

Reálný vekt. bundle  
s SO-symetrií

A  $H_{AB}$   
nedegezer.  $\Rightarrow$  exist  $H^{-1AB}$   
symetrické  $H_{AB} = H_{BA}$   
křivkán a směr indexů  
 $H^{AB} = H^{-1AB}$

transpozice  
 $\phi^A \rightarrow \phi^T_A = H_{AB} \phi^B$   
 $\phi_A \rightarrow \phi^{TA} = H^{AB} \phi_B$   
 $\chi^A_B \rightarrow \chi^{TA}_B = H^{AM} H_{BN} \chi^N_M$

$$(\phi, \psi) = \phi^A \psi^B H_{AB}$$

$$= \phi^T \cdot \psi = \phi \cdot \psi^T$$

$$(\phi, \chi \cdot \psi) = (\chi^T \cdot \phi, \psi)$$

signatura metriky  
( $\underbrace{- \dots -}_m \underbrace{+ \dots +}_p$ )

AM  
hladké metriky  $H \in \text{Vect } A^0_2 M$   
lokálně (ultralokálně)  
isomorfní metr. na A  
vektorový SO-bundle  
metrický bundle

Def <sup>lokální</sup> kalibrační SO-transformace

R ultralokálně křivka na Sect AM  
 $R \in \text{Vect } A^1_1 M$   
 $\phi \rightarrow \tilde{\phi} = R \cdot \phi$   
 $(\tilde{\phi}, \tilde{\psi}) = (\phi, \psi)$   
zachování orientaci

rozšíření na  $A^2_2 M$   
 $\chi^A_B \rightarrow \tilde{\chi}^A_B = R^A_C \dots R^N_B \chi^N_M$   
 $(\tilde{\phi} \otimes \tilde{\psi}) = \tilde{\phi} \otimes \tilde{\psi} \quad \tilde{C} \tilde{\chi} = \tilde{C} \chi$   
 $\tilde{H}_{AB} = H_{AB} \quad H_{AB} = R^C_A R^D_B H_{CD} \quad R^T \cdot R = \mathbb{1}$   
 $\tilde{E}_{A_1 \dots A_n} = E_{A_1 \dots A_n} \quad \det R = 1$

# Reálný vekt. bundle s SO-symetrií

A  $H_{AB}$

nedegezer.  $\Rightarrow$  exist  $H^{AB}$   
symetrické  $H_{AB} = H_{BA}$

zvětán a smič. indexů

$$H^{AB} = H^{-1AB}$$

transpozice

$$\phi^A \rightarrow \phi^T_A = H_{AB} \phi^B$$

$$\phi_A \rightarrow \phi^{TA} = H^{AB} \phi_B$$

$$\chi^A_B \rightarrow \chi^{TA}_B = H^{AM} H_{BN} \chi^N_M$$

$$(\phi, \psi) = \phi^{\text{A}} \psi^{\text{B}} H_{\text{AB}}$$

$$= \phi^{\text{T}} \cdot \psi = \phi \cdot \psi^{\text{T}}$$

$$(\phi, \mathcal{X} \cdot \psi) = (\mathcal{X}^{\text{T}} \cdot \phi, \psi)$$

signature metriky

$$\underbrace{(- \dots -)}_m \underbrace{(+ \dots +)}_p$$

AM

hladké metrické  $H \in \text{Vect } A_2^0 M$

lokálně (ultralokálně)

isomorfní metr. na  $A$

vektorový  $SO$ -bundle

metrický bundle

Def Kalibrační SO-transformace

$R$  ultralokální izometrie na  $\text{Sect } \mathbb{A}M$

$R \in \text{Sect } \mathbb{A}'_1 M$

$$\phi \rightarrow \tilde{\phi} = R \cdot \phi$$

$$\Rightarrow (\tilde{\phi}, \tilde{\psi}) = (\phi, \psi)$$

$\Rightarrow$  zachování orientaci

Def <sup>lokální</sup> kalibrační SO-transformace

$R$  ultralokálně na  $\text{Sect } \mathbb{A}^1 M$

$$R \in \text{Sect } \mathbb{A}^1 M$$

$$\phi \rightarrow \tilde{\phi} = R \cdot \phi$$

$$\bullet (\tilde{\phi}, \tilde{\psi}) = (\phi, \psi)$$

= zachování orientaci

pozitivní na  $\mathbb{A}^2 M$

$$\chi_{\mathbb{B}}^{\mathbb{A}} \longrightarrow \tilde{\chi}_{\mathbb{B}}^{\mathbb{A}} = R_{\mathbb{B}}^{\mathbb{A}} \cdots R_{\mathbb{B}}^{-1 \mathbb{N}} \cdots \chi_{\mathbb{N}}^{\mathbb{M}}$$

$$(\tilde{\phi} \otimes \tilde{\psi}) = \tilde{\phi} \otimes \tilde{\psi} \quad \mathbb{C} \tilde{\chi} = \mathbb{C} \chi$$

$$\tilde{H}_{\mathbb{A}\mathbb{B}} = H_{\mathbb{A}\mathbb{B}} \quad H_{\mathbb{A}\mathbb{B}} = R_{\mathbb{A}}^{\mathbb{M}} R_{\mathbb{B}}^{\mathbb{N}} H_{\mathbb{M}\mathbb{N}} \quad R^T \cdot R = \mathbb{1}$$

$$\tilde{\varepsilon}_{\mathbb{A}_1 \dots \mathbb{A}_D} = \varepsilon_{\mathbb{A}_1 \dots \mathbb{A}_D} \quad \det R = 1$$

Def <sup>lokalni</sup> kalibracni SO-transformace

$R$  ultraloka na zobe na Sect  $AM$

$$R \in \text{Sect } A_1^1 M$$

$$\phi \rightarrow \tilde{\phi} = R \cdot \phi$$

$$\langle \tilde{\phi}, \tilde{\psi} \rangle = \langle \phi, \psi \rangle$$

- zachovani orientaci

rozvizi na  $A_2^2 M$

$$\chi_B^A \rightarrow \tilde{\chi}_B^A = R_B^A \cdot R_{R^A}^{\tilde{A}} \cdot \chi_{\tilde{B}}^{\tilde{A}}$$

$$\langle \tilde{\phi}, \tilde{\psi} \rangle = \langle \phi, \psi \rangle \quad \tilde{C} \tilde{\chi} = C \chi$$

$$\tilde{H}_{\tilde{A}\tilde{B}} = H_{AB} \quad H_{AB} = R_A^C R_B^D H_{CD} \quad R^T \cdot R = \mathbb{1}$$

$$\tilde{e}_{\tilde{A}\tilde{B}\tilde{C}\tilde{D}} = e_{ABCD} \quad \det R = 1$$

Lokalni kalibr. grupa

Lokalni kalibr. Lieova alg

$$R_\alpha = \mathbb{1} + \alpha \Omega + O(\alpha^2)$$

$\Omega \in \text{Sect } A_1^1 M$

$$[\Omega, \Delta] = \Omega \cdot \Delta - \Delta \cdot \Omega$$

$$\Omega_B^M H_{MB} + \Omega_B^M H_{AM} = 0$$

$$\Omega_{BA} + \Omega_{AB} = 0 \quad \Omega + \Omega^T = 0$$

$$\Omega^T = -\Omega$$

$$R_\alpha = \exp(\alpha \Omega)$$

Trivializace SO-bundlu

Def triv  $E_A$  je konst.  $\Delta$  mitr. struktura je-l:

$$H_{AB} = (E_A, E_B) = \text{konst} \quad (\text{na } M)$$

$P_{\tilde{A}}$ : volba ortonormalni baze

$$H_{AB} = \begin{bmatrix} -1 & & 0 \\ & -1 & \\ 0 & & +1 \dots +1 \end{bmatrix}$$

$P_{\tilde{A}}$ : signature  $(- \dots - + \dots +)$   $m=p$

$$H_{AB} = \begin{bmatrix} 0 & -1 & & \\ 1 & 0 & & \\ & & 0 & -1 \\ & & & \dots \end{bmatrix}$$

Remena trivializace

$$E_A \rightarrow \tilde{E}_A = R \cdot E_A$$

$$H_{\tilde{A}\tilde{B}} = (\tilde{E}_A, \tilde{E}_B) = (E_A, E_B) = H_{AB}$$

$$\phi \in AM \quad \phi = \phi^A E_A = \tilde{\phi}^{\tilde{A}} \tilde{E}_A$$

$\phi^A$   $\tilde{\phi}^{\tilde{A}}$  pasivni kalibr. transf

$$\phi^A = R_{\tilde{A}}^A \tilde{\phi}^{\tilde{A}} \quad \tilde{E}_{\tilde{A}} = E_A R_{\tilde{A}}^A$$

$$H_{\tilde{A}\tilde{B}} = R_{\tilde{A}}^N R_{\tilde{B}}^M H_{NM} = H_{AB}$$

Lorentz kalibr. grupa

Lorentz kalibr. Lieova alga

$$R_\alpha = \underline{1} + \alpha \Omega + O(\alpha^2)$$

$$\Omega \in \text{Vect } A^1_1 M$$

$$[\Omega, A] = \Omega \cdot A - A \cdot \Omega$$

$$\Omega^M_A H_{MB} + \Omega^M_B H_{AM} = 0$$

$$\Omega_{BA} + \Omega_{AB} = 0 \quad \Omega + \Omega^T = 0$$

$$\Omega^T = -\Omega$$

$$R_\alpha = \exp(\alpha \Omega)$$



# Trivializace SO-bundlu

Def. triv.  $E_A$  je konv.  $D$   
metr. strukturou  $g \cdot l$ :

$$H_{AB} = (E_A, E_B) = \text{konst} \\ (\text{na } M)$$

$P_{\mathbb{R}}$ : volba ortonormální báze

$$H_{AB} = \begin{bmatrix} -1 & & & 0 \\ & -1 & & \\ & & \ddots & \\ 0 & & & +1 \\ & & & & +1 \\ & & & & & \ddots \end{bmatrix}$$

