Reálný vekt. bundle s SO-symetrií

A $\qquad H_{\underline{AB}}$

nedegener. $\Rightarrow$ exist $H^{-1\,\underline{AB}}$

symetrické $\qquad H_{\underline{AB}} = H_{\underline{BA}}$

zvedání a snižování indexů

$$H^{\underline{AB}} = H^{-1\,\underline{AB}}$$

transpozice

$$\phi^A \rightarrow \phi^T{}_A = H_{\underline{AB}}\,\phi^B$$

$$\phi_A \rightarrow \phi^{TA} = H^{\underline{AB}}\,\phi_B$$

$$X^A{}_B \rightarrow X^{TA}{}_B = H^{AM} H_{BN}\, X^N{}_M$$

$$(\phi, \psi) = \phi^A \psi^B H_{\underline{AB}}$$

$$= \phi^T \cdot \psi = \phi \cdot \psi^T$$

$$(\phi, X \cdot \psi) = (X^T \cdot \phi, \psi)$$

signatura metriky

$$(\underbrace{- \cdots -}_{m}\ \underbrace{+ \cdots +}_{p})$$

$A M$

hladká metrika $H \in \mathrm{Sect}\, A^0_2 M$

lokálně (ultralokálně) isomorfní metr. na A

vektorový SO-bundle

metrický bundle

Def Lokální Kalibrační SO-transformace

R ultralokální zobr. na Sect AM

$$R \in \mathrm{Sect}\, A^1_1 M$$

$$\phi \rightarrow \tilde{\phi} = R \cdot \phi$$

- $(\tilde{\phi}, \tilde{\psi}) = (\phi, \psi)$
- zachování orientaci

rozšíření na $A^q_r M$

$$X^{A\dots}_{B\dots} \longrightarrow \tilde{X}^{A\dots}_{B\dots} = R^A{}_K \cdots R^{-1N}{}_B \cdots X^{M\dots}_{N\dots}$$

$$\widetilde{(\phi \otimes \psi)} = \tilde{\phi} \otimes \tilde{\psi} \qquad C\tilde{X} = \widetilde{CX}$$

$$\tilde{H}_{AB} = H_{AB} \qquad H_{AB} = R^M{}_A R^N{}_B H_{MN} \qquad R^T \cdot R = \mathbb{1}$$

$$\tilde{\varepsilon}_{A_1 \dots A_D} = \varepsilon_{A_1 \dots A_D} \qquad \det R = 1$$

Reálný vekt. bundle
s SO-symetrií

A $\quad H_{\underline{AB}}$

nedegener. $\Rightarrow$ exist $H^{-1\,\underline{AB}}$

symetrické $\quad H_{\underline{AB}} = H_{\underline{BA}}$

zvedání a snižování indexů

$$H^{\underline{AB}} = H^{-1\,\underline{AB}}$$

transposice

$$\phi^{\underline{A}} \to \phi^{T}{}_{\underline{A}} = H_{\underline{AB}}\,\phi^{\underline{B}}$$

$$\phi_{\underline{A}} \to \phi^{T\underline{A}} = H^{\underline{AB}}\,\phi_{\underline{B}}$$

$$\chi^{\underline{A}}{}_{\underline{B}} \to \chi^{T\underline{A}}{}_{\underline{B}} = H^{\underline{AM}}\,H_{\underline{BN}}\,\chi^{\underline{N}}{}_{\underline{M}}$$

$$(\phi, \psi) = \phi^{A} \psi^{B} H_{\underline{AB}}$$

$$= \phi^{T} \cdot \psi = \phi \cdot \psi^{T}$$

$$(\phi, \chi \cdot \psi) = (\chi^{T} \cdot \phi, \psi)$$

signatura metriky

$$(\underbrace{- \cdots -}_{m} \underbrace{+ \cdots +}_{p})$$

$\mathbb{A}M$

hladké metriky $H \in \mathrm{Sect}\, \mathbb{A}^0_2 M$

lokální (ultralokální)

isomorfní metr. na $\mathbb{A}$

vektorový SO-bundle

metrický bundle

Def Kalibrační SO-transformace

R ultralok lin zobr na $\mathrm{Sect}\, AM$

$$R \in \mathrm{Sect}\, A^1_1 M$$

$$\phi \to \tilde{\phi} = R \cdot \phi$$

- $(\tilde{\phi}, \tilde{\psi}) = (\phi, \psi)$
- zachovává orientaci

Def Lokální Kalibrační SO-transformace

R ultralok lin zobr na Sect AM

$R \in \mathrm{Sect}\, A^1_1 M$

$\phi \to \tilde{\phi} = R \cdot \phi$

- $(\tilde{\phi}, \tilde{\psi}) = (\phi, \psi)$
- zachovává orientaci

rozšíření na $A^q_p M$

$X^{A\dots}_{B\dots} \longrightarrow \tilde{X}^{A\dots}_{B\dots} = R^{A}_{M} \dots R^{-1N}_{B} \dots X^{M\dots}_{N\dots}$

$\widetilde{(\phi \otimes \psi)} = \tilde{\phi} \otimes \tilde{\psi} \qquad C\tilde{X} = \widetilde{CX}$

$\tilde{H}_{AB} = H_{AB} \qquad H_{AB} = R^{M}_{A} R^{N}_{B} H_{MN} \qquad R^T \cdot R = \mathbb{1}$

$\tilde{\varepsilon}_{A_1 \dots A_D} = \varepsilon_{A_1 \dots A_D} \qquad \det R = 1$

Def Lokální kalibrační SO-transformace

R ultralokální zobr. na Sect $\mathcal{A}M$

$R \in \mathrm{Sect}\, \mathcal{A}^1_1 M$

$\phi \to \tilde{\phi} = R \cdot \phi$

- $(\tilde{\phi}, \tilde{\psi}) = (\phi, \psi)$
- zachovává orientaci

rozšíření na $\mathcal{A}^k_l M$

$\chi^{A\ldots}_{B\ldots} \longrightarrow \tilde{\chi}^{A\ldots}_{B\ldots} = R^A{}_C \cdots R^{-1N}{}_B \cdots \chi^{M\ldots}_{N\ldots}$

$\widetilde{(\phi \otimes \psi)} = \tilde{\phi} \otimes \tilde{\psi} \qquad C\tilde{\chi} = \widetilde{C\chi}$

$\tilde{H}_{AB} = H_{AB} \qquad H_{AB} = R^M{}_A R^N{}_B H_{MN} \qquad R^T R = \mathbb{1}$

$\tilde{\varepsilon}_{A_1 \ldots A_D} = \varepsilon_{A_1 \ldots A_D} \qquad \det R = 1$

Lokální kalibr. grupa

Lokální kalibr. Lieova alg.

$R_\alpha = \mathbb{1} + \alpha \Omega + O(\alpha^2)$

$\Omega \in \mathrm{Sect}\, \mathcal{A}^1_1 M$

$[\Omega, \Delta] = \Omega \cdot \Delta - \Delta \cdot \Omega$

$\Omega^M{}_B H_{MB} + \Omega^M{}_B H_{AM} = 0$

$\Omega_{BA} + \Omega_{AB} = 0 \qquad \Omega + \Omega^T = 0$

$\Omega^T = -\Omega$

$R_\alpha = \exp(\alpha \Omega)$

Trivializace SO-bundlu

Def triv. E_A je konz. s metr. strukturou j.t.:

$H_{AB} = (E_A, E_B) = \text{konst}$ (na M)

Př. volba ortonormální báze

$$H_{AB} = \begin{bmatrix} -1 & & & & 0 \\ & -1 & & & \\ & & \ddots & & \\ & & & +1 & \\ 0 & & & & +1 \end{bmatrix}$$

Př. signatura $(-\ldots-+\ldots+)$ $\quad m = p$

$$H_{AB} = \begin{bmatrix} 0 & -1 & & \\ 1 & 0 & & \\ & & 0 & -1 \\ & & 1 & 0 \\ & & & & \ddots \end{bmatrix}$$

změna trivializace

$E_A \to \tilde{E}_A = R \cdot E_A$

$H_{\tilde{A}\tilde{B}} = (\tilde{E}_A, \tilde{E}_B) = (E_A, E_B) = H_{AB}$

$\phi \in \mathcal{A}M \qquad \phi = \phi^A E_A = \phi^{\tilde{A}} \tilde{E}_A$

$\phi^A \qquad \phi^{\tilde{A}}$

pasivní kalibr. transf.

$\phi^A = R^A{}_{\tilde{M}} \phi^{\tilde{M}} \qquad \tilde{E}_{\tilde{M}} = E_A R^A{}_{\tilde{M}}$

$H_{\tilde{A}\tilde{B}} = R^M{}_{\tilde{A}} R^N{}_{\tilde{B}} H_{MN} = H_{AB}$

Lokální kalibr. grupa

Lokální kalibr. Lieova alg

$$R_\alpha = \mathbb{1} + \alpha\Omega + \mathcal{O}(\alpha^2)$$

$$\Omega \in \mathrm{Sect}\, \mathbf{A}^1_1 M$$

$$[\Omega, \Delta] = \Omega \cdot \Delta - \Delta \cdot \Omega$$

$$\Omega^{\underline{M}}{}_{\underline{A}} H_{\underline{MB}} + \Omega^{\underline{M}}{}_{\underline{B}} H_{\underline{AM}} = 0$$

$$\Omega_{\underline{BA}} + \Omega_{\underline{AB}} = 0 \qquad \Omega + \Omega^T = 0$$

$$\Omega^T = -\Omega$$

$$R_\alpha = \exp(\alpha\Omega)$$

Trivializace SO-bundlu

Def triv. E_A je kompat. s metr. strukturou j.-l.:

$$H_{AB} = (E_A, E_B) = \text{konst}$$

(na M)

Př.: volba ortonormální báze

$$H_{AB} = \begin{bmatrix} -1 & & & & & \\ & -1 & & & 0 & \\ & & \ddots & & & \\ & & & +1 & & \\ & 0 & & & +1 & \\ & & & & & \ddots \end{bmatrix}$$

