$\tilde{D}\phi - D\phi = A\cdot\phi$$

$$\tilde{D}_{\underline{m}}\phi^{\underline{A}} - D_{\underline{m}}\phi^{\underline{A}} = A_{\underline{m}}{}^{\underline{A}}{}_{\underline{B}}\,\phi^{\underline{B}}$$

$$\tilde{D}T - DT = AT$$

$$A_{\underline{m}}T^{\underline{AB}\dots}_{\underline{CD}\dots} = A_{\underline{m}}{}^{\underline{A}}{}_{\underline{M}}\,T^{\underline{MB}\dots}_{\underline{CD}\dots} + A_{\underline{m}}{}^{\underline{B}}{}_{\underline{M}}\,T^{\underline{AM}}_{\underline{CD}} + \dots$$
$$- A_{\underline{m}}{}^{\underline{M}}{}_{\underline{C}}\,T^{\underline{AB}\dots}_{\underline{MD}} - A_{\underline{m}}{}^{\underline{M}}{}_{\underline{D}}\,T^{\underline{AB}\dots}_{\underline{CM}} - \dots$$

Př. $A_{\underline{m}}M^{\underline{A}}{}_{\underline{B}} = A_{\underline{m}}{}^{\underline{A}}{}_{\underline{N}}M^{\underline{N}}{}_{\underline{B}} - A_{\underline{m}}{}^{\underline{N}}{}_{\underline{B}}M^{\underline{A}}{}_{\underline{N}}$

$$A_{\underline{m}}M = [A_{\underline{m}}, M]$$

trivializace

$E_{\underline{A}}$ báze v AM

$$\partial E_{\underline{A}} = 0 \qquad \partial E^{\underline{A}} = 0$$

$$\phi = \phi^{\underline{A}}E_{\underline{A}}$$

$$\partial\phi = (d\phi^{\underline{A}})\otimes E_{\underline{A}}$$

$$\partial_m\phi^{\underline{A}} = (\partial\phi)^{\underline{A}}_{m} = \phi^{\underline{A}}{}_{,m}$$

$$D = \partial + A \qquad A_{\underline{m}}{}^{\underline{A}}{}_{\underline{B}}$$

$\mathbb{M}$ pseudoderivace $A^{\otimes}_{e}M$

$$\mathbb{M} : \operatorname{Sect} A^{\otimes}_{e}M \to \operatorname{Sect} A^{\otimes}_{e}M$$

$$\mathbb{M}(\phi + r\psi) = \mathbb{M}\phi + r\,\mathbb{M}\psi$$

$$\mathbb{M}(\phi\psi) = (\mathbb{M}\phi)\psi + \phi(\mathbb{M}\psi)$$

$$\mathbb{M}\,C\phi = C\,\mathbb{M}\phi$$

$$\mathbb{M}f = 0$$

$$\mathbb{M}T^{\underline{A}\dots}_{\underline{B}\dots} = M^{\underline{A}}{}_{\underline{N}}T^{\underline{N}\dots}_{\underline{B}\dots} + \dots - M^{\underline{N}}{}_{\underline{B}}T^{\underline{A}\dots}_{\underline{N}\dots} - \dots$$

$$\mathbb{M}\phi^{\underline{A}} = M^{\underline{A}}{}_{\underline{N}}\phi^{\underline{N}}$$

vztah kov. derivací

$D, \tilde{D}$ kov. der. na AM

$$\tilde{D}\phi - D\phi = A \cdot \phi$$

$$\tilde{D}_{\underline{m}}\phi^{\underline{A}} - D_{\underline{m}}\phi^{\underline{A}} = A_{\underline{m}}{}^{\underline{A}}{}_{\underline{B}}\phi^{\underline{B}}$$

$$\tilde{D}T - DT = AT$$

$$A_{\underline{m}}T^{\underline{AB}\dots}_{\underline{CD}\dots} = A_{\underline{m}}{}^{\underline{A}}{}_{\underline{M}}T^{\underline{MB}\dots}_{\underline{CD}\dots} + A_{\underline{m}}{}^{\underline{B}}{}_{\underline{M}}T^{\underline{AM}}_{\underline{CD}} + \dots$$
$$- A_{\underline{m}}{}^{\underline{M}}{}_{\underline{C}}T^{\underline{AB}\dots}_{\underline{MD}\dots} - A_{\underline{m}}{}^{\underline{M}}{}_{\underline{D}}T^{\underline{AB}\dots}_{\underline{CM}\dots} - \dots$$

Př: $A_{\underline{m}}M^{\underline{A}}{}_{\underline{B}} = A_{\underline{m}}{}^{\underline{A}}{}_{\underline{N}}M^{\underline{N}}{}_{\underline{B}} - A_{\underline{m}}{}^{\underline{N}}{}_{\underline{B}}M^{\underline{A}}{}_{\underline{N}}$

$$A_{\underline{m}}M = [A_{\underline{m}}, M]$$

trivializace

E_A báze v $\mathbb{A}^M$

$\partial E_A = 0 \qquad \partial E^A = 0$

$\phi = \phi^A E_A$

$\partial\phi = (d\phi^A) \otimes E_A$

$\partial_m \phi^A = (\partial\phi)^A_m = \phi^A{}_{|m}$

$D = \partial + \mathbb{A} \qquad \mathbb{A}_{\underline{m}}{}^{\underline{A}}{}_{\underline{B}}$

$\mathbb{M}$ pseudoderivace $\mathbb{A}_e^{\varepsilon} M$

$$\mathbb{M} : \mathcal{S}ect\, \mathbb{A}_e^{\varepsilon} M \longrightarrow \mathcal{S}ect\, \mathbb{A}_e^{\varepsilon} M$$

$$\mathbb{M}(\phi + r\psi) = \mathbb{M}\phi + r\,\mathbb{M}\psi$$

$$\mathbb{M}(\phi\psi) = (\mathbb{M}\phi)\psi + \phi(\mathbb{M}\psi)$$

$$\mathbb{M}\, G\phi = G\,\mathbb{M}\phi$$

$$\mathbb{M} f = 0$$

$$\mathbb{M} T^{\underline{A}\dots}_{\underline{B}\dots} = M^{\underline{A}}{}_{\underline{N}} T^{\underline{N}\dots}_{\underline{B}\dots} + \dots - M^{\underline{N}}{}_{\underline{B}} T^{\underline{A}\dots}_{\underline{N}\dots} - \dots$$

$$\mathbb{M}\phi^{\underline{A}} = M^{\underline{A}}{}_{\underline{N}}\phi^{\underline{N}}$$