$P_{\mathbb{C}}$ : signature  $(- \dots - + \dots +)$   $m = p$

$$H_{AB} = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & \ddots \end{bmatrix}$$

Γεμενα trivializace

$$E_A \rightarrow \tilde{E}_A = R \cdot E_A$$

$$H_{\tilde{A}\tilde{B}} = (\tilde{E}_A, \tilde{E}_B) = (E_A, E_B) = H_{AB}$$

$$\phi \in \mathcal{AM} \quad \phi = \phi^A E_A = \phi^{\tilde{A}} \tilde{E}_A$$

$$\phi^A \quad \phi^{\tilde{A}}$$

pasivni kalibra. transf.

$$\phi^A = R^A_{\tilde{A}} \phi^{\tilde{A}} \quad \tilde{E}_{\tilde{A}} = E_A R^A_{\tilde{A}}$$

$$H_{\tilde{A}\tilde{B}} = R^M_{\tilde{A}} R^N_{\tilde{B}} H_{MN} = H_{AB}$$

Kon der ma SO-bundel  
(SO-kov. derivace)

Def.  $A_M$  SO-bundle  
 $D$  kov. derivace

$$DH = 0$$

$$\Downarrow D_\xi(\phi, \psi) = (D_\xi \phi, \psi) + (\phi, D_\xi \psi)$$

Lemma  
 $\Downarrow$  kov. trivializace  $E_n$  kov.  $s H$

$$\partial H = 0$$

$$\partial_m H_{AB} = \partial_m (H_{MN} E_A^M E_B^N) = 0$$

Věta

$D$  je kov. oH dána  
vekt. pot  $A_m^k$  vůči  $\partial$

$$\Downarrow D = \partial + A_m$$
$$\Downarrow A_m^T = -A_m$$

ditž

$$DH = \partial H = 0$$

$$\Rightarrow 0 = A_m^k H_{AB} = -A_m^{k|A} H_{k|B} - A_m^{k|B} H_{|A}^k$$
$$A_m + A_m^T = 0$$

Věta

$D$  je kov. oH  
 $F_m^k$  tenzor kv.  $D$   
 $\Downarrow F_m^T = -F_m$

ditž

$$DH = 0 \Rightarrow D DH = 0$$

$$\Rightarrow 0 = [D_m D_n - D_n D_m + T_m^k D_k] H_{AB} = F_m^k H_{AB}$$

$$\Rightarrow F_m + F_m^T = 0$$

Kov der ma SO-Bundel  
(SO-Kov. derivative)

Def. AM SO-Bundle  
D Kov. derivative

$$DH = 0$$

$\Leftrightarrow$

$$D_{\xi}(\phi, \psi) = (D_{\xi}\phi, \psi) + (\phi, D_{\xi}\psi)$$

Lemma Kov. trivialisace  $\Gamma_A$  Kov.  $\psi H$

$$\partial H = 0$$

$$\partial_{\underline{m}} H_{\underline{AB}} = \partial_{\underline{m}} (H_{MN} E_{\underline{A}}^M E_{\underline{B}}^N) = 0$$

Věta

$D$  je konz. oH dána  
vekt. pot  $A_m^k$  vici  $\odot$

$$D = \odot + A_m$$

$$\Downarrow A_m^T = -A_m$$

dikž.

$$DH = \partial H = 0$$

$$\Rightarrow 0 = A_m H_{AB} = -A_m^k{}_A H_{kB} - A_m^k{}_B H_{Ak}$$

$$A_m + A_m^T = 0$$

Vite

$D$  je konz. oH

$\underline{F}_{mn}^k$  tenzor ži.  $D$

$$\Downarrow \underline{F}_{mn}^T = -\underline{F}_{mn}$$

ditz:

$$D \cdot H = 0 \Rightarrow DDH = 0$$

$$\Rightarrow 0 = [D_n D_n - D_n D_m + T_{mn}^k D_k] H_{AB} = \underline{F}_{mn} H_{AB}$$

$$\Rightarrow \underline{F}_{mn} + \underline{F}_{mn}^T = 0$$

Def Kalibr. transf. zw. der.  
 meijne transf.  $R$   
 $\phi \rightarrow \tilde{\phi} = R \cdot \phi$

man zw. der transf. kovariante

$$\tilde{D}\tilde{\phi} = \widetilde{D\phi}$$

Verten

$$\tilde{D} = D + \Lambda \quad \Lambda = -(DR) \cdot R^{-1}$$

direkt

$$\begin{aligned} \tilde{D}(R \cdot \phi) &= D(R \cdot \phi) + \Lambda \cdot R \cdot \phi \\ &= (DR) \cdot \phi + \underline{R \cdot (D\phi)} + \Lambda \cdot R \cdot \phi = \underline{R \cdot (D\phi)} \end{aligned}$$

$$\Lambda \cdot R = -DR$$

Verten  $A, \tilde{A}$  sowie kal. transf.  $A_{\mu\nu}^k$   
 nicht stetig  $\partial$  dann verhält sich  $A_{\mu\nu}, \tilde{A}_{\mu\nu}$

$$\begin{aligned} \tilde{A}_{\mu\nu} &= R \cdot A \cdot R^{-1} - (\partial R) \cdot R^{-1} \\ &= A - (DR) \cdot R^{-1} = A - R^{-1} \cdot (\tilde{D}R) \end{aligned}$$

direkt:

$$\begin{aligned} D\phi &= \partial\phi + A \cdot \phi & \tilde{D}\phi &= \partial\phi + \tilde{A} \cdot \phi \\ \tilde{D}\phi &= D\phi - (DR) \cdot R^{-1} \cdot \phi \\ &= \partial\phi + (A - (DR) \cdot R^{-1}) \cdot \phi \end{aligned}$$

$$\begin{aligned} \tilde{A} &= A - (DR) \cdot R^{-1} \\ &= A - (\partial R) \cdot R^{-1} - \cancel{A \cdot R \cdot R^{-1}} + R \cdot A \cdot R^{-1} \\ &= R \cdot A \cdot R^{-1} - (\partial R) \cdot R^{-1} \end{aligned}$$

$$D \Leftrightarrow \tilde{D} \quad A \Leftrightarrow \tilde{A} \quad R \Leftrightarrow R^{-1}$$

$$A = \tilde{A} - (\tilde{D}R^{-1}) \cdot R \quad \tilde{A} = A + (\tilde{D}R) \cdot R^{-1} = \tilde{A} - R^{-1} \cdot DR$$

Verten  $F, \tilde{F}$  transf. zw.  $D, \tilde{D}$

$$\tilde{F} = R \cdot F \cdot R^{-1}$$

$$\tilde{F}_{\mu\nu}^k = R_{\mu}^k \cdot R_{\nu}^{\mu} \cdot F_{\mu\nu}^k$$

$$\tilde{D}\tilde{\phi} = \widetilde{D\phi}$$

$$\tilde{D}\tilde{D}\tilde{\phi} = \widetilde{D D\phi}$$

$$\tilde{F} \cdot \tilde{\phi} = \widetilde{F \cdot \phi}$$

$$\tilde{F} \cdot R \cdot \phi = R \cdot F \cdot \phi$$

$$\tilde{F} = R \cdot F \cdot R^{-1}$$

Def Kalibr. transf kov. der.

meine transf  $R$

$$\phi \rightarrow \tilde{\phi} = R \cdot \phi$$

von kov. der transf ~~korrespondierende~~

$$\tilde{D}\tilde{\phi} = \widetilde{D\phi}$$

bedenken

$$\tilde{D} = D + \Lambda \quad \Lambda = -(DR) \cdot R^{-1}$$

dieses.

$$\tilde{D}(R \cdot \phi) = D(R \cdot \phi) + \Lambda \cdot R \cdot \phi$$

$$= (DR) \cdot \phi + \underline{R \cdot (D\phi)} + \Lambda \cdot R \cdot \phi = \underline{R \cdot (D\phi)}$$

$$\Lambda \cdot R = -DR$$



Věta

$D, \tilde{D}$  souvisí kal. transf.  $A_m^k$   
nůci stejné  $\partial$  dávny vešit,  $A_m, \tilde{A}_m$

$$\begin{aligned}\tilde{A}_m &= R \cdot A \cdot R^{-1} - (\partial R) \cdot R^{-1} \\ &= A - (DR) \cdot R^{-1} = A - R^{-1} \cdot (\tilde{D}R)\end{aligned}$$

důk:

$$D\phi = \partial\phi + A \cdot \phi \quad \tilde{D}\phi = \partial\phi + \tilde{A} \cdot \phi$$

$$\tilde{D}\phi = D\phi - (DR) \cdot R^{-1} \cdot \phi$$

$$= \partial\phi + (A - (DR) \cdot R^{-1}) \cdot \phi$$

$$\tilde{A} = A - (DR) \cdot R^{-1}$$

$$= A - (\partial R) \cdot R^{-1} - \cancel{A \cdot R \cdot R^{-1}} + R \cdot A \cdot R^{-1}$$

$$= R \cdot A \cdot R^{-1} - (\partial R) \cdot R^{-1}$$

$$D \Leftrightarrow \tilde{D} \quad A \Leftrightarrow \tilde{A} \quad R \Leftrightarrow R^{-1}$$

$$A = \tilde{A} - (\tilde{D}R^{-1}) \cdot R \quad \tilde{A} = A + (\tilde{D}R^{-1}) \cdot R = \tilde{A} - R^{-1} \cdot DR$$