Př. signatura $(- \dots - + \dots +)$ $\quad m = p$

$$H_{AB} = \begin{bmatrix} 0 & 1 & & & \\ 1 & 0 & & & \\ & & 0 & 1 & \\ & & 1 & 0 & \\ & & & & \ddots \end{bmatrix}$$

změna trivializace

$$E_A \to \tilde{E}_A = R \cdot E_A$$

$$H_{\tilde{A}\tilde{B}} = (\tilde{E}_A, \tilde{E}_B) = (E_A, E_B) = H_{AB}$$

$$\phi \in AM \qquad \phi = \phi^A E_A = \phi^{\tilde{A}} \tilde{E}_A$$

$$\phi^A \qquad \phi^{\tilde{A}}$$

pasivní kalibr. transf.

$$\phi^A = R^A{}_{\tilde{M}} \phi^{\tilde{M}} \qquad \tilde{E}_{\tilde{A}} = E_A R^A{}_{\tilde{A}}$$

$$H_{\tilde{A}\tilde{B}} = R^M{}_{\tilde{A}} R^N{}_{\tilde{B}} H_{MN} = H_{AB}$$

Kon. der. na SO-bundlech
(SO-kov. derivace)

Def. AM SO-bundle
D kov. derivace

$$DH = 0$$
$$\Updownarrow$$
$$D_\xi(\phi,\psi) = (D_\xi\phi,\psi) + (\phi, D_\xi\psi)$$

Lemma
∂ souř. trivializace E_n konz. s H

$$\partial H = 0$$

$$\partial_m H_{\underline{AB}} = \partial_m (H_{MN} E^M_{\underline{A}} E^N_{\underline{B}}) = 0$$

Věta
D je konz. s H dáná
vekt. pot. $A_m{}^k{}_l$ vůči ∂

$$D = \partial + A_m$$
$$\Downarrow$$
$$A_m^T = -A_m$$

důk.

$$DH = \partial H = 0$$

$$\Rightarrow 0 = A_m H_{\underline{AB}} = -A_m{}^k{}_{\underline{A}} H_{\underline{kB}} - A_m{}^k{}_{\underline{B}} H_{\underline{Ak}}$$

$$A_m + A_m^T = 0$$

Věta
D je konz. s H
$F_{mn}{}^k{}_l$ tenzor kř. D
$$\Downarrow$$
$$F_{mn}^T = -F_{mn}$$

důk:

$$D H = 0 \Rightarrow D D H = 0$$

$$\Rightarrow 0 = [D_m D_n - D_n D_m + T_{mn}^k D_k] H_{\underline{AB}} = F_{mn} H_{\underline{AB}}$$

$$\Rightarrow F_{mn} + F_{mn}^T = 0$$

Kov. der. na SO-bundlech

(SO-kov. derivace)

Def. AM SO-Bundle

D kov. derivace

$$DH = 0$$

$$\Updownarrow$$

$$D_\xi(\phi,\psi) = (D_\xi\phi,\psi) + (\phi, D_\xi\psi)$$

Lemma ∂ ortn. trivializace E_A kons. s H

$$\partial H = 0$$

$$\partial_{\underline{m}} H_{\underline{AB}} = \partial_{\underline{m}}\left(H_{MN} E^M_{\underline{A}} E^N_{\underline{B}}\right) = 0$$

Věta

D je transz. o H dána vekt. pot $A_{\underline{m}}{}^{\underline{k}}{}_{\underline{l}}$ vůči ∂

$$D = \partial + A_{\underline{m}}$$

$$\Downarrow$$

$$A_{\underline{m}}^T = - A_{\underline{m}}$$

důk.

$$DH = \partial H = 0$$

$$\Rightarrow 0 = A_{\underline{m}} H_{\underline{AB}} = - A_{\underline{m}}{}^{\underline{K}}{}_{\underline{A}} H_{\underline{KB}} - A_{\underline{m}}{}^{\underline{K}}{}_{\underline{B}} H_{\underline{AK}}$$

$$A_{\underline{m}} + A_{\underline{m}}^T = 0$$

Věta

D je konz. o H

$F_{\underline{mn}}{}^{\underline{k}}{}_{\underline{l}}$ tenzor síl. D

$\Downarrow$

$$F_{\underline{mn}}^{T} = -F_{\underline{mn}}$$

důk:

$$D \cdot H = 0 \Rightarrow DDH = 0$$

$$\Rightarrow 0 = [D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}} + T_{\underline{mn}}{}^{\underline{k}} D_{\underline{k}}] H_{\underline{AB}} = F_{\underline{mn}} H_{\underline{AB}}$$

$$\Rightarrow F_{\underline{mn}} + F_{\underline{mn}}^{T} = 0$$

Def Kalibr. transf kov. der.

mějme transf R

$$\phi \to \tilde\phi = R\cdot\phi$$

na kov. der transf rozšíříme

$$\tilde D\tilde\phi = \widetilde{D\phi}$$

Věta

$$\tilde D = D + \Lambda \qquad \Lambda = -(DR)\cdot R^{-1}$$

důkaz.

$$\tilde D(R\cdot\phi) = D(R\cdot\phi) + \Lambda\cdot R\cdot\phi$$
$$= (DR)\cdot\phi + \underline{R\cdot(D\phi)} + \Lambda\cdot R\cdot\phi = \underline{R\cdot(D\phi)}$$
$$\Lambda\cdot R = -DR$$

Věta $\quad A_{\underline m}{}^{\underline k}{}_{\underline l}$

$D, \tilde D$ souvis kal. transf

vůči stejné ∂ dány vekt pot $A_{\underline m}, \tilde A_{\underline m}$

$$\tilde A_{\underline m} = R\cdot A\cdot R^{-1} - (\partial R)\cdot R^{-1}$$
$$= A - (DR)\cdot R^{-1} = A - R^{-1}\cdot(\tilde D R)$$

důk:

$$D\phi = \partial\phi + A\cdot\phi \qquad \tilde D\phi = \partial\phi + \tilde A\cdot\phi$$
$$\tilde D\phi = D\phi - (DR)\cdot R^{-1}\phi$$
$$= \partial\phi + (A - (DR)\cdot R^{-1})\cdot\phi$$
$$\tilde A = A - (DR)\cdot R^{-1}$$
$$= \cancel{A} - (\partial R)\cdot R^{-1} - \cancel{A\cdot R\cdot R^{-1}} + R\cdot A\cdot R^{-1}$$
$$= R\cdot A\cdot R^{-1} - (\partial R)\cdot R^{-1}$$

$$D \leftrightarrow \tilde D \qquad A \leftrightarrow \tilde A \qquad R \leftrightarrow R^{-1}$$
$$A = \tilde A - (\tilde D R^{-1})\cdot R \qquad \tilde A = A + (\tilde D R^{-1})\cdot R = \tilde A - R^{-1} D R$$

Věta

$F, \tilde F$ tenz. křiv $D, \tilde D$

$$\Downarrow \quad \tilde F = R\cdot F\cdot R^{-1}$$
$$\tilde F_{\underline{mn}}{}^{\underline k}{}_{\underline l} = R^{\underline k}{}_{\underline M}\, R^{-1\,\underline N}{}_{\underline l}\, F_{\underline{mn}}{}^{\underline M}{}_{\underline N}$$

$$\tilde D\tilde\phi = \widetilde{D\phi}$$
$$\tilde D\tilde D\tilde\phi = \widetilde{DD\phi}$$
$$\tilde F\cdot\tilde\phi = \widetilde{F\cdot\phi}$$
$$\tilde F\cdot R\cdot\phi = R\cdot F\cdot\phi$$
$$\tilde F = R\cdot F\cdot R^{-1}$$

Def Kalibr. transf kov. der.

miejne transf R

$$\phi \to \tilde{\phi} = R \cdot \phi$$

na kov. der transf rozšíříme

$$\tilde{D}\tilde{\phi} = \widetilde{D\phi}$$

Věta

$$\tilde{D} = D + \Lambda \qquad \Lambda = -(DR) \cdot R^{-1}$$

důkaz.

$$\tilde{D}(R \cdot \phi) = D(R \cdot \phi) + \Lambda \cdot R \cdot \phi$$

$$= (DR) \cdot \phi + \underline{R \cdot (D\phi)} + \Lambda \cdot R \cdot \phi = \underline{R \cdot (D\phi)}$$

$$\Lambda \cdot R = -DR$$

Věta

$D, \tilde{D}$ souvis kal. transf. $\quad A_{m}{}^{k}{}_{l}$

vůči stejné ∂ dány vekt. pot. $A_m, \tilde{A}_m$

$$\tilde{A}_m = R \cdot A \cdot R^{-1} - (\partial R) \cdot R^{-1}$$

$$= A - (DR) \cdot R^{-1} = A - R^{-1} \cdot (\tilde{D} R)$$

důk:

$$D\phi = \partial\phi + A \cdot \phi \quad \tilde{D}\phi = \partial\phi + \tilde{A} \cdot \phi$$

$$\tilde{D}\phi = D\phi - (DR) \cdot R^{-1} \cdot \phi$$

$$= \partial\phi + (A - (DR) \cdot R^{-1}) \cdot \phi$$

$$\tilde{A} = A - (DR) \cdot R^{-1}$$

$$= \cancel{A} - (\partial R) \cdot R^{-1} - \cancel{A \cdot R \cdot R^{-1}} + R \cdot A \cdot R^{-1}$$

$$= R \cdot A \cdot R^{-1} - (\partial R) \cdot R^{-1}$$

$$D \leftrightarrow \tilde{D} \quad A \leftrightarrow \tilde{A} \quad R \leftrightarrow R^{-1}$$

$$A = \tilde{A} - (\tilde{D} R^{-1}) \cdot R \qquad \tilde{A} = A + (\tilde{D} R^{-1}) \cdot R = \tilde{A} - R^{-1} D R$$

