Konex na SO-bundlech

(SO-kov. derivace)

Def. AM SO-bundle

D kov. derivace

$$DH = 0$$

$\Updownarrow$

$$D_\xi(\phi,\psi) = (D_\xi\phi,\psi) + (\phi, D_\xi\psi)$$

Lemma

∂ souř. trivializace E_n konst. H

$$\partial H = 0$$

$$\partial_m H_{\underline{AB}} = \partial_m (H_{MN} E^M_{\underline{A}} E^N_{\underline{B}}) = 0$$

Věta

D je kov. $\circ H$ dáná

vekt. pot. $A_{m}{}^{k}{}_{l}$ vůči ∂

$$D = \partial + A_m$$

$\Downarrow$

$$A_m^T = -A_m$$

důk.

$$DH = \partial H = 0$$

$$\Rightarrow 0 = A_m H_{\underline{AB}} = -A_m{}^{k}{}_{\underline{A}} H_{\underline{kB}} - A_m{}^{k}{}_{\underline{B}} H_{\underline{Ak}}$$

$$A_m + A_m^T = 0$$

Věta

D je kov. $\circ H$

$F_{\underline{mn}}{}^{k}{}_{l}$ tenzor kř. D

$\Downarrow$

$$F_{\underline{mn}}^T = -F_{\underline{mn}}$$

důk.

$$D H = 0 \Rightarrow DDH = 0$$

$$\Rightarrow 0 = [D_m D_n - D_n D_m + T_{\underline{mn}}{}^{k} D_k] H_{\underline{AB}} = F_{\underline{mn}} H_{\underline{AB}}$$

$$\Rightarrow F_{\underline{mn}} + F_{\underline{mn}}^T = 0$$

Kov. der. na SO-bundlech

(SO-kov. derivace)

Def. AM SO-bundle

D kov. derivace

$$DH = 0$$

$$\Updownarrow$$

$$D_\xi(\phi,\psi) = (D_\xi \phi,\psi) + (\phi, D_\xi \psi)$$

Lemma ∂ souř. trivializace E_A konz. s H

$$\partial H = 0$$

$$\partial_{\underline{m}} H_{\underline{A}\underline{B}} = \partial_{\underline{m}}\left(H_{MN} E^M_{\underline{A}} E^N_{\underline{B}}\right) = 0$$

Věta

D je tranz. a H dána
vekt. pot. $A_{\underline{m}}{}^{\underline{k}}{}_{\underline{l}}$ vůči ∂

$$D = \partial + A_{\underline{m}}$$

$$\Downarrow$$

$$A_{\underline{m}}^{T} = - A_{\underline{m}}$$

důk.

$$DH = \partial H = 0$$

$$\Rightarrow 0 = A_{\underline{m}} H_{\underline{AB}} = - A_{\underline{m}}{}^{\underline{K}}{}_{\underline{A}} H_{\underline{KB}} - A_{\underline{m}}{}^{\underline{K}}{}_{\underline{B}} H_{\underline{AK}}$$

$$A_{\underline{m}} + A_{\underline{m}}^{T} = 0$$

Věta

D je komp. s H

$F_{\underline{mn}}{}^{\underline{k}}{}_{\underline{l}}$ tenzor křiv. D

$\Downarrow$

$$F^T_{\underline{mn}} = -F_{\underline{mn}}$$

důk:

$$D \cdot H = 0 \Rightarrow DDH = 0$$

$$\Rightarrow 0 = [D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}} + T_{\underline{mn}}^{\underline{k}} D_{\underline{k}}] H_{\underline{AB}} = F_{\underline{mn}} H_{\underline{AB}}$$

$$\Rightarrow F_{\underline{mn}} + F^T_{\underline{mn}} = 0$$

Def Kalibr. transf. kov. der.

mějme transf R

$$\phi \to \tilde{\phi} = R\cdot\phi$$

na kov. der transf rozšíříme

$$\tilde{D}\tilde{\phi} = \widetilde{D\phi}$$

Věta

$$\tilde{D} = D + \Lambda \qquad \Lambda = -(DR)\cdot R^{-1}$$

důkaz.

$$\tilde{D}(R\cdot\phi) = D(R\cdot\phi) + \Lambda\cdot R\cdot\phi$$

$$= (DR)\cdot\phi + \underline{R\cdot(D\phi)} + \Lambda\cdot R\cdot\phi = \underline{R\cdot(D\phi)}$$

$$\Lambda\cdot R = -DR$$

Věta $\quad A^{\underline{k}}_{\underline{l}}$

$D, \tilde{D}$ souvis. kal. transf.