Def 2

$F, F^{\sim}$  linear forms  $D, D^{\sim}$

$$\Leftrightarrow F = R \cdot F \cdot R^{-1}$$

$$\underline{F}_{m \times n}^{\sim k} = R_{m \times n}^k \underline{R}^{-1N} \underline{F}_{m \times n}^{\sim N}$$

$$\underline{D}^{\sim} \underline{\phi}^{\sim} = \underline{D} \underline{\phi}$$

$$\underline{D}^{\sim} \underline{D}^{\sim} \underline{\phi}^{\sim} = \underline{D} \underline{D} \underline{\phi}$$

$$\underline{F}^{\sim} \cdot \underline{\phi}^{\sim} = \underline{F} \cdot \underline{\phi}$$

$$\underline{F}^{\sim} \cdot R \cdot \underline{\phi} = R \cdot F \cdot \underline{\phi}$$

$$\underline{F}^{\sim} = R \cdot F \cdot R^{-1}$$

Komplexní vekt. bundl s  
hermitovskou strukturou  
(U-bundly)

$E_L = \underline{E}$  komplex. vekt. pr.  $\phi^A$   
 $E_R = \overline{E}$  sdružený kom. vekt. pr.  $\phi^{\overline{A}}$

$$E_L \leftrightarrow E_R$$

antilineární  $\overline{\alpha\phi} = \alpha^* \overline{\phi}$

$$\overline{\overline{\phi}} = \phi$$

$$E_{\lambda_L \lambda_R} = \underbrace{E_L \otimes \dots \otimes E_L}_{\lambda_L} \otimes \underbrace{E_L^* \otimes \dots \otimes E_L^*}_{\lambda_L} \otimes \underbrace{E_R \otimes \dots \otimes E_R}_{\lambda_R} \otimes \underbrace{E_R^* \otimes \dots \otimes E_R^*}_{\lambda_R}$$

antilineární operace

$$A : EM \rightarrow EM$$

$$(A\phi)^{\overline{A}} = A^{\overline{A}}_{\overline{B}} \overline{\phi}^{\overline{B}}$$

Skalární součin

$$\langle \cdot, \cdot \rangle_L \quad \langle \cdot, \cdot \rangle_R$$

generování pro hermit. str.  $h_{\overline{A}\overline{B}}$

$$\langle \phi_1, \phi_2 \rangle_L = \overline{\phi}_1^{\overline{A}} \phi_2^{\overline{B}} h_{\overline{A}\overline{B}}$$

$$\langle \psi_1, \psi_2 \rangle_R = \overline{\psi}_1^{\overline{A}} \psi_2^{\overline{B}} \overline{h}_{\overline{A}\overline{B}}$$

1)  $h$  nedegeener  $h^{-1\overline{A}\overline{B}}$

$$2) \overline{h}_{\overline{A}\overline{B}} = h_{\overline{B}\overline{A}}$$

3)  $h$  pos. def.  $\langle \phi, \phi \rangle_L > 0 \quad \phi \neq 0$

$$+ : E_{00}^{10} \leftrightarrow E_{10}^{00}$$

$$E \leftrightarrow E^*$$

$$(\phi^{\dagger})_{\overline{A}} = \overline{\phi}^{\overline{B}} h_{\overline{B}\overline{A}}$$

$$\phi^{\dagger\dagger} = \phi$$

$$\langle \phi_1, \phi_2 \rangle_L = \phi_1^{\dagger} \cdot \phi_2$$

$$+ : E_{10}^{10} \rightarrow E_{10}^{10}$$

$$(\chi^{\dagger})^{\overline{A}}_{\overline{B}} = \overline{h}^{\overline{A}\overline{C}} h_{\overline{C}\overline{B}} \chi^{\overline{E}}_{\overline{D}}$$

$$\langle \phi_1, \chi\phi_2 \rangle_L = \langle \chi^{\dagger}\phi_1, \phi_2 \rangle_L$$

$$+ : E_{00}^{01} \leftrightarrow E_{01}^{00}$$

$$\overline{E} \leftrightarrow \overline{E}^*$$

$$(\psi^{\dagger})_{\overline{B}} = \overline{\psi}^{\overline{A}} \overline{h}_{\overline{A}\overline{B}}$$

$$\psi^{\dagger\dagger} = \psi$$

$$\langle \psi_1, \psi_2 \rangle_R = \psi_1^{\dagger} \cdot \psi_2$$

$$+ : E_{01}^{01} \rightarrow E_{01}^{01}$$

$$\langle \psi_1, \chi\psi_2 \rangle_R = \langle \chi^{\dagger}\psi_1, \psi_2 \rangle_R$$

# Komplexní vekt. bundl s hermitovskou strukturou (U-bundly)

$E_L = E$  komplex. vekt.  $\mathbb{R}$ .  $\phi^A$   
 $E_R = \bar{E}$  sdružený komplex. vekt.  $\mathbb{R}$ .  $\phi^{\bar{A}}$

$$\bar{\quad} : E_L \leftrightarrow E_R$$

$\bar{\quad}$  antilineární  $\overline{\alpha\phi} = \alpha^* \bar{\phi}$

$$\overline{\overline{\phi}} = \phi$$

$$\phi_{\substack{A \dots B \dots \\ C \dots \bar{D} \dots}}$$

$$E_{\substack{l_L \ l_R \\ l_L \ l_R}} = \underbrace{E_L \otimes \dots \otimes E_L}_{l_L} \otimes \underbrace{E_L^* \otimes \dots \otimes E_L^*}_{l_L} \otimes \underbrace{E_R \otimes \dots \otimes E_R}_{l_R} \otimes \underbrace{E_R^* \otimes \dots \otimes E_R^*}_{l_R}$$

antilinear operation

$$a : EM \rightarrow EM$$

$$(a\phi)^A = A^A_B \bar{\phi}^B$$

Skalarinė daugyba

$$\langle , \rangle_L \quad \langle , \rangle_R$$

generuoti. nuo hermit. str.  $h_{\bar{A}B}$

$$\langle \phi_1, \phi_2 \rangle_L = \bar{\phi}_1^{\bar{A}} \phi_2^B h_{\bar{A}B}$$

$$\langle \psi_1, \psi_2 \rangle_R = \bar{\psi}_1^A \psi_2^{\bar{B}} h_{A\bar{B}}$$

1)  $h$  nedegeneruoti  $h^{-1\bar{A}B}$

2)  $\bar{h}_{\bar{A}B} = h_{\bar{B}A}$

3)  $h$  pos def.  $\langle \phi, \phi \rangle_L > 0 \quad \phi \neq 0$

$$+ : E_{00}^{10} \leftrightarrow E_{10}^{00}$$

$$E \leftrightarrow E^*$$

$$(\phi^+)_{\bar{A}} = \bar{\phi}^{\bar{A}} h_{\bar{A}A}$$

$$\phi^{++} = \phi$$

$$\langle \phi_1, \phi_2 \rangle_L = \phi_1^+ \cdot \phi_2$$

$$+ : E_{10}^{10} \rightarrow E_{10}^{10}$$

$$(\chi^+)_{\bar{B}} = \bar{h}^{\bar{A}A} h_{\bar{A}B} \bar{\chi}^{\bar{A}}$$

$$\langle \phi_1, \chi \cdot \phi_2 \rangle_L = \langle \chi^+ \phi_1, \phi_2 \rangle_L$$

$$+ : E_{00}^{01} \leftrightarrow E_{01}^{00}$$

$$\bar{E} \leftrightarrow \bar{E}^*$$

$$(\psi^+)_{\bar{A}} = \bar{\psi}^{\bar{A}} \bar{h}_{\bar{A}A}$$

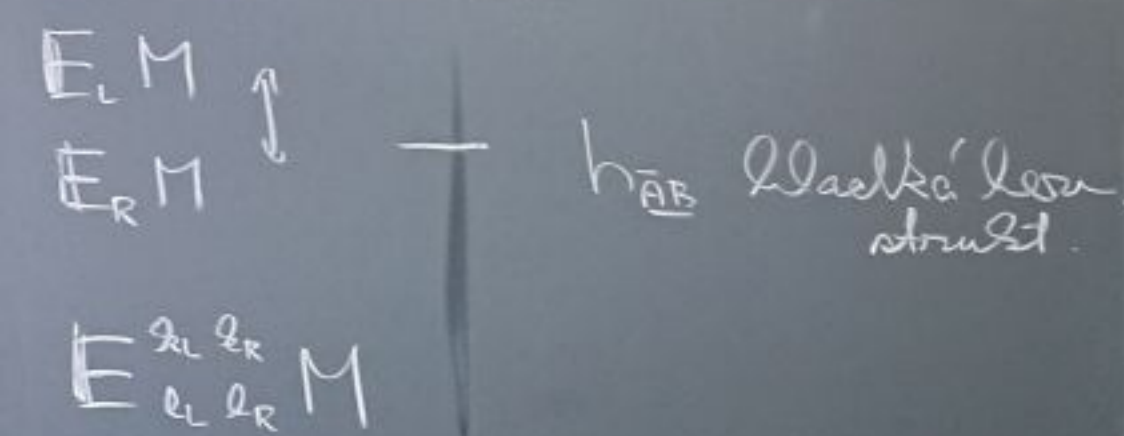
$$\psi^{++} = \psi$$

$$\langle \psi_1, \psi_2 \rangle_R = \psi_1^+ \cdot \psi_2$$

$$+ : E_{01}^{01} \rightarrow E_{01}^{01}$$

$$\langle \psi_1, \chi \cdot \psi_2 \rangle_R = \langle \chi^+ \psi_1, \psi_2 \rangle_R$$