Věta

$F, \tilde{F}$ tenz. kvůli $D, \tilde{D}$

$$\Downarrow \quad \tilde{F} = R \cdot F \cdot R^{-1}$$

$$\tilde{F}_{\underline{m}}{}^{\underline{K}}{}_{\underline{L}} = R^{\underline{K}}{}_{\underline{M}} \, R^{-1\,\underline{N}}{}_{\underline{L}} \, F_{\underline{m}}{}^{\underline{M}}{}_{\underline{N}}$$

$$\tilde{D} \tilde{\Phi} = \widetilde{D \Phi}$$

$$\tilde{D} \tilde{D} \tilde{\Phi} = \widetilde{D D \Phi}$$

$$\tilde{F} \cdot \tilde{\Phi} = \widetilde{F \cdot \Phi}$$

$$\tilde{F} \cdot R \cdot \Phi = R \cdot F \cdot \Phi$$

$$\tilde{F} = R \cdot F \cdot R^{-1}$$

Komplexní vekt. bundl s hermitovskou strukturou (U-bundly)

$\mathbb{E}_L = \mathbb{E}$ komplex. vekt. pr. ϕ^{A}

$\mathbb{E}_R = \bar{\mathbb{E}}$ sdružený kom. vekt. pr. $\phi^{\bar{A}}$

$\bar{\ } : \mathbb{E}_L \longleftrightarrow \mathbb{E}_R$

$\bar{\ }$ antilineární $\overline{\alpha\phi} = \alpha^* \bar{\phi}$

$\bar{\bar{\phi}} = \phi$ $\quad \phi^{A \dots \bar{B} \dots}_{C \dots \bar{D} \dots}$

$$\mathbb{E}^{k_L k_R}_{l_L l_R} = \underbrace{\mathbb{E}_L \otimes \dots \otimes}_{k_L} \underbrace{\mathbb{E}^*_L \otimes \dots \otimes}_{l_L} \underbrace{\mathbb{E}_R \otimes \dots \otimes}_{k_R} \underbrace{\mathbb{E}^*_R \otimes \dots}_{l_R}$$

antilin. operace

$$a : \mathbb{E}M \to \mathbb{E}M$$

$$(a\phi)^{A} = A^{A}{}_{\bar{M}} \bar{\phi}^{\bar{M}}$$

Skalární součin

$\langle\ ,\ \rangle_L \quad \langle\ ,\ \rangle_R$

generov. pro herm. str. $h_{\bar{A}B}$

$$\langle \phi_1, \phi_2 \rangle_L = \bar{\phi}^{\bar{A}}_1 \phi^{B}_2 h_{\bar{A}B}$$

$$\langle \psi_1, \psi_2 \rangle_R = \bar{\psi}^{A}_1 \psi^{\bar{B}}_2 \bar{h}_{A\bar{B}}$$

1) h nedegener. $\quad h^{-1 A\bar{B}}$

2) $\bar{h}_{A\bar{B}} = h_{\bar{B}A}$

3) h poz. def. $\langle \phi, \phi \rangle > 0 \quad \phi \neq 0$

$\dagger : \mathbb{E}^{10}_{00} \leftrightarrow \mathbb{E}^{00}_{10}$

$\mathbb{E} \leftrightarrow \mathbb{E}^*$

$$(\phi^\dagger)_{\bar{A}} = \bar{\phi}^{\bar{M}} h_{\bar{M}A}$$

$$\phi^{\dagger\dagger} = \phi$$

$$\langle \phi_1, \phi_2 \rangle_L = \phi_1^\dagger \cdot \phi_2$$

$\dagger : \mathbb{E}^{10}_{10} \to \mathbb{E}^{10}_{10}$

$$(\mathcal{X}^\dagger)^{A}{}_{B} = h^{-1\bar{N}A} h_{\bar{B}B} \bar{\mathcal{X}}^{\bar{B}}{}_{\bar{N}}$$

$$\langle \phi_1, \mathcal{X} \cdot \phi_2 \rangle_L = \langle \mathcal{X}^\dagger \cdot \phi_1, \phi_2 \rangle_L$$

$\dagger : \mathbb{E}^{01}_{00} \leftrightarrow \mathbb{E}^{00}_{01}$

$\bar{\mathbb{E}} \leftrightarrow \bar{\mathbb{E}}^*$

$$(\psi^\dagger)_{\bar{A}} = \bar{\psi}^{M} \bar{h}_{M\bar{A}}$$

$$\psi^{\dagger\dagger} = \psi$$

$$\langle \psi_1, \psi_2 \rangle_R = \psi_1^\dagger \cdot \psi_2$$

$\dagger \ \mathbb{E}^{01}_{01} \to \mathbb{E}^{01}_{01}$

$$\langle \psi_1, \mathcal{X} \cdot \psi_2 \rangle_R = \langle \mathcal{X}^\dagger \cdot \psi_1, \psi_2 \rangle$$

Komplexní vekt. bundl s hermitovskou strukturou (U-bundly)

$\mathbb{E}_L = \mathbb{E}$ komplex. vekt. pr. $\phi^{\underline{A}}$

$\mathbb{E}_R = \overline{\mathbb{E}}$ sdružený kom. vekt. pr. $\phi^{\bar{\underline{A}}}$

$\overline{} : \mathbb{E}_L \longleftrightarrow \mathbb{E}_R$

$\overline{}$ antilineární $\overline{\alpha\phi} = \alpha^* \overline{\phi}$

$\overline{\overline{\phi}} = \phi$ $\qquad \phi^{\underline{A}\dots\bar{\underline{B}}\dots}_{\underline{C}\dots\bar{\underline{D}}\dots}$

$$\mathbb{E}^{k_L\, k_R}_{l_L\, l_R} = \underbrace{\mathbb{E}_L \otimes \dots \otimes}_{k_L} \underbrace{\mathbb{E}^*_L \otimes \dots \otimes}_{l_L} \underbrace{\mathbb{E}_R \otimes \dots \otimes}_{k_R} \underbrace{\mathbb{E}^*_R \otimes \dots}_{l_R}$$

antilin. operace

$$\mathcal{A} : \mathbb{E}M \to \bar{\mathbb{E}}M$$

$$(\mathcal{A}\phi)^{\underline{A}} = A^{\underline{A}}{}_{\bar{\underline{n}}} \bar{\phi}^{\bar{\underline{n}}}$$

Skalární součin

$$\langle\, , \rangle_L \qquad \langle\, , \rangle_R$$

generov. pomo herm. str. $h_{\bar{\underline{A}}\underline{B}}$

$$\langle \phi_1, \phi_2 \rangle_L = \bar{\phi}_1^{\bar{\underline{A}}} \phi_2^{\underline{B}} h_{\bar{\underline{A}}\underline{B}}$$

$$\langle \psi_1, \psi_2 \rangle_R = \bar{\psi}_1^{\underline{A}} \psi_2^{\bar{\underline{B}}} \bar{h}_{\underline{A}\bar{\underline{B}}}$$

1) h nedegener. $\quad h^{-1\bar{\underline{A}}\underline{B}}$

2) $\bar{h}_{\underline{A}\bar{\underline{B}}} = h_{\bar{\underline{B}}\underline{A}}$

3) h poz. def. $\quad \langle \phi, \phi \rangle_L > 0 \quad \phi \neq 0$

$$\dagger : \mathbb{E}^{10}_{00} \leftrightarrow \mathbb{E}^{00}_{10}$$

$$\mathbb{E} \leftrightarrow \mathbb{E}^*$$

$$(\phi^\dagger)_{\underline{A}} = \bar{\phi}^{\underline{\bar{M}}} h_{\underline{\bar{M}}\underline{A}}$$

$$\phi^{\dagger\dagger} = \phi$$

$$\langle \phi_1, \phi_2 \rangle_L = \phi_1^\dagger \cdot \phi_2$$

$$\dagger : \mathbb{E}^{10}_{10} \to \mathbb{E}^{10}_{10}$$

$$(\mathcal{X}^\dagger)^{\underline{A}}{}_{\underline{B}} = h^{-1\,\underline{\bar{N}}\underline{A}} h_{\underline{\bar{M}}\underline{B}} \bar{\mathcal{X}}^{\underline{\bar{M}}}{}_{\underline{\bar{N}}}$$

$$\langle \phi_1, \mathcal{X} \cdot \phi_2 \rangle_L = \langle \mathcal{X}^\dagger \phi_1, \phi_2 \rangle_L$$

$$\dagger : \mathbb{E}^{01}_{00} \leftrightarrow \mathbb{E}^{00}_{01}$$

$$\bar{\mathbb{E}} \leftrightarrow \bar{\mathbb{E}}^*$$

$$(\psi^\dagger)_{\underline{\bar{A}}} = \bar{\psi}^{\underline{M}} \bar{h}_{\underline{M}\underline{\bar{A}}}$$

$$\psi^{\dagger\dagger} = \psi$$

$$\langle \psi_1, \psi_2 \rangle_R = \psi_1^\dagger \cdot \psi_2$$

$$\dagger \; \mathbb{E}^{01}_{01} \to \mathbb{E}^{01}_{01}$$

$$\langle \psi_1, \mathcal{X} \cdot \psi_2 \rangle_R = \langle \mathcal{X}^\dagger \psi_1, \psi_2 \rangle$$

Komplexní vekt bundl s hermitovskou strukturou (U- bundy)

$\mathbb{E}_L M$ $\updownarrow$ $\mathbb{E}_R M$ — $h_{\bar{A}B}$ hladká herm. strukt.