vůči stejné ∂ dány vekt. pot. $A, \tilde{A}$

$$\tilde{A} = R\cdot A\cdot R^{-1} - (\partial R)\cdot R^{-1}$$

$$= A - (DR)\cdot R^{-1} = A - R^{-1}\cdot(\tilde{D}R)$$

důk:

$$D\phi = \partial\phi + A\cdot\phi \qquad \tilde{D}\phi = \partial\phi + \tilde{A}\cdot\phi$$

$$\tilde{D}\phi = D\phi - (DR)\cdot R^{-1}\phi$$

$$= \partial\phi + (A - (DR)\cdot R^{-1})\cdot\phi$$

$$\tilde{A} = A - (DR)\cdot R^{-1}$$

$$= \cancel{A} - (\partial R)\cdot R^{-1} - \cancel{A\cdot R\cdot R^{-1}} + R\cdot A\cdot R^{-1}$$

$$= R\cdot A\cdot R^{-1} - (\partial R)\cdot R^{-1}$$

$$D \leftrightarrow \tilde{D} \quad A \leftrightarrow \tilde{A} \quad R \leftrightarrow R^{-1}$$

$$A = \tilde{A} - (\tilde{D}R^{-1})\cdot R \qquad \tilde{A} = A + (\tilde{D}R^{-1})\cdot R = \tilde{A} - R^{-1}DR$$

Věta

$F, \tilde{F}$ tenz. křiv. $D, \tilde{D}$

$\Downarrow$

$$\tilde{F} = R\cdot F\cdot R^{-1}$$

$$\tilde{F}_{\underline{mn}}{}^{\underline{k}}{}_{\underline{l}} = R^{\underline{k}}{}_{\underline{M}} R^{-1\underline{N}}{}_{\underline{l}} F_{\underline{mn}}{}^{\underline{M}}{}_{\underline{N}}$$

$$\tilde{D}\tilde{\phi} = \widetilde{D\phi}$$

$$\tilde{D}\tilde{D}\tilde{\phi} = \widetilde{DD\phi}$$

$$\tilde{F}\cdot\tilde{\phi} = \widetilde{F\cdot\phi}$$

$$\tilde{F}\cdot R\cdot\phi = R\cdot F\cdot\phi$$

$$\tilde{F} = R\cdot F\cdot R^{-1}$$

Def Kalibr. transf kov. der.

místní transf R

$$\phi \to \tilde{\phi} = R\cdot\phi$$

na kov. der transf rozšíříme

$$\tilde{D}\tilde{\phi} = \widetilde{D\phi}$$

Věta

$$\tilde{D} = D + \Lambda \qquad \Lambda = -(DR)\cdot R^{-1}$$

důkaz.

$$\tilde{D}(R\cdot\phi) = D(R\cdot\phi) + \Lambda\cdot R\cdot\phi$$

$$= (DR)\cdot\phi + \underline{R\cdot(D\phi)} + \Lambda\cdot R\cdot\phi = \underline{R\cdot(D\phi)}$$

$$\Lambda\cdot R = -DR$$

Věta

$D, \tilde{D}$ souvis. kal. transf. $\quad A_m{}^k{}_l$

vůči stejné ∂ dány vzt. pot $A_m, \tilde{A}_m$

$$\tilde{A}_m = R \cdot A \cdot R^{-1} - (\partial R) \cdot R^{-1}$$

$$= A - (DR) \cdot R^{-1} = A - R^{-1} \cdot (\tilde{D} R)$$

důk:

$$D\phi = \partial\phi + A \cdot \phi \qquad \tilde{D}\phi = \partial\phi + \tilde{A} \cdot \phi$$

$$\tilde{D}\phi = D\phi - (DR) \cdot R^{-1} \cdot \phi$$

$$= \partial\phi + (A - (DR) \cdot R^{-1}) \cdot \phi$$

$$\tilde{A} = A - (DR) \cdot R^{-1}$$

$$= \cancel{A} - (\partial R) \cdot R^{-1} - \cancel{A \cdot R \cdot R^{-1}} + R \cdot A \cdot R^{-1}$$

$$= R \cdot A \cdot R^{-1} - (\partial R) \cdot R^{-1}$$

$$D \leftrightarrow \tilde{D} \qquad A \leftrightarrow \tilde{A} \qquad R \leftrightarrow R^{-1}$$

$$A = \tilde{A} - (\tilde{D} R^{-1}) \cdot R \qquad \tilde{A} = A + (\tilde{D} R^{-1}) \cdot R = \tilde{A} - R^{-1} D R$$

Věta

$F, \tilde{F}$ tenz. křiv. $D, \tilde{D}$

$\Downarrow$

$$\tilde{F} = R \cdot F \cdot R^{-1}$$

$$\tilde{F}_{\underline{mn}}{}^{\underline{K}}{}_{\underline{L}} = R^{\underline{K}}{}_{\underline{M}} R^{-1\underline{N}}{}_{\underline{L}} F_{\underline{mn}}{}^{\underline{M}}{}_{\underline{N}}$$

$$\tilde{D}\tilde{\phi} = \widetilde{D\phi}$$

$$\tilde{D}\tilde{D}\tilde{\phi} = \widetilde{DD\phi}$$

$$\tilde{F} \cdot \tilde{\phi} = \widetilde{F \cdot \phi}$$

$$\tilde{F} \cdot R \cdot \phi = R \cdot F \cdot \phi$$

$$\tilde{F} = R \cdot F \cdot R^{-1}$$