Komplexní vekt. bundl s hermitovskou strukturou (U-bundly)



Unitární Galilej. transf.

ultralok. lin. operace

$$U: \text{Vect } E_L M \rightarrow \text{Vect } E_L M$$

$$U \in \text{Vect } E_{\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}} M$$

$$\phi \rightarrow \tilde{\phi} = U \cdot \phi \quad \phi \in \text{Vect } E_L M$$

$$\bar{U} \in \text{Vect } E_{\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}} M \quad \psi \in \text{Vect } E_R M$$

$$\psi \rightarrow \tilde{\psi} = \bar{U} \cdot \psi$$

$$\langle \tilde{\phi}_1, \tilde{\phi}_2 \rangle_L = \langle \phi_1, \phi_2 \rangle_L \quad \langle \tilde{\psi}_1, \tilde{\psi}_2 \rangle_R = \langle \psi_1, \psi_2 \rangle_R$$

$$h_{\bar{B}A} = \tilde{h}_{\bar{B}A} = \bar{U}^{-1 \bar{N}}_{\bar{B}} U^{\bar{M}}_{\bar{A}} h_{\bar{N}M}$$

$$U^\dagger \cdot U = \mathbb{1} = U \cdot U^\dagger$$

Ložní Gal. Lieova algebra

$$U_\alpha = \mathbb{1} + i\alpha u + O(\alpha^2)$$

$$iu - iu^\dagger = 0$$

$$u^\dagger = u$$

$$U_\alpha = \exp(i\alpha u)$$

Trivializ. konzist. s hermit. str.

$$E_A \quad \bar{E}_A$$

$$\langle E_A, E_B \rangle_L = \text{konst} \quad \langle \bar{E}_A, \bar{E}_B \rangle_R = \text{konst}$$

Pr. orthonom. báze

$$\langle E_A, E_B \rangle_L = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$$

Komplexní vekt. bundl s  
hermitovskou strukturou  
(U-bundly)

$$E_L M$$

$$E_R M$$



—

$$h_{\bar{A}B}$$

hlačka levo  
strukt.

$$E \begin{matrix} \mathcal{L}_L & \mathcal{L}_R \\ \mathcal{L}_L & \mathcal{L}_R \end{matrix} M$$



Unitary Galilei transf.

ultralocal lin. operace

$$U: \text{Sect } E_L M \rightarrow \text{Sect } E_L M$$

$$U \in \text{Sect } E_{10}^{10} M$$

$$\phi \rightarrow \tilde{\phi} = U \cdot \phi \quad \phi \in \text{Sect } E_L M$$

$$\bar{U} \in \text{Sect } E_{01}^{01} M$$

$$\psi \rightarrow \tilde{\psi} = \bar{U} \cdot \psi \quad \psi \in \text{Sect } E_R M$$

$$\langle \tilde{\phi}_1, \tilde{\phi}_2 \rangle_L = \langle \phi_1, \phi_2 \rangle_L \quad \langle \tilde{\psi}_1, \tilde{\psi}_2 \rangle_R = \langle \psi_1, \psi_2 \rangle_R$$

$$h_{\bar{B}\bar{A}} = \tilde{h}_{\bar{B}\bar{A}} = \bar{U}^{-1\bar{N}}_{\bar{B}} U^{\bar{M}}_{\bar{A}} h_{\bar{N}\bar{M}}$$

$$U^\dagger \cdot U = \mathbb{1} = U \cdot U^\dagger$$

Lorentz'nal. Lieova algebra

$$U_\alpha = \mathbb{1} + i\alpha u + \mathcal{O}(\alpha^2)$$

$$iu - iu^\dagger = 0$$

$$u^\dagger = u$$

$$U_\alpha = \exp(i\alpha u)$$

Trivializ. konzist. s herm. str

$$E_A \quad \bar{E}_A$$

$$\langle E_A, E_B \rangle_L = \text{konst} \quad \langle \bar{E}_A, \bar{E}_B \rangle_R = \text{konst}$$

Pr. ortogonal. baze

$$\langle E_A, E_B \rangle_L = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

Kov. der. konzistentní s  
hermitovskou strukturou

Def pozitivní  $D \in \text{EM}$  na  $\bar{E}M$

$$\bar{D}\bar{\phi} = \overline{D\phi}$$

společná der na  $E_{\ell_1 \ell_2}^{\ell_1 \ell_2} M$

$$D = D \oplus \bar{D}$$

celková der je reálná

$$\bar{D} = D$$

Def Kov. der  $D$  je konzistentní s  
hermit strukturou

$$Dh = 0$$

$$D_{\xi} \langle \phi_1, \phi_2 \rangle_L = \langle D_{\xi} \phi_1, \phi_2 \rangle_L + \langle \phi_1, D_{\xi} \phi_2 \rangle_L$$

Lem  
trivial. konzistentní str

$$\partial h = 0$$

$$\begin{matrix} M \\ \bar{M} \end{matrix} \rightarrow$$

$$\begin{aligned} M \chi_{\underline{C} \bar{D}}^{\underline{A} \bar{B}} &= M_{\underline{K}}^{\underline{A}} \chi_{\underline{C} \bar{D}}^{\underline{K} \bar{B}} + \dots - M_{\underline{C}}^{\underline{K}} \chi_{\underline{K} \bar{D}}^{\underline{A} \bar{B}} \\ &+ \bar{M}_{\underline{K}}^{\bar{B}} \chi_{\underline{C} \bar{D}}^{\underline{A} \bar{K}} + \dots - \bar{M}_{\bar{D}}^{\bar{K}} \chi_{\underline{C} \bar{K}}^{\underline{A} \bar{B}} \end{aligned}$$

Konzistentní pseudoder. na  $E_{\ell_1 \ell_2}^{\ell_1 \ell_2} M$

$$\bar{M} \bar{\chi} = \overline{M \chi}$$

společné pseudoder

$$M = M \oplus \bar{M}$$

$$M = \bar{M}$$

$$M \phi^{\underline{A}} = M_{\underline{B}}^{\underline{A}} \phi^{\underline{B}} \quad \bar{M} \psi^{\bar{A}} = \bar{M}_{\bar{B}}^{\bar{A}} \psi^{\bar{B}}$$

Kov. der. konzistentní s

hermitovskou strukturou

Def. rozšíření  $D \supseteq \mathbb{E} \Gamma_{mc} \bar{\mathbb{E}} M$

$$\bar{D} \bar{\phi} = \overline{D \phi}$$

společná der. na  $\mathbb{E} \begin{matrix} \mathbb{R}_L & \mathbb{R}_R \\ \mathbb{R}_L & \mathbb{R}_R \end{matrix} M$

$$D = D \oplus \bar{D}$$

celková der. je reálná

$$\bar{D} = D$$

Def Kov. der  $D$  je Kov.  $\sigma$   
hermit strukt

$$Dh = 0$$

$$D_{\xi} \langle \phi_1, \phi_2 \rangle_L = \langle D_{\xi} \phi_1, \phi_2 \rangle_L + \langle \phi_1, D_{\xi} \phi_2 \rangle_L$$

Lemma

trivial. Kov.  $\sigma$  hermit str

$$\partial h = 0$$

Rozložení - pseudosym. na  $E_{\mathbb{R}}^{\mathbb{R}} M$

$$\overline{M} \overline{X} = \overline{MX}$$

symetrické jádrem

$$M = M \oplus \overline{M}$$

$$M = \overline{M}$$

$$M \phi^A = M^A_{\underline{B}} \phi^B \quad \overline{M} \psi^{\overline{A}} = \overline{M}^{\overline{A}}_{\overline{B}} \psi^{\overline{B}}$$

$$M \chi_{\underline{C} \overline{D}}^{\overline{A} \underline{B}} = M^{\overline{A}}_{\underline{K}} \chi_{\underline{C} \overline{D}}^{\underline{K} \overline{B}} + \dots - M^{\underline{K}}_{\underline{C}} \chi_{\underline{K} \overline{D}}^{\overline{A} \overline{B}} - \dots \\ + \overline{M}^{\overline{B}}_{\overline{K}} \chi_{\underline{C} \overline{D}}^{\overline{A} \underline{K}} + \dots - \overline{M}^{\underline{K}}_{\overline{D}} \chi_{\underline{C} \overline{K}}^{\overline{A} \underline{B}} - \dots$$