$\mathbb{E}^{\mathfrak{e}_L \mathfrak{e}_R}_{\mathfrak{e}_L \mathfrak{e}_R} M$

Unitární kalibr. transf

ultralok. lin operace

$U: \mathrm{Sect}\, \mathbb{E}_L M \to \mathrm{Sect}\, \mathbb{E}_L M$

$U \in \mathrm{Sect}\, \mathbb{E}^{10}_{10} M$

$\phi \to \tilde{\phi} = U \cdot \phi \qquad \phi \in \mathrm{Sect}\, \mathbb{E}_L M$

$\bar{U} \in \mathrm{Sect}\, \mathbb{E}^{01}_{01} M$

$\psi \to \tilde{\psi} = \bar{U} \cdot \psi \qquad \psi \in \mathrm{Sect}\, \mathbb{E}_R M$

$\langle \tilde{\phi}_1, \tilde{\phi}_2 \rangle_L = \langle \phi_1, \phi_2 \rangle_L \qquad \langle \tilde{\psi}_1, \tilde{\psi}_2 \rangle_R = \langle \psi_1, \psi_2 \rangle_R$

$h_{\underline{\bar{B}A}} = \tilde{h}_{\underline{\bar{B}A}} = \bar{U}^{-1 \bar{N}}{}_{\bar{B}}\, U^{-1 M}{}_{A}\, h_{\bar{N}M}$

$U^\dagger \cdot U = \mathbb{1} = U \cdot U^\dagger$

Lokální kal. Lieova algebra

$U_\alpha = \mathbb{1} + i\alpha u + O(\alpha^2)$

$iu - iu^\dagger = 0$

$u^\dagger = u$

$U_\alpha = \exp(i\alpha u)$

Trivializ. konzist. s herm. str

$E_A \qquad \bar{E}_A$

$\langle E_A, E_B \rangle_L = \mathrm{konst} \quad \langle \bar{E}_A, \bar{E}_B \rangle_R = \mathrm{konst}$

Př. ortonom. báze

$$\langle E_A, E_B \rangle_L = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

Komplexní vekt. bundl s hermitouskou strukturou

(U-bundy)

$\mathbb{E}_L M$

$\updownarrow$

$\mathbb{E}_R M$

$h_{\underline{\bar{A} B}}$ hladká herm. strukt.

$\mathbb{E}^{\ell_L \ell_R}_{\ell_L \ell_R} M$

Unitární kalibr. transf

ultralok. lin operace

$$U : \mathrm{Sect}\, \mathbb{E}_L M \to \mathrm{Sect}\, \mathbb{E}_L M$$

$$U \in \mathrm{Sect}\, \mathbb{E}^{10}_{10} M$$

$$\phi \to \tilde{\phi} = U \cdot \phi \qquad \phi \in \mathrm{Sect}\, \mathbb{E}_L M$$

$$\bar{U} \in \mathrm{Sect}\, \mathbb{E}^{01}_{01} M$$

$$\psi \to \tilde{\psi} = \bar{U} \cdot \psi \qquad \psi \in \mathrm{Sect}\, \mathbb{E}_R M$$

$$\langle \tilde{\phi}_1, \tilde{\phi}_2 \rangle_L = \langle \phi_1, \phi_2 \rangle_L \qquad \langle \tilde{\psi}_1, \tilde{\psi}_2 \rangle_R = \langle \psi_1, \psi_2 \rangle_R$$

$$h_{\underline{\bar{B}A}} = \tilde{h}_{\underline{\bar{B}A}} = \bar{U}^{-1\underline{\bar{N}}}{}_{\underline{\bar{B}}}\, U^{-1\underline{M}}{}_{\underline{A}}\, h_{\underline{\bar{N}M}}$$

$$U^{\dagger} \cdot U = \mathbb{1} = U \cdot U^{\dagger}$$

Lokální Zal. Lieova algebra

$$U_\alpha = \mathbb{1} + i\alpha u + \mathcal{O}(\alpha^2)$$

$$iu - iu^+ = 0$$

$$u^+ = u$$

$$U_\alpha = \exp(i\alpha u)$$

Trivializ. konzist. s herm. str

$$E_A \qquad \bar{E}_A$$

$$\langle E_A, E_B \rangle_L = \text{konst} \quad \langle \bar{E}_A, \bar{E}_B \rangle_R = \text{konst}$$

Př. ortonom. báze

$$\langle E_A, E_B \rangle_L = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

Kov. der. konzistentní s hermitovskou strukturou

Def rozšíření D z EM na $\bar{E}M$

$$\bar{D}\bar{\phi} = \overline{D\phi}$$

společná der na $E^{\mathfrak{k}_1 \mathfrak{k}_2}_{\mathfrak{l}_1 \mathfrak{l}_2} M$

$$\mathsf{D} = D \oplus \bar{D}$$

celková der je reálná

$$\bar{\mathsf{D}} = \mathsf{D}$$

Def Kov. der D je kompat s hermit strukt.

$$\mathsf{D} h = 0$$

$$D_\xi \langle \phi_1, \phi_2 \rangle_L = \langle D_\xi \phi_1, \phi_2 \rangle_L + \langle \phi_1, D_\xi \phi_2 \rangle_L$$

Lem

triviál. kov. a herm str

$$\partial h = 0$$

Rozšíření pseudoder. na $E^{\mathfrak{k}_1 \mathfrak{k}_2}_{\mathfrak{l}_1 \mathfrak{l}_2} M$

$$\bar{M} \bar{\mathcal{X}} = \overline{M \mathcal{X}}$$

společné působení

$$\mathsf{M} = M \oplus \bar{M}$$

$$\bar{\mathsf{M}} = \mathsf{M}$$

$$M \phi^A = M^A{}_B \phi^B \qquad \bar{M} \psi^{\bar{A}} = \bar{M}^{\bar{A}}{}_{\bar{B}} \psi^{\bar{B}}$$

$$\begin{aligned} M &\rightarrow \\ \bar{M} &\rightarrow \end{aligned} \quad \mathsf{M} \mathcal{X}^{A \ldots \bar{B} \ldots}_{C \ldots \bar{D} \ldots} = M^A{}_K \mathcal{X}^{K \ldots \bar{B} \ldots}_{C \ldots \bar{D} \ldots} + \ldots - M^K{}_C \mathcal{X}^{A \ldots \bar{B} \ldots}_{K \ldots \bar{D} \ldots} - \ldots$$

$$+ \bar{M}^{\bar{B}}{}_{\bar{K}} \mathcal{X}^{A \ldots \bar{K} \ldots}_{C \ldots \bar{D} \ldots} + \ldots - \bar{M}^{\bar{K}}{}_{\bar{D}} \mathcal{X}^{A \ldots \bar{B} \ldots}_{C \ldots \bar{K} \ldots} - \ldots$$

Kov. der. konzistentní s hermitovskou strukturou

Def rozšíření D z $\mathbb{E}M$ na $\bar{\mathbb{E}}M$

$$\bar{D}\bar{\phi} = \overline{D\phi}$$

společná der na $\mathbb{E}^{k_1 k_2}_{l_1 l_2} M$

$$\mathcal{D} = D \oplus \bar{D}$$

celková der je reálná:

$$\bar{\mathcal{D}} = \mathcal{D}$$

Def Kov. odv. D je združ. s hermit. strukt.

$$Dh = 0$$

$$D_\xi \langle \phi_1, \phi_2 \rangle_L = \langle D_\xi \phi_1, \phi_2 \rangle_L + \langle \phi_1, D_\xi \phi_2 \rangle_L$$

Lem

triviál. kov. z hol. str.

$$\partial h = 0$$

Rozšíření pseudoder. na $E^{q_1 q_2}_{l_1 l_2} M$

$$\bar{\mathbb{M}}\, \bar{X} = \overline{\mathbb{M} X}$$

společné působení

$$\mathbb{M} = \mathbb{M} \oplus \bar{\mathbb{M}}$$

$$\mathbb{M} = \bar{\mathbb{M}}$$

$$\mathbb{M}\phi^{\underline{A}} = M^{\underline{A}}{}_{\underline{B}}\,\phi^{\underline{B}} \qquad \bar{\mathbb{M}}\psi^{\bar{A}} = \bar{M}^{\bar{A}}{}_{\bar{B}}\,\psi^{\bar{B}}$$

$$\mathbb{M}\,\chi^{\underline{A}\ldots\bar{B}\ldots}_{\underline{C}\ldots\bar{D}\ldots} = M^{\underline{A}}{}_{\underline{K}}\,\chi^{\underline{K}\ldots\bar{B}\ldots}_{\underline{C}\ldots\bar{D}\ldots} + \ldots - M^{\underline{K}}{}_{\underline{C}}\,\chi^{\underline{A}\ldots\bar{B}\ldots}_{\underline{K}\ldots\bar{D}\ldots} - \ldots$$ ← působení $\mathbb{M}$

$$+ \bar{M}^{\bar{B}}{}_{\bar{K}}\,\chi^{\underline{A}\ldots\bar{K}\ldots}_{\underline{C}\ldots\bar{D}\ldots} + \ldots - \bar{M}^{\bar{K}}{}_{\bar{D}}\,\chi^{\underline{A}\ldots\bar{B}\ldots}_{\underline{C}\ldots\bar{K}\ldots} - \ldots$$ ← působení $\bar{\mathbb{M}}$

$D \quad \tilde{D}$ kovar. s heren. str.