← jádrem  $M$

← jádrem  $\overline{M}$

$D \tilde{D}$  komutativ hermitisch

$D - \tilde{D}$  pseudodifferenzial generiert

rozdiel tenz  $iA_m^k$

$$D_m \phi^A - \tilde{D}_m \phi^A = iA_m^A{}^B \phi^B$$

$$D_m \psi^{\bar{A}} - \tilde{D}_m \psi^{\bar{A}} = -i\bar{A}_m^{\bar{A}}{}^{\bar{B}} \psi^{\bar{B}}$$

plund  $\tilde{D} = \partial$   $A_m$  vekt. potencial

Vektor  $A_m^+ = A_m$

div  $Dh = \tilde{D}h = 0$

tenzor kovariant:  $D$

$$iF_{mn}^A \phi^B = [D_m D_n - D_n D_m + T_{mn}^k D_k] \phi^A = \overset{D}{d}_m \overset{D}{d}_n \phi^A$$

$$-i\bar{F}_{mn}^{\bar{A}} \psi^{\bar{B}} = [D_m D_n - D_n D_m + T_{mn}^k D_k] \psi^{\bar{A}} = \overset{D}{d}_m \overset{D}{d}_n \psi^{\bar{A}}$$

Vektor

$$F^+ = F$$

$$Fh = 0$$

kalibracni transform

$$\phi \rightarrow \tilde{\phi} = U \cdot \phi$$

$$\psi \rightarrow \tilde{\psi} = \bar{U} \cdot \psi$$

$$D \rightarrow \tilde{D} = D + \Lambda$$

$$\Lambda = -(DU) \cdot U^\dagger$$

$$D = \partial + iA \quad \tilde{D} = \partial + i\tilde{A}$$

$$\tilde{A} = U \cdot A \cdot U^\dagger + i(\partial U) \cdot U^{-1}$$

$$= A + i(DU) \cdot U^{-1}$$

$$= A + U^{-1} \cdot (\tilde{D}U)$$

$$\tilde{F} = U \cdot F \cdot U^{-1} \quad \bar{U}^{-1} = U^\dagger$$

$D \tilde{D}$  komutativ hermitisch

$D - \tilde{D}$  pseudodifferenzial gener.

rozdiel tenz  $iA_m^k$

$$D_m \phi^A - \tilde{D}_m \phi^A = iA_m^A{}_B \phi^B$$

$$D_m \psi^{\bar{B}} - \tilde{D}_m \psi^{\bar{B}} = -i\bar{A}_m^{\bar{A}}{}_{\bar{B}} \psi^{\bar{B}}$$

plund  $\tilde{D} = 0$   $A_m$  vekt. potenciál

Väta

$$A_m^+ = A_m$$

die  $Dh = \tilde{D}h = 0$



tenzor křivosti  $D$

$$i F_{\underline{mn}B}^A \phi^B = [D_{\underline{m}D_{\underline{n}}} - D_{\underline{n}D_{\underline{m}}} + T_{\underline{mn}}^{\underline{k}} D_{\underline{k}}] \phi^A = \overset{D}{d_{\underline{m}}} \overset{D}{d_{\underline{n}}} \phi^A$$

$$-i \bar{F}_{\underline{mn} \bar{B}}^{\bar{A}} \psi^{\bar{B}} = [D D - D D + T D] \psi^{\bar{A}} = \overset{D}{d_{\underline{m}}} \overset{D}{d_{\underline{n}}} \psi^{\bar{A}}$$

Věta

$$F^{\dagger} = F$$

$$F h = 0$$

Kalibracīm' transformācija

$$\phi \rightarrow \tilde{\phi} = U \cdot \phi$$

$$\psi \rightarrow \tilde{\psi} = \bar{U} \cdot \psi$$

$$D \rightarrow \tilde{D} = D + \Lambda$$

$$\Lambda = - (DU) \cdot U^\dagger$$

$$D = \partial + iA \quad \tilde{D} = \partial + i\tilde{A}$$

$$\tilde{A} = U \cdot A \cdot U^\dagger + i(\partial U) \cdot U^{-1}$$

$$= A + i(DU) \cdot U^{-1}$$

$$= A + U^{-1} \cdot (\tilde{D}U)$$

$$\tilde{F} = U \cdot F \cdot U^{-1} \quad \bar{U}^{-1} = U^\dagger$$

Lineární  $U(1)$ -bundl

$U(1)$ -nabitá pole

$$\mathbb{C} \quad \bar{\mathbb{C}} \quad \begin{matrix} \mathbb{C}^{\mathbb{Z}_L \mathbb{Z}_R} \\ \mathbb{C}^{\mathbb{L} \mathbb{L}_R} \end{matrix} \quad h_{\bar{A}B}$$

$$\mathbb{C}^{\mathbb{Z}+1} \leftrightarrow \mathbb{C}^{\mathbb{Z}}$$

$$\phi_{\dots \bar{B}}^{\dots A} \leftrightarrow \phi_{\dots \bar{M}}^{\dots M} \quad \phi_{\dots \bar{B}}^{\dots A} = \phi_{\dots \bar{M}}^{\dots M} \mathbb{1}_{\bar{B}}^A$$

$$\mathbb{C}^{\mathbb{Z}} \rightarrow \begin{cases} \mathbb{C}^{\mathbb{Z}0} \\ \mathbb{C}^0 \end{cases} \quad \mathbb{C}^{\mathbb{Z}\mathbb{Z}} \rightarrow \begin{cases} \mathbb{C}^{\mathbb{Z}\mathbb{Z}0} \\ \mathbb{C}^{0\mathbb{L}} \end{cases}$$

$$\mathbb{C}^{\mathbb{Z}+1 \mathbb{Z}+1} \leftrightarrow \mathbb{C}^{\mathbb{Z}\mathbb{L}}$$

$$\phi_{\dots \bar{B} \dots \bar{B}}^{\dots A \dots A} \leftrightarrow \phi_{\dots \bar{M} \dots \bar{N}}^{\dots M \dots N} \quad h_{\bar{N}M}$$

$$= (\phi_{\dots \bar{M} \dots \bar{N}}^{\dots M \dots N}) h_{\bar{N}M}$$

$$\Psi_{\bar{A}} \leftrightarrow \phi_{\bar{B}} = \Psi_{\bar{A}} h_{\bar{A}B}$$

$$\Psi_{\bar{A}} \leftrightarrow \phi^{\bar{B}} = \Psi_{\bar{A}} h^{\bar{A}B}$$

$$\mathbb{C}^{\mathbb{Z}_L \mathbb{Z}_R} \rightarrow \begin{cases} \mathbb{C}^{\mathbb{Z}0} \\ \mathbb{C} \\ \mathbb{C}^{0\mathbb{Z}} \end{cases} \quad \begin{matrix} \mathbb{C}^{\mathbb{Z}} \\ \mathbb{C}^0 \\ \mathbb{C}^{-\mathbb{Z}} \end{matrix}$$

$$\mathbb{C}^m \quad \begin{matrix} \phi & \psi \\ m & m \end{matrix} \rightarrow m+m$$

μάθημα κεντρική

$$- : \mathbb{C}^m \rightarrow \mathbb{C}^{-m}$$

$$\bar{\bar{\phi}} = \phi$$

$$(\phi, \psi) = \underbrace{\bar{\phi}}_0 \psi \in \mathbb{C}$$

$U(1)$ -bundl

$$\mathbb{C}^m M$$

$U(1)$ -kalibraien transf

$$U \in \text{Vect } \mathbb{C}^1 M = \text{Vect } \mathbb{C}^0 M = \mathbb{F} M$$

$$U^* U = 1 \quad U = \exp(iu) \quad \begin{matrix} u \in \mathbb{F} M \\ u(x) \in \mathbb{R} \end{matrix}$$

$$\phi \in \mathbb{C}^1 M \quad \phi \rightarrow \hat{\phi} = \exp(iu) \phi$$

$$\phi \in \mathbb{C}^m M \quad \phi \rightarrow \hat{\phi} = \exp(imu) \phi$$

trivializace kous olov. str

$$\mathbb{C}^1 M \quad E \quad \bar{E} E = 1 \quad \phi = \varphi E \quad \varphi \in \mathbb{F} M$$

$$\mathbb{C}^m M \quad E^m \quad \psi = \psi E^m \quad \psi \in \mathbb{F} M$$

# Lineární $U(1)$ -bundl

$U(1)$ -nabitá pole

$$\mathbb{C} \quad \bar{\mathbb{C}} \quad \mathbb{C} \begin{matrix} \mathbb{Z}_L & \mathbb{Z}_R \\ l_L & l_R \end{matrix} \quad h_{\bar{A}B}$$

$$\mathbb{C} \begin{matrix} \mathbb{Z}+1 \\ l+1 \end{matrix} \leftrightarrow \mathbb{C} \begin{matrix} \mathbb{Z} \\ l \end{matrix}$$

$$\phi \begin{matrix} \dots A \\ \dots B \end{matrix} \leftrightarrow \phi \begin{matrix} \dots M \\ \dots \bar{M} \end{matrix} \quad \phi \begin{matrix} \dots A \\ \dots B \end{matrix} = \phi \begin{matrix} \dots M \\ \dots \bar{M} \end{matrix} \begin{matrix} \uparrow A \\ \downarrow B \end{matrix}$$

$$\mathbb{C} \begin{matrix} \mathbb{Z} \\ l \end{matrix} \rightarrow \left\{ \begin{matrix} \mathbb{C} \begin{matrix} \mathbb{Z} \\ 0 \end{matrix} \\ \mathbb{C} \begin{matrix} \mathbb{Z} \\ l \end{matrix} \end{matrix} \right. \quad \mathbb{C} \begin{matrix} \mathbb{Z} \\ l \end{matrix} \rightarrow \left\{ \begin{matrix} \mathbb{C} \begin{matrix} \mathbb{Z} \\ 0 \end{matrix} \\ \mathbb{C} \begin{matrix} \mathbb{Z} \\ l \end{matrix} \end{matrix} \right.$$

$$\mathbb{C} \begin{matrix} \mathbb{Z}+1 & l+1 \\ p & q \end{matrix} \leftrightarrow \mathbb{C} \begin{matrix} \mathbb{Z} & l \\ p & q \end{matrix}$$

$$\phi \begin{matrix} \dots A \dots \bar{B} \\ \dots \end{matrix} \mapsto \phi \begin{matrix} \dots M \dots \bar{N} \\ \dots \end{matrix} \quad h_{\bar{N}M}$$

$$= \left( \phi \begin{matrix} \dots M \dots \bar{N} \\ \dots \end{matrix} h_{\bar{N}M} \right) h_{\bar{A}B}$$

$$\psi^{\bar{A}} \leftrightarrow \phi_{\underline{B}} = \psi^{\bar{A}} h_{\underline{A}\underline{B}}$$

$$\psi_{\underline{A}} \leftrightarrow \phi^{\bar{B}} = \psi_{\underline{A}} h^{\bar{A}\bar{B}}$$

$$\begin{matrix} \mathbb{C}^{z_L z_R} \\ \mathbb{C}^{l_L l_R} \end{matrix} \rightarrow \left\{ \begin{matrix} \mathbb{C}^{z 0} \\ \mathbb{C}^{0 0} \\ \mathbb{C}^{0 z} \\ \mathbb{C}^{0 0} \end{matrix} \right. \quad \begin{matrix} \mathbb{C}^{z} \\ \mathbb{C}^0 \\ \mathbb{C}^{-z} \end{matrix}$$

$$\begin{matrix} \mathbb{C}^m & \phi & \psi & \rightarrow & m+m \\ & m & m & & \end{matrix}$$

μάθημα κεντρική

$$- : \mathbb{C}^m \rightarrow \mathbb{C}^{-m}$$

$$\bar{\phi} = \phi$$

$$\underbrace{(\phi, \psi)}_{m \quad m} = \underbrace{\bar{\phi}}_0 \psi \in \mathbb{C}$$

$U(1)$ -bundl

$\mathbb{C}^m M$

$U(1)$ -Kalibriertransf.

$$U \in \text{Vect } \mathbb{C}^1 M = \text{Vect } \mathbb{C}^0 M = \mathcal{F}M$$

$$U^* U = 1 \quad U = \exp(iu) \quad \begin{array}{l} u \in \mathcal{F}M \\ u(x) \in \mathbb{R} \end{array}$$

$$\phi \in \mathbb{C}^1 M \quad \phi \rightarrow \hat{\phi} = \exp(iu) \phi$$

$$\phi \in \mathbb{C}^m M \quad \phi \rightarrow \hat{\phi} = \exp(imu) \phi$$

trivialisierung  $\text{Komb. observ. ste.}$

$$\mathbb{C}^1 M \quad E \quad \bar{E} E = 1 \quad \phi = \varphi E \quad \varphi \in \mathcal{F}M$$

$$\mathbb{C}^m M \quad E^m \quad \psi = \psi E^m \quad \psi \in \mathcal{F}M$$

$U(1)$ -Kovar. derivace

$$\mathbb{C}^m M \quad \bar{\phi} \quad \phi \psi$$

$$D(\phi \psi) = (D\phi) \psi + \phi (D\psi)$$

$$D\bar{\phi} = \overline{D\phi}$$

$$D, \tilde{D} \quad \phi \in \mathbb{C}^m M$$

$$D_m \phi - \tilde{D}_m \phi = i A_m \phi \quad A_m \in \mathbb{C}^m M$$

$$\psi \in \mathbb{C}^m M \quad D_m \psi - \tilde{D}_m \psi = im A_m \psi$$

Arnold  $E \quad \partial E = 0$

$$D\phi = \partial\phi + iA_m \phi \quad A_m \text{ vekt. potenciál}$$

$$A_m^x = A_m$$

tenzor křivosti

$$\phi \in \mathbb{C}^m M \quad i F_{mn} \phi = [D_m D_n - D_n D_m + T_{mn}^k D_k] \phi = \partial_m \partial_n \phi$$

$$\psi \in \mathbb{C}^m M \quad im F_{mn} \psi = [D_m D_n - D_n D_m + T D] \psi = \partial_m \partial_n \psi$$

$$F_{mn} = \partial_m A_n - \partial_n A_m + i \underbrace{[A_m, A_n]}_0 = \partial_m A_n \quad \mathbb{C}^0 M$$

$$D = \partial + iA$$

$$g^{mn} D_m F_{an} = J_a \quad \nabla_m F^{am} = J^a$$

$$- g^{mn} D_m D_n \phi + M^2 \phi = 0$$

$$J_a = mi (\bar{\phi} D_a \phi - \phi D_a \bar{\phi}) \quad \phi \in \mathbb{C}^m M$$

$$\phi \in \mathbb{C}^m M \quad \phi \rightarrow \tilde{\phi} = U^m \phi$$

$$U = \exp(iu)$$

$$D \rightarrow \tilde{D} = D + \Lambda$$

$$\Lambda = -(DU) \cdot U^{-1} = -idu$$

$$\tilde{D}\phi = D\phi - im du \phi$$

$$A \rightarrow \tilde{A} = U \cdot A \cdot U^{-1} + i(\partial U) \cdot U^{-1} = A - du$$

$$F \rightarrow \tilde{F} = U \cdot F \cdot U^{-1} = F$$

# $U(1)$ -Kovar. derivace

$$\mathbb{C}^m M \quad \bar{\phi} \quad \phi \psi$$

$$D(\phi \psi) = (D\phi) \psi + \phi(D\psi)$$

$$D\bar{\phi} = \overline{D\phi}$$

$$D, \tilde{D} \quad \phi \in \mathbb{C}^m M$$

$$D_m \phi - \tilde{D}_m \phi = i A_m \phi \quad A_m \in \mathbb{C}^m M$$

$$\psi \in \mathbb{C}^m M \quad D_m \psi - \tilde{D}_m \psi = i m A_m \psi$$

$$\text{Anivial } E \quad \partial E = 0$$

$$D\phi = \partial\phi + i A_m \phi \quad A_m \text{ vekt. potencial}$$

$$A_m^* = A_m$$



tenzor krivosti

$$\phi \in \mathbb{C}^1 M \quad i F_{\underline{mn}} \phi = [D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}} + T_{\underline{mn}}^k D_k] \phi = \overset{D}{\partial}_{\underline{m}} \overset{D}{\partial}_{\underline{n}} \phi$$

$$\psi \in \mathbb{C}^n M \quad i_m F_{\underline{mn}} \psi = [D D - D D + T D] \psi = \overset{D}{\partial}_{\underline{m}} \overset{D}{\partial}_{\underline{n}} \psi$$

$$F_{\underline{mn}} = \overset{\partial}{\partial}_{\underline{m}} A_{\underline{n}} + i \underbrace{[A_{\underline{m}}, A_{\underline{n}}]}_0 = \overset{\partial}{\partial}_{\underline{m}} A_{\underline{n}}$$

$$D = \partial + iA$$

$$\phi \in \mathbb{C}^n M \quad \phi \rightarrow \tilde{\phi} = U^m \phi$$

$$U = \exp(iu)$$

$$D \rightarrow \tilde{D} = D + \Lambda$$

$$\Lambda = -(DU) \cdot U^{-1} = -i du$$

$$\tilde{D} \phi = D\phi - im du \phi$$

$$\begin{aligned} A \rightarrow \tilde{A} &= U \cdot A \cdot U^{-1} + i(DU) \cdot U^{-1} \\ &= A - du \end{aligned}$$

$$F \rightarrow \tilde{F} = U \cdot F \cdot U^{-1} = F$$

$$F_{mn} = \partial_m A_n - \partial_n A_m + i \underbrace{[A_m, A_n]}_0 = d_m A_n \quad \mathbb{C}^0 M$$

$$D = \partial + iA$$

$$g^{mn} D_m F_{an} = J_a \quad \nabla_m F^{am} = J^a$$

$$- g^{mn} D_m D_n \phi + M^2 \phi = 0$$

$$J_a = mi (\bar{\phi} D_a \phi - \phi D_a \bar{\phi}) \quad \phi \in \mathbb{C}^n M$$

$G$  holomorfne  $Ad$

$\mathfrak{g}$  holomorfne  $ad$

$k$  mede gener.

$\text{Der } \mathfrak{g} = \text{ad } \mathfrak{g}$

$\rightarrow$  algebra operatoru  
Anvaru  $ad_m$   $m \in \mathfrak{g}$

prostor alg. der. tj. oper.  $\delta$   
 $\delta(m, n) = [\delta m, n] + [m, \delta n]$

vekt pr.  $A$

repr. gr.  $G$  na  $A$

repr. alg  $\mathfrak{g}$  na  $A$

$T_{A, B}^A$

$$t_m^A|_B = m^{\sim} t_{\alpha}^A|_B$$

$$A = A_0 \oplus \bigoplus_{\mathfrak{k}} A_{\mathfrak{k}}$$

$P_{\mathfrak{k}}$  projektor na  $A_{\mathfrak{k}}$

$$t_{\alpha}^A|_B = \bigoplus_{\mathfrak{k}} t_{\mathfrak{k}}^A|_B$$

$$t_{\mathfrak{k}} = P_{\mathfrak{k}} \cdot t \cdot P_{\mathfrak{k}}$$

$t_{\mathfrak{k}}$  ireducibilni

$$[S, t_m] = 0 \Rightarrow S = \sum_{\mathfrak{k}} A_{\mathfrak{k}} P_{\mathfrak{k}}$$

$\forall m \in \mathfrak{g}$

$G$  polynomiál  $Ad$

$\mathfrak{g}$  polynomiál  $ad$

$k$  medegener.