$D - \tilde{D}$ pseudoder. gener.

rozdíl tenz $iA_{\underline{m}}{}^{\underline{k}}{}_{\underline{l}}$

$$D_{\underline{m}}\phi^{\underline{A}} - \tilde{D}_{\underline{m}}\phi^{\underline{A}} = iA_{\underline{m}}{}^{\underline{A}}{}_{\underline{B}}\phi^{\underline{B}}$$

$$D_{\underline{m}}\psi^{\bar{\underline{A}}} - \tilde{D}_{\underline{m}}\psi^{\bar{\underline{A}}} = -i\bar{A}_{\underline{m}}{}^{\bar{\underline{A}}}{}_{\bar{\underline{B}}}\psi^{\bar{\underline{B}}}$$

pokud $\tilde{D} = \partial$ $\quad A_{\underline{m}}$ vekt. potenciál

Věta
$$A_{\underline{m}}^{+} = A_{\underline{m}}$$

důk $Dh = \tilde{D}h = 0$

tenzor křivosti: D

$$iF_{\underline{mn}}{}^{\underline{A}}{}_{\underline{B}}\phi^{\underline{B}} = \left[D_{\underline{m}}D_{\underline{n}} - D_{\underline{n}}D_{\underline{m}} + T_{\underline{mn}}{}^{\underline{k}}D_{\underline{k}}\right]\phi^{\underline{A}} = {}^{D}d_{\underline{2}}{}^{D}d_{\underline{n}}\,\phi^{\underline{A}}$$

$$-i\bar{F}_{\underline{mn}}{}^{\bar{\underline{A}}}{}_{\bar{\underline{B}}}\psi^{\bar{\underline{B}}} = \left[DD - DD + TD\right]\psi^{\bar{\underline{A}}} = {}^{D}d_{\underline{2}}{}^{D}d_{\underline{n}}\,\psi^{\bar{\underline{A}}}$$

Věta
$$F^{+} = F$$

$$Fh = 0$$

kalibrační transf

$$\phi \to \tilde{\phi} = U\cdot\phi$$
$$\psi \to \tilde{\psi} = \bar{U}\cdot\psi$$

$$D \to \tilde{D} = D + \Lambda$$

$$\Lambda = -(DU)\cdot U^{+}$$

$$D = \partial + iA \qquad \tilde{D} = \partial + i\tilde{A}$$

$$\tilde{A} = U\cdot A\cdot U^{+} + i(\partial U)\cdot U^{-1}$$
$$= A + i(DU)\cdot U^{-1}$$
$$= A + U^{-1}\cdot(\tilde{D}U)$$

$$\tilde{F} = U\cdot F\cdot U^{-1} \qquad U^{-1} = U^{+}$$

$D \; \tilde{D}$ konz. s herm. str.

$D - \tilde{D}$ pseudoder. gener.

rozdíl tenz. $i A_{\underline{m}}{}^{\underline{K}}{}_{\underline{L}}$

$$D_{\underline{m}}\phi^{\underline{A}} - \tilde{D}_{\underline{m}}\phi^{\underline{A}} = i A_{\underline{m}}{}^{\underline{A}}{}_{\underline{B}}\phi^{\underline{B}}$$

$$D_{\underline{m}}\psi^{\bar{\underline{A}}} - \tilde{D}_{\underline{m}}\psi^{\bar{\underline{A}}} = -i \bar{A}_{\underline{m}}{}^{\bar{\underline{A}}}{}_{\bar{\underline{B}}}\psi^{\bar{\underline{B}}}$$

příkl. $\tilde{D} = \partial$ $\quad A_{\underline{m}}$ vekt. potenciál

Věta

$$A_{\underline{m}}^{+} = A_{\underline{m}}$$

důk. $Dh = \tilde{D}h = 0$

tenzor křivosti D

$$i F_{\underline{mn}\,\underline{B}}{}^{\underline{A}} \phi^{\underline{B}} = \left[D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}} + T_{\underline{mn}}{}^{\underline{k}} D_{\underline{k}} \right] \phi^{\underline{A}} = {}^{D}d_{\underline{m}}\, {}^{D}d_{\underline{n}}\, \phi^{\underline{A}}$$

$$-i \bar{F}_{\underline{mn}}{}^{\bar{\underline{A}}}{}_{\bar{\underline{B}}} \psi^{\bar{\underline{B}}} = \left[D D - D D + T D \right] \psi^{\bar{\underline{A}}} = {}^{D}d_{\underline{m}}\, {}^{D}d_{\underline{n}}\, \psi^{\bar{\underline{A}}}$$

Věta

$$F^{\dagger} = F$$

$$F h = 0$$

Kalibrační transf.

$$\phi \to \tilde{\phi} = U \cdot \phi$$

$$\psi \to \tilde{\psi} = \bar{U} \cdot \psi$$

$$D \to \tilde{D} = D + \Lambda$$

$$\Lambda = -(DU) \cdot U^{\dagger}$$

$$D = \partial + iA \qquad \tilde{D} = \partial + i\tilde{A}$$

$$\tilde{A} = U \cdot A \cdot U^{\dagger} + i(\partial U) \cdot U^{-1}$$

$$= A + i(DU) \cdot U^{-1}$$

$$= A + U^{-1} \cdot (\tilde{D}U)$$

$$\tilde{F} = U \cdot F \cdot U^{-1} \qquad U^{-1} = U^{\dagger}$$

Linearní $U(1)$-bundl

$U(1)$-nabitá pole

$\mathbb{C} \quad \overline{\mathbb{C}} \quad \mathbb{C}^{q_L \, q_R}_{l_L \, l_R} \qquad h_{\underline{\bar{A}B}}$

$\mathbb{C}^{q+1}_{l+1} \longleftrightarrow \mathbb{C}^{q}_{l}$

$\phi^{\dots \underline{A}}_{\dots \underline{B}} \longleftrightarrow \phi^{\dots \underline{M}}_{\dots \underline{M}} \qquad \phi^{\dots \underline{A}}_{\dots \underline{B}} = \phi^{\dots \underline{M}}_{\dots \underline{M}} \, \mathbb{1}^{\underline{A}}_{\underline{B}}$

$\mathbb{C}^{q}_{l} \rightarrow \begin{cases} \mathbb{C}^{q}_{0} \\ \mathbb{C}^{0}_{l} \end{cases} \qquad \mathbb{C}^{q\,l} \rightarrow \begin{cases} \mathbb{C}^{q\,0} \\ \mathbb{C}^{0\,l} \end{cases}$

$\mathbb{C}^{q+1 \; l+1}_{p\,q} \longleftrightarrow \mathbb{C}^{q\,l}_{p\,q}$

$\phi^{\dots \underline{A} \dots \underline{\bar{B}}}_{\dots \; \dots} \longleftrightarrow \phi^{\dots \underline{M} \dots \underline{\bar{N}}}_{\dots \; \dots} \, h_{\underline{\bar{N}M}}$

$= \left(\phi^{\dots \underline{M} \dots \underline{\bar{N}}}_{\dots \; \dots} \, h_{\underline{\bar{N}M}} \right) h^{-1 \underline{\bar{B}A}}$

$\psi^{\underline{\bar{A}}} \leftrightarrow \phi_{\underline{B}} = \psi^{\underline{\bar{A}}} h_{\underline{\bar{A}B}}$

$\psi_{\underline{\bar{A}}} \leftrightarrow \phi^{\underline{B}} = \psi_{\underline{\bar{A}}} h^{-1 \underline{\bar{A}B}}$

$\mathbb{C}^{q_L \, q_R}_{l_L \, l_R} \rightarrow \begin{cases} \mathbb{C}^{q\,0}_{0\,0} & \mathbb{C}^{q} \\ \mathbb{C} & \mathbb{C}^{0} \\ \mathbb{C}^{0\,q}_{0\,0} & \mathbb{C}^{-q} \end{cases}$

$\mathbb{C}^{m} \quad \underset{m}{\phi} \, \underset{m}{\psi} \rightarrow m+m$

nábojové sdružení

$\overline{} : \mathbb{C}^{m} \rightarrow \mathbb{C}^{-m}$

$\overline{\overline{\phi}} = \phi$

$(\underset{m}{\phi}, \underset{m}{\psi}) = \underbrace{\overline{\phi} \psi}_{0} \in \mathbb{C}$

$U(1)$-bundl

$\mathbb{C}^{m} M$

$U(1)$-kalibrační transf.

$U \in \mathrm{Sect}\, \mathbb{C}^{1}_{1} M = \mathrm{Sect}\, \mathbb{C}^{0} M = \mathcal{F}M$

$U^{*} U = 1 \qquad U = \exp(iu) \qquad u \in \mathcal{F}M, \; u(x) \in \mathbb{R}$

$\phi \in \mathbb{C}^{1} M \qquad \phi \rightarrow \tilde{\phi} = \exp(iu)\, \phi$

$\phi \in \mathbb{C}^{m} M \qquad \phi \rightarrow \tilde{\phi} = \exp(imu)\, \phi$

trivializace kovz obec. atd.

$\mathbb{C}^{1} M \quad E \qquad \bar{E} E = 1 \qquad \phi = \varphi E \quad \varphi \in \mathcal{F}M$

$\mathbb{C}^{m} M \quad E^{m} \qquad\qquad \psi = \varphi E^{m} \quad \varphi \in \mathcal{F}M$