$\text{Der } \mathfrak{g} = \text{ad } \mathfrak{g}$

$\rightarrow$  algebra operátoru  
Annam  $ad_m$   $m \in \mathfrak{g}$

$\rightarrow$  prostor  $\text{alg. deriv.}$   $\dagger_j$  oper.  $\delta$   
 $\delta[m, n] = [\delta m, n] + [m, \delta n]$

vekt. pr.  $A$

repr. gr.  $G$  na  $A$

repr. alg  $\mathcal{G}$  na  $A$

$$T_{A, B}^A$$

$$t_{m, B}^A = m^\alpha t_{\alpha, B}^A$$

$$A = A_0 \oplus \bigoplus_k A_k$$

$P_k$  projektor na  $A_k$

$$t_{\alpha, B}^A = \bigoplus_k t_{k, \alpha, B}^A$$

$$t_k = P_k \cdot t \cdot P_k$$

$t_k$  irreducibilni

$$[S, t_m] = 0 \Rightarrow$$

$$S = \sum_k A_k P_k$$

$\forall m \in \mathcal{G}$

# Lokální kalibrační grupa a algebra

$GM$   $h \in \text{Vect } GM$   
 $h(x) \in G_x M$

$\mathfrak{g}M$   $m \in \text{Vect } \mathfrak{g}M$   
 $m(x) \in \mathfrak{g}_x M$

$$[m, m]^x = m^k m^l C_{kl}^x$$

$$\text{ad}_m m = [m, m]$$

$$\text{ad}_m^k = m^l C_{kl}^x$$

Kovarov. der. na bundlu Lieov alg.

Def. mějme bundl. Lieov alg  $\mathfrak{g}M$   
kov. der.  $\mathcal{D}$  je kovar. s alg. str.

$$\mathcal{D}c = 0$$

$$\mathcal{D}[m, m] = [\mathcal{D}m, m] + [m, \mathcal{D}m]$$

Lema

$$\mathcal{D}k = 0$$

Def. Trivializace kovar. s alg. str.

$e_\alpha$  báze  $\mathfrak{g}M$

$C_{\alpha\beta}^x = \text{konst}$  (konst.  $C$  vůči  $e_\alpha$ )

Lema

$\partial$  kovar. der. trivial  $e_\alpha$   
kovar. s alg. str.

$$\partial e_\alpha = 0$$

$$\Downarrow \partial c = 0 \quad \partial k = 0 \quad \partial \text{kovar. s alg. str.}$$

# Lokální kalibrační grupa a algebra

$$GM \quad h \in \text{Sect } GM$$

$$h(x) \in G_x M$$

$$\mathfrak{g}M \quad m \in \text{Sect } \mathfrak{g}M$$

$$m(x) \in \mathfrak{g}_x M$$

$$[m, m]^\alpha = m^k m^l C_{kl}^\alpha$$

$$\text{ad}_m m = [m, m]$$

$$\text{ad}_m^\alpha \mathfrak{f} = m^k C_{kl}^\alpha \mathfrak{f}$$



Konv. der. ma bunden Liouy alg.

Def  
miejm bundl. L alg  $\mathfrak{g}^M$   
konv. der.  $D$  je konv. s alg str.

$$Dc = 0$$

$$\Downarrow D[m, m] = [Dm, m] + [m, Dm]$$

Lema

$$Dk = 0$$

Def Anisialisace konv. s alg str.

$e_\alpha$  báze  $\mathfrak{g}^M$

$C_{\alpha\beta}^{\gamma} = \text{konst}$  (konj. C mezi  $e_\alpha$ )

Lemma

① sowie der trivial  $e_x$   
konver  $\rightarrow$  alg. str.

$$\partial e_x = 0$$

$\Downarrow$

$$\partial c = 0$$

$$\partial k = 0$$

① konver  $\rightarrow$  alg str

Viete:

$\tilde{D}, \tilde{D}$  kov. der me  $\mathfrak{g}M$

$$D = \tilde{D} + A \quad A_{\mu\nu}^{\alpha} \in \mathbb{R}$$

$$A \cdot [m, m] = [A \cdot m, m] + [m, A \cdot m]$$

$$\Rightarrow \exists \mathcal{R} \quad A = \text{ad}_{\mathcal{R}}$$

$$\mathcal{R}_{\mu\nu}^{\alpha} \quad A_{\mu\nu}^{\alpha} = \mathcal{R}_{\mu\nu}^{\alpha} C_{\alpha\beta}^{\gamma}$$

Lemma

$\partial$  sowie der.

$$\equiv A = \text{ad}_{\mathcal{R}}$$

$$D = \partial + A \quad \text{je kov. u. abg. str.}$$

Viete

me  $\mathfrak{g}M$

$$\begin{aligned} D_{\mu} m^{\nu} &= \partial_{\mu} m^{\nu} + A_{\mu\alpha}^{\nu} m^{\alpha} \\ &= \partial_{\mu} m^{\nu} + [\mathcal{R}_{\mu}, m]^{\nu} \end{aligned}$$

$$B \in \mathfrak{g} \text{ der } \mathfrak{g}_1 M \quad B_{\mu}^{\alpha} = \text{ad}_{B^{\alpha}} = B^{\alpha} C_{\alpha\beta}^{\gamma}$$

$$D_{\mu} B_{\nu}^{\alpha} = \partial_{\mu} B_{\nu}^{\alpha} + [A_{\mu}, B]^{\alpha}_{\nu}$$

$$(D_{\mu} B^{\alpha}) C_{\alpha\beta}^{\gamma} = (\partial_{\mu} B^{\alpha} + [\mathcal{R}_{\mu}, B]^{\alpha}) C_{\alpha\beta}^{\gamma}$$

$$\begin{aligned} \hat{F}_{\mu\nu}^{\alpha} &= \partial_{\mu} \mathcal{R}_{\nu}^{\alpha} - \partial_{\nu} \mathcal{R}_{\mu}^{\alpha} + [\mathcal{R}_{\mu}, \mathcal{R}_{\nu}]^{\alpha} \\ &= \partial_{\mu} \mathcal{R}_{\nu}^{\alpha} - \partial_{\nu} \mathcal{R}_{\mu}^{\alpha} + \mathcal{R}_{\mu}^{\beta} \mathcal{R}_{\nu}^{\gamma} C_{\beta\gamma}^{\alpha} \\ T &= 0 \end{aligned}$$

Viete:

Viete  $F_{\mu\nu}^{\alpha}$  tensor für der  $D_{\mu}$

$$F_{\mu\nu} c = 0$$

$$F_{\mu\nu} \cdot [m, m] = [F_{\mu\nu} \cdot m, m] + [m, F_{\mu\nu} \cdot m]$$

$$\Downarrow \quad \hat{F} = \text{ad}_{\hat{F}}$$

$$F_{\mu\nu}^{\alpha} = \hat{F}_{\mu\nu}^{\alpha} C_{\alpha\beta}^{\gamma}$$

$$F_{\mu\nu} B^{\alpha} = \hat{F}_{\mu\nu} B^{\alpha} = [\hat{F}, B]^{\alpha}$$

$$F_{\mu\nu} B_{\alpha}^{\beta} = [\hat{F}_{\mu\nu}, B]^{\alpha}_{\beta} = [\hat{F}, B]^{\alpha}_{\beta} C_{\alpha\beta}^{\gamma}$$

$D$  kov. der. dann nicht pot  $\mathcal{R}$  u.  $\partial$

$$\hat{F}_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + [A_{\mu}, A_{\nu}]$$

$$\hat{F}_{\mu\nu} = \partial_{\mu} \mathcal{R}_{\nu} - \partial_{\nu} \mathcal{R}_{\mu} + [\mathcal{R}_{\mu}, \mathcal{R}_{\nu}]$$

Viele:

$D, \tilde{D}$  kov. der ma  $\mathfrak{g}M$

$$D = \tilde{D} + A \quad A_{m \times \mathbb{R}}$$

$$A \cdot [m, m] = [A \cdot m, m] + [m, A \cdot m]$$

$$\Rightarrow \exists \mathfrak{X} \quad A = \text{ad}_{\mathfrak{X}}$$
$$\mathfrak{X}_{m \times \mathbb{R}} \quad A_{m \times \mathbb{R}} = \mathfrak{X}_{m \times \mathbb{R}} C_{\mathbb{R} \times \mathbb{R}}$$

Lemma

$\partial$  sowie der.

$$\Leftrightarrow A = \text{ad}_{\mathfrak{X}}$$

$$D = \partial + A \quad \text{je kov. u. alg. str.}$$

Let  $e$

$m \in \text{Lie } \mathfrak{g}_1 M$

$$\begin{aligned} D_m m^{\text{R}} &= \partial_m m^{\text{R}} + A_{\text{B}}^{\text{R}} m^{\text{B}} \\ &= \partial_m m^{\text{R}} + [\mathcal{F}_m, m]^{\text{R}} \end{aligned}$$

$$B \in \text{Lie } \mathfrak{g}_1 M \quad B^{\text{R}}_{\text{B}} = \text{ad}_B^{\text{R}} = B^{\text{K}} C_{\text{KB}}^{\text{R}}$$

$$D_m B^{\text{R}}_{\text{B}} = \partial_m B^{\text{R}}_{\text{B}} + [A_m, B]^{\text{R}}_{\text{B}}$$

$$(D_m B^{\text{K}}) C_{\text{KB}}^{\text{R}} = (\partial_m B^{\text{K}} + [\mathcal{F}_m, B]^{\text{K}}) C_{\text{KB}}^{\text{R}}$$