Linearní $U(1)$-bundl

$U(1)$-nabitá pole

$$\mathbb{C} \quad \overline{\mathbb{C}} \quad \mathbb{C}^{k_L\, k_R}_{l_L\, l_R} \qquad h_{\bar{\underline{A}}\underline{B}}$$

$$\mathbb{C}^{k+1}_{l+1} \longleftrightarrow \mathbb{C}^{k}_{l}$$

$$\phi^{\dots\underline{A}}_{\dots\underline{B}} \longleftrightarrow \phi^{\dots\underline{M}}_{\dots\underline{M}} \qquad \phi^{\dots\underline{A}}_{\dots\underline{B}} = \phi^{\dots\underline{M}}_{\dots\underline{M}}\, \mathbb{1}^{\underline{A}}_{\underline{B}}$$

$$\mathbb{C}^{k}_{l} \longrightarrow \begin{cases} \mathbb{C}^{k}_{0} \\ \mathbb{C}^{0}_{l} \end{cases} \qquad \mathbb{C}^{kl} \longrightarrow \begin{cases} \mathbb{C}^{k0} \\ \mathbb{C}^{0l} \end{cases}$$

$$\mathbb{C}^{k+1\, l+1}_{pq} \longleftrightarrow \mathbb{C}^{kl}_{pq}$$

$$\phi^{\dots\underline{A}\dots\bar{\underline{B}}}_{\dots\ \dots} \longleftrightarrow \phi^{\dots\underline{M}\dots\bar{\underline{N}}}_{\dots\ \dots}\, h_{\bar{\underline{N}}\underline{M}}$$

$$= \left(\phi^{\dots\underline{M}\dots\bar{\underline{N}}}_{\dots\dots}\, h_{\bar{\underline{N}}\underline{M}}\right) h^{-1\,\bar{\underline{B}}\underline{A}}$$

$$\psi^{\bar{A}} \leftrightarrow \phi_{\underline{B}} = \psi^{\bar{A}} h_{\bar{A}\underline{B}}$$

$$\psi_{\bar{A}} \leftrightarrow \phi^{\underline{B}} = \psi_{\bar{A}} h^{-1\bar{A}\underline{B}}$$

$$\mathbb{C}^{q_L q_R}_{l_L l_R} \longrightarrow \begin{cases} \mathbb{C}^{q0}_{00} & \mathbb{C}^{q} \\ \mathbb{C} & \mathbb{C}^{0} \\ \mathbb{C}^{0q}_{00} & \mathbb{C}^{-q} \end{cases}$$

$$\mathbb{C}^{m} \qquad \underset{m}{\phi}\,\underset{m}{\psi} \to m+m$$

nábojové sdružení

$$\bar{\ } : \mathbb{C}^{m} \to \mathbb{C}^{-m}$$

$$\bar{\bar{\phi}} = \phi$$

$$(\underset{m}{\phi}, \underset{m}{\psi}) = \underbrace{\bar{\phi}\psi}_{0} \in \mathbb{C}$$

$U(1)$-bundl

$\mathbb{C}^m M$

$U(1)$-kalibrační transf.

$U \in \mathrm{Sect}\, \mathbb{C}^1_1 M = \mathrm{Sect}\, \mathbb{C}^0 M = \mathcal{F} M$

$U^* U = 1 \qquad U = \exp(iu) \qquad u \in \mathcal{F} M$

$u(x) \in \mathbb{R}$

$\phi \in \mathbb{C}^1 M \qquad \phi \to \tilde{\phi} = \exp(iu)\, \phi$

$\phi \in \mathbb{C}^m M \qquad \phi \to \tilde{\phi} = \exp(imu)\, \phi$

trivializace kovz. obr. sta

$\mathbb{C}^1 M \qquad E \qquad \bar{E} E = 1 \qquad \phi = \varphi E \qquad \varphi \in \mathcal{F} M$

$\mathbb{C}^m M \qquad E^m \qquad \psi = \varphi E^m \qquad \varphi \in \mathcal{F} M$

$U(1)$ - Kovar. derivace

$\mathbb{C}^n M \quad \overline{\Phi} \qquad \phi\,\psi$

$$D(\phi\psi) = (D\phi)\psi + \phi(D\psi)$$

$$D\overline{\phi} = \overline{D\phi}$$

$D, \tilde{D} \qquad \phi \in \mathbb{C}^1 M$

$$D_m\phi - \tilde{D}_m\phi = iA_m\phi \qquad A_m{}^k{}_k \in \mathbb{C}^0 M$$

$\psi \in \mathbb{C}^n M \qquad D_m\psi - \tilde{D}_m\psi = imA_m\psi$

triviál $E \qquad \partial E = 0$

$$D\phi = \partial\phi + iA_m\phi \qquad A_m \text{ vekt. potenciál}$$

$$A_m^* = A_m$$

tenzor křivosti

$\phi \in \mathbb{C}^1 M \qquad iF_{mn}\phi = [D_m D_n - D_n D_m + T^k_{mn} D_k]\phi = {}^D d_m {}^D d_n \phi$

$\psi \in \mathbb{C}^n M \qquad imF_{mn}\psi = [DD - DD + TD]\psi = {}^D d_m {}^D d_n \psi$

$$F_{mn} = {}^\partial d_m A_n + i\underbrace{[A_m, A_n]}_{0} = d_m A_n \qquad \mathbb{C}^0 M$$

$D = \partial + iA$

$$g^{mn} D_m F_{an} = J_a \qquad \nabla_m F^{am} = J^a$$

$$-g^{mn} D_m D_n \phi + M^2\phi = 0$$

$$J_a = mi(\overline{\phi} D_a \phi - \phi D_a \overline{\phi}) \qquad \phi \in \mathbb{C}^n M$$

$\phi \in \mathbb{C}^n M \qquad \phi \to \tilde{\phi} = U^n \phi$

$$U = \exp(iu)$$

$$D \to \tilde{D} = D + \Lambda$$

$$\Lambda = -(DU)\cdot U^{-1} = -i\,du$$

$$\tilde{D}\phi = D\phi - in\,du\,\phi$$

$$A \to \tilde{A} = U\cdot A\cdot U^{-1} + i(\partial U)\cdot U^{-1}$$

$$= A - du$$

$$F \to \tilde{F} = U\cdot F\cdot U^{-1} = F$$

$U(1)$-Kovar. derivace

$\mathbb{C}^m M \quad \overline{\phi} \quad \phi\psi$

$$D(\phi\psi) = (D\phi)\psi + \phi(D\psi)$$

$$D\overline{\phi} = \overline{D\phi}$$

$D, \tilde{D} \qquad \phi \in \mathbb{C}^1 M$

$$D_{\underline{m}}\phi - \tilde{D}_{\underline{m}}\phi = iA_{\underline{m}}\phi \qquad A_{\underline{m}} \in \mathbb{C}^0 M$$

$$\psi \in \mathbb{C}^m M \quad D_{\underline{m}}\psi - \tilde{D}_{\underline{m}}\psi = imA_{\underline{m}}\psi$$

Trivial $E \quad \partial E = 0$

$$D\phi = \partial\phi + iA_{\underline{m}}\phi \qquad A_{\underline{m}} \text{ vekt potenciál}$$

$$A_{\underline{m}}^{*} = A_{\underline{m}}$$

tenzor křivosti

$\phi \in \mathbb{C}^1 M \qquad i F_{\underline{mn}} \phi = [D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}} + T^{\underline{k}}_{\underline{mn}} D_{\underline{k}}]\phi = {}^{D}d_{\underline{m}} {}^{D}d_{\underline{n}} \phi$

$\psi \in \mathbb{C}^n M \qquad i m F_{\underline{mn}} \psi = [DD - DD + TD]\psi = {}^{D}d_{\underline{n}} {}^{D}d_{\underline{n}} \psi$

$$F_{\underline{mn}} = {}^{\partial}d_{\underline{m}} A_{\underline{n}} + i \underbrace{[A_{\underline{m}}, A_{\underline{n}}]}_{0} = d_{\underline{m}} A_{\underline{n}}$$

$D = \partial + iA$

$\phi \in C^n M \quad \phi \to \tilde{\phi} = U^m \phi$

$U = \exp(iu)$

$D \to \tilde{D} = D + \Lambda$

$\Lambda = -(DU) \cdot U^{-1} = -i\,du$

$\tilde{D}\phi = D\phi - im\,du\,\phi$

$A \to \tilde{A} = U \cdot A \cdot U^{-1} + i(\partial U) \cdot U^{-1}$

$= A - du$

$F \to \tilde{F} = U \cdot F \cdot U^{-1} = F$

$$F_{\underline{mn}} = {}^{\partial} d_{\underline{m}} A_{\underline{n}} + i\underbrace{[A_{\underline{m}}, A_{\underline{n}}]}_{0} = d_{\underline{m}} A_{\underline{n}} \qquad C^0 M$$

$$D = \partial + iA$$

$$g^{\underline{mn}} D_{\underline{m}} F_{\underline{an}} = J_{\underline{a}} \qquad \nabla_{\underline{n}} F^{\underline{an}} = J^{\underline{a}}$$

$$-g^{\underline{mn}} D_{\underline{m}} D_{\underline{n}} \phi + M^2 \phi = 0$$

$$J_{\underline{a}} = mi(\bar{\phi} D_{\underline{a}} \phi - \phi D_{\underline{a}} \bar{\phi}) \qquad \phi \in \mathbb{C}^n M$$

G pologrostá Ad

$\mathfrak{g}$ pologrostá ad

k nedegener.

$\mathrm{Der}\,\mathfrak{g} = \mathrm{ad}_{\mathfrak{g}}$

↳ algebra operátorů lineárn ad_m $m \in \mathfrak{g}$

↳ prostor alg. der. tj. oper. δ

$\delta[m, n] = [\delta m, n] + [m, \delta n]$

vekt. pr. A

repr. gr. G na A $\quad T_{\underline{a}}{}^{\underline{A}}{}_{\underline{B}}$

repr. alg $\mathfrak{g}$ na A $\quad t_m{}^{\underline{A}}{}_{\underline{B}} = m^{\underline{a}}\, t_{\underline{a}}{}^{\underline{A}}{}_{\underline{B}}$

$A = A_0 \oplus \bigoplus_{r} A_r$ $\quad P_r$ projektor na A_r

$t_{\underline{a}}{}^{\underline{A}}{}_{\underline{B}} = \bigoplus t_{r\underline{a}}{}^{\underline{A}}{}_{\underline{B}}$ $\quad t_r = P_r \cdot t \cdot P_r$

t_r ireducibilní

$[S, t_m] = 0 \quad \forall m \in \mathfrak{g} \quad \Rightarrow \quad S = \sum_r A_r P_r$

G poloprostá Ad

$\mathfrak{g}$ poloprostá ad

k nedegener.

$\mathrm{Der}\,\mathfrak{g} = \mathrm{ad}_{\mathfrak{g}}$

↳ algebra operátorů tvaru ad_m $m \in \mathfrak{g}$

↳ prostor alg. der. tj. oper. δ

$$\delta[m,n] = [\delta m, n] + [m, \delta n]$$

vekt. pr. A

repr. gr. G na A $\quad T_{g}{}^{\underline{A}}{}_{\underline{B}}$

repr. alg $\mathfrak{g}$ na A $\quad t_{m}{}^{\underline{A}}{}_{\underline{B}} = m^{\alpha}\, t_{\underline{\alpha}}{}^{\underline{A}}{}_{\underline{B}}$

$$A = A_0 \oplus \bigoplus_r A_r$$

P_r projektor na A_r

$$t_{\underline{\alpha}}{}^{\underline{A}}{}_{\underline{B}} = \bigoplus t_{r\underline{\alpha}}{}^{\underline{A}}{}_{\underline{B}}$$

$$t_r = P_r \cdot t \cdot P_r$$

t_r ireducibilní

$$[S, t_m] = 0 \quad \Rightarrow \quad S = \sum_r \lambda_r P_r$$

$\forall m \in \mathfrak{g}$

Lokální kalibrační grupa a algebra

GM $\qquad h \in \mathrm{Sect}\, GM$

$\qquad\qquad h(x) \in G_x M$

gM $\qquad m \in \mathrm{Sect}\, gM$

$\qquad\qquad m(x) \in g_x M$

$$[m,n]^\alpha = m^\mu n^\nu c_{\mu\nu}{}^\alpha$$

$$\mathrm{ad}_m n = [m,n]$$

$$\mathrm{ad}_m{}^\alpha{}_\beta = m^\kappa c_{\kappa\beta}{}^\alpha$$

Kovar. der. na bundlu Lieovy alg.

Def

mějme bundl. L. alg gM

kov. der. D je konz. s alg. str.

$$Dc = 0$$

$$\Updownarrow$$

$$D[m,n] = [Dm,n] + [m,Dn]$$

Lema

$$Dk = 0$$

Def Trivializace konz. s alg. str.

e_α báze v gM

$c_{\alpha\beta}{}^\gamma = \text{konst}$ (konz. c vůči e_α)

Lema

∂ souř. der. trivial e_α

konz. s alg. str.

$$\partial e_\alpha = 0$$

$\Downarrow$

$\partial c = 0 \qquad \partial$ konz. s alg. str.

$$\partial k = 0$$

Lokální kalibrační grupa a algebra

$\mathbb{G}M \qquad h \in \mathcal{S}ect\, \mathbb{G}M$

$h(x) \in \mathbb{G}_x M$

$\mathfrak{g}M \qquad m \in \mathcal{S}ect\, \mathfrak{g}M$

$m(x) \in \mathfrak{g}_x M$

$$[m,n]^\alpha = m^\mu n^\nu c_{\mu\nu}{}^\alpha$$

$$\mathrm{ad}_m n = [m,n]$$

$$\mathrm{ad}_m{}^\alpha{}_\beta = m^\kappa c_{\kappa\beta}{}^\alpha$$

Kovar. der. na bundle Lieovy alg.

Def

mějme bundl. L. alg gM

kov. der. D je kons. s alg. str.

$$DC = 0$$

$$\Updownarrow$$

$$D[m,n] = [Dm,n] + [m,Dn]$$

Lemma

$$Dk = 0$$

Def trivializace kons. s alg. str.

e_α báze v gM

$C_{\alpha\beta}{}^{\gamma} = \mathrm{konst}$ (komp. C vůči e_α)

Lema

∂ savr. der. trivial e_α
konz. s alg. str.

$$\partial e_\alpha = 0$$

$$\Downarrow$$

$$\partial c = 0 \qquad \partial \text{ konz. s alg. str.}$$

$$\partial k = 0$$

Věta:

$D, \tilde{D}$ kov. der na $\mathfrak{g}M$

$$D = \tilde{D} + \mathbf{A} \qquad A_{\underline{m}}{}^{\alpha}{}_{\beta}$$

$$A \cdot [m, n] = [A \cdot m, n] + [m, A \cdot n]$$

$$\Rightarrow \exists\, \mathfrak{A} \qquad A = \mathrm{ad}_{\mathfrak{A}}$$

$$\mathfrak{A}_{\underline{m}}{}^{\kappa} \qquad A_{\underline{m}}{}^{\alpha}{}_{\beta} = \mathfrak{A}_{\underline{m}}{}^{\lambda} c_{\lambda\beta}{}^{\alpha}$$

Lemma

∂ souř. der.

$A = \mathrm{ad}_{\mathfrak{A}}$

$\Downarrow$ $D = \partial + \mathbf{A}$ je kovar. s alg. str.

Věta

$m \in$ Sect $\mathfrak{g}M$

$$D_{\underline{m}} m^{\alpha} = \partial_{\underline{m}} m^{\alpha} + A_{\underline{m}}{}^{\alpha}{}_{\beta} m^{\beta}$$
$$= \partial_{\underline{m}} m^{\alpha} + [\mathfrak{A}_{\underline{m}}, m]^{\alpha}$$

$B \in$ Sect $\mathfrak{g}^1_1 M \qquad B^{\alpha}{}_{\beta} = \mathrm{ad}_B{}^{\alpha}{}_{\beta} = B^{\kappa} c_{\kappa\beta}{}^{\alpha}$

$$D_{\underline{m}} B^{\alpha}{}_{\beta} = \partial_{\underline{m}} B^{\alpha}{}_{\beta} + [A_{\underline{m}}, B]^{\alpha}{}_{\beta}$$

$$(D_{\underline{m}} B^{\kappa}) c_{\kappa\beta}{}^{\alpha} = (\partial_{\underline{m}} B^{\kappa} + [\mathfrak{A}_{\underline{m}}, B]^{\kappa}) c_{\kappa\beta}{}^{\alpha}$$

$$\mathfrak{F}_{\underline{mn}}{}^{\kappa} = \partial_{\underline{m}} \wedge \mathfrak{A}_{\underline{n}}{}^{\kappa} + [\mathfrak{A}_{\underline{m}}, \mathfrak{A}_{\underline{n}}]^{\kappa}$$
$$= \partial_{\underline{m}} \mathfrak{A}_{\underline{n}}{}^{\kappa} - \partial_{\underline{n}} \mathfrak{A}_{\underline{m}}{}^{\kappa} + \mathfrak{A}_{\underline{m}}{}^{\mu} \mathfrak{A}_{\underline{n}}{}^{\nu} c_{\mu\nu}{}^{\kappa}$$
$$T = 0$$

Věta $F_{\underline{mn}}{}^{\alpha}{}_{\beta}$ tenz. kř. der $D_{\underline{m}}$

$$F_{\underline{mn}}\, c = 0$$

$$F_{\underline{mn}} \cdot [m, n] = [F_{\underline{mn}} \cdot m, n] + [m, F_{\underline{mn}} \cdot n]$$

$\Downarrow$

$$\exists\, \mathfrak{F}_{\underline{mn}} \qquad F = \mathrm{ad}_{\mathfrak{F}}$$

$$F_{\underline{mn}}{}^{\alpha}{}_{\beta} = \mathfrak{F}_{\underline{mn}}{}^{\kappa} c_{\kappa\beta}{}^{\alpha}$$

$$F_{\underline{mn}} B^{\kappa} = F_{\underline{mn}}{}^{\kappa}{}_{\lambda} B^{\lambda} = [\mathfrak{F}, B]^{\kappa}$$

$$F_{\underline{mn}} B^{\alpha}{}_{\beta} = [F_{\underline{mn}}, B]^{\alpha}{}_{\beta} = [\mathfrak{F}, B]^{\kappa} c_{\kappa\beta}{}^{\alpha}$$

Věta: D kov. der. dané vekt. pot. $\mathfrak{A}$ vůči ∂

$$F_{\underline{mn}} = \partial_{\underline{m}} A_{\underline{n}} + [A_{\underline{m}}, A_{\underline{n}}]$$

$$\mathfrak{F}_{\underline{mn}} = \partial_{\underline{m}} \mathfrak{A}_{\underline{n}} + [\mathfrak{A}_{\underline{m}}, \mathfrak{A}_{\underline{n}}]$$

Věta:

$\mathcal{D}, \tilde{\mathcal{D}}$ kov. der. na $\mathfrak{g}M$

$$\mathcal{D} = \tilde{\mathcal{D}} + A \qquad A_{\underline{m}\,\beta}{}^{\alpha}$$

$$A\cdot[m,n] = [A\cdot m, n] + [m, A\cdot n]$$

$$\Rightarrow \exists\, \mathfrak{k} \qquad A = \mathrm{ad}_{\mathfrak{k}}$$

$$\mathfrak{k}_{\underline{m}}{}^{k} \qquad A_{\underline{m}\beta}{}^{\alpha} = \mathfrak{k}_{\underline{m}}{}^{\delta}\, c_{\delta\beta}{}^{\alpha}$$

Lemma

∂ souř. der.

$A = \mathrm{ad}_{\mathfrak{k}}$

$\Downarrow$

$\mathcal{D} = \partial + A$ je kompat. s alg. str.