Wieder  $F_{mn}^\alpha$  tensor für der  $D_m$

$$F_{mn} C = 0$$

$$F_{mn} \cdot [m, m] = [F_{mn} \cdot m, m] + [m, F_{mn} \cdot m]$$

$$\Downarrow \exists F_{mn} \quad F = \text{adj}_F$$

$$F_{mn}^\alpha = F_{mn}^k C_{k\beta}^\alpha$$

$$F_{mn} B^k = F_{mn}^k B^\alpha = [F, B]^k$$

$$F_{mn} B^\alpha = [F_{mn}, B]^\alpha = [F, B]^k C_{k\beta}^\alpha$$

Wieder:  
D Kov. der, dann nicht pot  $\mathcal{R}$  nicht  $\circ$

$$F_{mn} = \partial_m A_n - [A_m, A_n]$$

$$F_{mn} = \partial_m \mathcal{R}_n - [\mathcal{R}_m, \mathcal{R}_n]$$

$$\hat{F}_{mn}^k = \partial_m \mathcal{R}_n^k - [\mathcal{R}_m, \mathcal{R}_n]^k$$

$$= \partial_m \mathcal{R}_n^k - \partial_n \mathcal{R}_m^k + \mathcal{R}_m^l \mathcal{R}_n^j C_{lj}^k$$

$$T=0$$

# Asociované vekt. bundly

$\mathfrak{g}M$  bundl Lieovy alg

$AM$  vekt. bundl  $SO$ -bundl

$t_{\lambda}^A_B$  gen repr

$$m \in \mathfrak{g} \rightarrow M^A_B = t_{m^A}^B = m^K t_{K^A}^B$$

$$t_{[m,n]} = [t_m, t_n]$$

$$C_{\mu\nu}^{\kappa} t_{\kappa}^A_B = t_{\mu}^A t_{\nu}^B - t_{\nu}^A t_{\mu}^B$$

$$(T_m \phi, T_m \psi) = (\phi, \psi) \quad H_{AB}$$

$$t_m^T = -t_m$$

$$t_{\kappa AB} + t_{\kappa BA} = 0$$

rozšíření kov. der  
 $\mathbb{R} \mathfrak{g}M$  na  $AM$

$$D \text{ na } \mathfrak{g}M \quad Dc = 0$$

$$D \text{ na } AM \quad DH = 0$$

$$D_{\underline{m}} t_{\kappa}^A_B = 0$$

trivializace

$$e_{\alpha} \quad C_{\alpha\beta}^{\gamma} = \text{konst} \quad \partial c = 0$$

$$E_A \quad H_{AB} = \text{konst} \quad \partial H = 0$$

$$t_{\mu}^A_B = \text{konst} \quad \partial t = 0$$

$$\partial e_{\alpha} = 0, \partial E_A = 0$$

$$D_{\underline{m}} m^{\kappa} = \partial_{\underline{m}} m^{\kappa} + [A_{\underline{m}}, m]^{\kappa} \quad A_{\underline{m}}^{\kappa} = \mathcal{L}_{\underline{m}}^{\kappa} C_{\mu\nu}^{\kappa}$$

$$D_{\underline{m}} \phi^A = \partial_{\underline{m}} \phi^A + A_{\underline{m}}^A_B \phi^B$$

$$\underbrace{D_{\underline{m}} t_{\kappa}^A_B}_0 = \underbrace{\partial_{\underline{m}} t_{\kappa}^A_B}_0 - \mathcal{L}_{\underline{m}}^{\lambda} C_{\lambda\kappa}^{\gamma} t_{\gamma}^A_B + [A_{\underline{m}}, t_{\kappa}]^A_B$$

$$-\mathcal{L}_{\underline{m}}^{\lambda} [t_{\lambda}, t_{\kappa}]^A_B + [A_{\underline{m}}, t_{\kappa}]^A_B = 0$$

$$[A_{\underline{m}} - \mathcal{L}_{\underline{m}}^{\lambda} t_{\lambda}, t_{\kappa}]^A_B = 0$$

$$A_{\underline{m}}^A_B - \mathcal{L}_{\underline{m}}^{\lambda} t_{\lambda}^A_B = \sum_{\alpha} a_{\alpha} P_{\alpha}^A_B$$

$$A^T = -A \quad t_m^T = -t_m \quad P_{\alpha}^T = P_{\alpha}$$

$$\Rightarrow a_{\lambda} = 0 \quad A_{\underline{m}}^A_B = \mathcal{L}_{\underline{m}}^{\lambda} t_{\lambda}^A_B$$

# Asociované vekt. bundly

$\mathfrak{g}M$  bundl Lieovy algy

$AM$  vekt. bundl  $SO$ -bundl

$t_{\underline{k}}^{\underline{A}}_{\underline{B}}$  gen repr.

$$m \in \mathfrak{g} \rightarrow M_{\underline{B}}^{\underline{A}} = t_{m \underline{B}}^{\underline{A}} = m^{\underline{k}} t_{\underline{k}}^{\underline{A}}_{\underline{B}}$$

$$t_{[m, n]} = [t_m, t_n]$$

$$C_{\underline{m}\underline{n}}^{\underline{k}} t_{\underline{k}}^{\underline{A}}_{\underline{B}} = t_{\underline{k}}^{\underline{A}}_{\underline{C}} t_{\underline{n}}^{\underline{C}}_{\underline{B}} - t_{\underline{n}}^{\underline{A}}_{\underline{C}} t_{\underline{k}}^{\underline{C}}_{\underline{B}}$$

$$(T_m \phi, T_m \psi) = (\phi, \psi) \quad H_{\underline{AB}}$$

$$t_m^T = -t_m$$

$$t_{\underline{k}}^{\underline{A}}_{\underline{B}} + t_{\underline{k}}^{\underline{B}}_{\underline{A}} = 0$$



ποσitivity με τον det

$\mathbb{R} \text{ gm ma AM}$

$D \text{ ma gm } D_c = 0$

$D \text{ ma AM } DH = 0$

$$D_m t_{ik}^D = 0$$

trivialization

$e_\alpha$

$E_A$

$C_{\alpha\beta}^{\gamma} = \text{konst}$

$H_{AB} = \text{konst}$

$t_{\mu}^A{}^B = \text{konst}$

$$\partial C = 0$$

$$\partial H = 0$$

$$\partial t = 0$$

$$\partial e_\alpha = 0 \quad \partial E_A = 0$$

$$D_{\underline{m}} m^{\underline{k}} = \partial_{\underline{m}} m^{\underline{k}} + [\mathcal{R}_{\underline{m}}, m]^{\underline{k}} \quad A_{\underline{m} \underline{f}}^{\underline{a}} = \mathcal{R}_{\underline{m}}^{\underline{k}} C_{\underline{f}}^{\underline{a}}$$

$$D_{\underline{B}} \phi^{\underline{A}} = \partial_{\underline{m}} \phi^{\underline{A}} + A_{\underline{m} \underline{B}}^{\underline{A}} \phi^{\underline{B}}$$

$$\underbrace{D_{\underline{B}} t_{\underline{k} \underline{B}}^{\underline{A}}}_0 = \underbrace{\partial_{\underline{m}} t_{\underline{k} \underline{B}}^{\underline{A}}}_0 - \mathcal{R}_{\underline{m}}^{\underline{\lambda}} C_{\underline{\lambda} \underline{k}}^{\underline{\nu}} t_{\underline{\nu} \underline{B}}^{\underline{A}} + [A_{\underline{m}}, t_{\underline{k}}]^{\underline{A}}_{\underline{B}}$$

$$-\mathcal{R}_{\underline{B}}^{\underline{\lambda}} [t_{\underline{\lambda}}, t_{\underline{k}}]^{\underline{A}}_{\underline{B}} + [A_{\underline{m}}, t_{\underline{k}}]^{\underline{A}}_{\underline{B}} = 0$$

$$[A_{\underline{m}} - \mathcal{R}_{\underline{m}}^{\underline{\lambda}} t_{\underline{\lambda}}, t_{\underline{k}}]^{\underline{A}}_{\underline{B}} = 0$$

$$A_{\underline{m} \underline{B}}^{\underline{A}} - \mathcal{R}_{\underline{m}}^{\underline{\lambda}} t_{\underline{\lambda} \underline{B}}^{\underline{A}} = \sum_{\underline{\lambda}} \alpha_{\underline{\lambda}} P_{\underline{\lambda} \underline{B}}^{\underline{A}}$$

$$A^T = -A \quad t_{\underline{m}}^{\dagger} = -t_{\underline{m}} \quad P_{\underline{\lambda}}^T = P_{\underline{\lambda}}$$

$$\Rightarrow \alpha_{\underline{\lambda}} = 0 \quad A_{\underline{m} \underline{B}}^{\underline{A}} = \mathcal{R}_{\underline{m}}^{\underline{\lambda}} t_{\underline{\lambda} \underline{B}}^{\underline{A}}$$