Vieta

$m \in \mathcal{L}ed\, \mathfrak{g}\, M$

$$D_{\underline{m}} m^{\underline{\alpha}} = \partial_{\underline{m}} m^{\underline{\alpha}} + A_{\underline{m}}{}^{\underline{\alpha}}{}_{\underline{\beta}}\, m^{\underline{\beta}}$$

$$= \partial_{\underline{m}} m^{\underline{\alpha}} + [\mathcal{A}_{\underline{m}}, m]^{\underline{\alpha}}$$

$B \in \mathcal{L}ed\, \mathfrak{g}^1 M \qquad B^{\underline{\alpha}}{}_{\underline{\beta}} = ad_B{}^{\underline{\alpha}}{}_{\underline{\beta}} = B^{\underline{\kappa}} c_{\underline{\kappa}\underline{\beta}}{}^{\underline{\alpha}}$

$$D_{\underline{m}} B^{\underline{\alpha}}{}_{\underline{\beta}} = \partial_{\underline{m}} B^{\underline{\alpha}}{}_{\underline{\beta}} + [A_{\underline{m}}, B]^{\underline{\alpha}}{}_{\underline{\beta}}$$

$$(D_{\underline{m}} B^{\underline{\kappa}}) c_{\underline{\kappa}\underline{\beta}}{}^{\underline{\alpha}} = (\partial_{\underline{m}} B^{\underline{\kappa}} + [\mathcal{A}_{\underline{m}}, B]^{\underline{\kappa}})\, c_{\underline{\kappa}\underline{\beta}}{}^{\underline{\alpha}}$$

Věta $\mathcal{F}_{mn}{}^{\alpha}{}_{\beta}$ tenz. kov. der. D_m

$$\mathcal{F}_{mn}\, c = 0$$

$$\mathcal{F}_{mn}\cdot[m,n] = [\mathcal{F}_{mn}\cdot m, n] + [m, \mathcal{F}_{mn}\cdot n]$$

$$\Downarrow$$

$$\exists\, \hat{F}_{mn} \qquad \mathcal{F} = \mathrm{ad}_{\hat{F}}$$

$$\mathcal{F}_{mn}{}^{\alpha}{}_{\beta} = \hat{F}_{mn}{}^{\kappa}\, c_{\kappa\beta}{}^{\alpha}$$

$$\mathcal{F}_{mn} B^{\kappa} = \mathcal{F}_{mn}{}^{\kappa}{}_{\alpha} B^{\alpha} = [\hat{F}, B]^{\kappa}$$

$$\mathcal{F}_{mn} B^{\alpha}{}_{\beta} = [\mathcal{F}_{mn}, B]^{\alpha}{}_{\beta} = [\hat{F}, B]^{\kappa}\, c_{\kappa\beta}{}^{\alpha}$$

Věta: D kov. der. daná vekt. pot. $\hat{A}$ vůči ∂

$$\mathcal{F}_{mn} = \partial_m A_n + [A_m, A_n]$$

$$\hat{F}_{mn} = \partial_m \hat{A}_n + [\hat{A}_m, \hat{A}_n]$$

$$\hat{F}_{mn}^{\kappa} = \partial_m \wedge \hat{A}_n^{\kappa} + [\hat{A}_m, \hat{A}_n]^{\kappa}$$

$$= \partial_m \hat{A}_n^{\kappa} - \partial_n \hat{A}_m^{\kappa} + \hat{A}_m^{\mu} \hat{A}_n^{\nu}\, c_{\mu\nu}{}^{\kappa}$$

$$T = 0$$

Asociované vekt. bundly

gM bundl Lieovy alg

AM vekt. bundl SO-bundl

$t_{\kappa}{}^{A}{}_{B}$ gen repr

$m \in g \to M^{A}{}_{B} = t_{m}{}^{A}{}_{B} = m^{\kappa} t_{\kappa}{}^{A}{}_{B}$

$t_{[m,n]} = [t_m, t_n]$

$C_{\mu\nu}{}^{\kappa} t_{\kappa}{}^{A}{}_{B} = t_{\mu}{}^{A}{}_{C} t_{\nu}{}^{C}{}_{B} - t_{\nu}{}^{A}{}_{C} t_{\mu}{}^{C}{}_{B}$

$(T_m \phi, T_m \psi) = (\phi, \psi)$ $\quad H_{AB}$

$t_m{}^{T} = -t_m$

$t_{\kappa AB} + t_{\kappa BA} = 0$

rozšíření kov. der

z gM na AM

D na gM $\quad Dc = 0$

D na AM $\quad DH = 0$

$D_m t_{\kappa}{}^{A}{}_{B} = 0$

trivializace

$e_\alpha \quad C_{\alpha\beta}{}^{\gamma} = \text{konst} \quad \partial c = 0$

$E_A \quad H_{AB} = \text{konst} \quad \partial H = 0$

$t_{\mu}{}^{A}{}_{B} = \text{konst} \quad \partial t = 0$

$\partial e_\alpha = 0, \ \partial E_A = 0$

$D_m m^{\kappa} = \partial_m m^{\kappa} + [\Omega_m, m]^{\kappa} \quad A_m{}^{\alpha}{}_{\beta} = \Omega_m^{\kappa} C_{\kappa\beta}{}^{\alpha}$

$D_m \phi^{A} = \partial_m \phi^{A} + A_m{}^{A}{}_{B} \phi^{B}$

$\underbrace{D_m t_{\kappa}{}^{A}{}_{B}}_{0} = \underbrace{\partial_m t_{\kappa}{}^{A}{}_{B}}_{0} - \Omega_m^{\lambda} C_{\lambda\kappa}{}^{\nu} t_{\nu}{}^{A}{}_{B}$

$+ [A_m, t_\kappa]^{A}{}_{B}$

$-\Omega_m^{\lambda} [t_\lambda, t_\kappa]^{A}{}_{B} + [A_m, t_\kappa]^{A}{}_{B} = 0$

$[A_m - \Omega_m^{\lambda} t_\lambda, t_\kappa]^{A}{}_{B} = 0$

$A_m{}^{A}{}_{B} - \Omega_m^{\lambda} t_{\lambda}{}^{A}{}_{B} = \sum_{\ell} a_\ell P_{\ell}{}^{A}{}_{B}$

$A^T = -A \quad t_m^{T} = -t_m \quad P_\ell^{T} = P_\ell$

$\Rightarrow a_\ell = 0 \quad A_\mu{}^{A}{}_{B} = \Omega_\mu^{\lambda} t_{\lambda}{}^{A}{}_{B}$

Asociované vekt. bundly

$\mathfrak{g}M$ bundle Lieovy alg

AM vekt. bundle SO-bundle

$t_{\underline{\kappa}}{}^{\underline{A}}{}_{\underline{B}}$ gen repr.

$$m \in \mathfrak{g} \to M^{\underline{A}}{}_{\underline{B}} = t_m{}^{\underline{A}}{}_{\underline{B}} = m^{\underline{\kappa}}\, t_{\underline{\kappa}}{}^{\underline{A}}{}_{\underline{B}}$$

$$t_{[m,n]} = [t_m, t_n]$$

$$C_{\underline{\mu\nu}}{}^{\underline{\kappa}}\, t_{\underline{\kappa}}{}^{\underline{A}}{}_{\underline{B}} = t_{\underline{\mu}}{}^{\underline{A}}{}_{\underline{C}}\, t_{\underline{\nu}}{}^{\underline{C}}{}_{\underline{B}} - t_{\underline{\nu}}{}^{\underline{A}}{}_{\underline{C}}\, t_{\underline{\mu}}{}^{\underline{C}}{}_{\underline{B}}$$

$$(T_m \cdot \phi, T_m \cdot \psi) = (\phi, \psi) \qquad H_{\underline{AB}}$$

$$t_m{}^T = -t_m$$

$$t_{\underline{\kappa}\,\underline{AB}} + t_{\underline{\kappa}\,\underline{BA}} = 0$$

rozšíření kov. der

z gM na AM

D na gM $\qquad D c = 0$

D na AM $\qquad D H = 0$

$$D_{\underline{m}} t_{\underline{k}}{}^{A}{}_{\underline{B}} = 0$$

trivializace

$e_\alpha \qquad C_{\alpha\beta}{}^{\gamma} = \text{konst} \quad \partial C = 0$

$E_A \qquad H_{AB} = \text{konst} \quad \partial H = 0$

$t_{\mu}{}^{A}{}_{B} = \text{konst} \quad \partial t = 0$

$\partial e_\alpha = 0 \quad \partial E_A = 0$

$$\mathcal{D}_{\underline{m}} m^{\underline{k}} = \partial_{\underline{m}} m^{\underline{k}} + [\Omega_{\underline{m}}, m]^{\underline{k}} \qquad A_{\underline{m}}{}^{\underline{\alpha}}{}_{\underline{\beta}} = \Omega_{\underline{m}}{}^{\underline{k}} C_{\underline{k}\underline{\beta}}{}^{\underline{\alpha}}$$

$$D_{\underline{m}} \phi^{\underline{A}} = \partial_{\underline{m}} \phi^{\underline{A}} + A_{\underline{m}}{}^{\underline{A}}{}_{\underline{B}} \phi^{\underline{B}}$$

$$\underbrace{D_{\underline{m}} t_{\underline{k}}{}^{\underline{A}}{}_{\underline{B}}}_{0} = \underbrace{\partial_{\underline{m}} t_{\underline{k}}{}^{\underline{A}}{}_{\underline{B}}}_{0} - \Omega_{\underline{m}}{}^{\underline{\lambda}} C_{\underline{\lambda}\underline{k}}{}^{\underline{\nu}} t_{\underline{\nu}}{}^{\underline{A}}{}_{\underline{B}} + [A_{\underline{m}}, t_{\underline{k}}]^{\underline{A}}{}_{\underline{B}}$$

$$-\Omega_{\underline{m}}{}^{\underline{\lambda}} [t_{\underline{\lambda}}, t_{\underline{k}}]^{\underline{A}}{}_{\underline{B}} + [A_{\underline{m}}, t_{\underline{k}}]^{\underline{A}}{}_{\underline{B}} = 0$$

$$[A_{\underline{m}} - \Omega_{\underline{m}}{}^{\underline{\lambda}} t_{\underline{\lambda}}, t_{\underline{k}}]^{\underline{A}}{}_{\underline{B}} = 0$$

$$A_{\underline{m}}{}^{\underline{A}}{}_{\underline{B}} - \Omega_{\underline{m}}{}^{\lambda} t_{\lambda}{}^{\underline{A}}{}_{\underline{B}} = \sum_{\ell} a_{\ell} P_{\ell}{}^{\underline{A}}{}_{\underline{B}}$$

$$A^T = -A \qquad t_m^T = -t_m \qquad P_\ell^T = P_\ell$$

$$\Rightarrow a_\ell = 0 \qquad A_{\underline{m}}{}^{\underline{A}}{}_{\underline{B}} = \Omega_{\underline{m}}{}^{\lambda} t_{\lambda}{}^{\underline{A}}{}_{\underline{B}}$$