

Komplexní vekt. bundl s
hermitovskou strukturou
(U-bundly)

$E_L = \underline{E}$ komplex. vekt. pr. ϕ^A
 $E_R = \overline{E}$ sdružená kom. vekt. pr. $\phi^{\overline{A}}$

$$E_L \leftrightarrow E_R$$

antilineární $\overline{\alpha\phi} = \alpha^* \overline{\phi}$

$$\overline{\overline{\phi}} = \phi$$

$$\overline{\phi_{A \dots B \dots}} = \phi_{\overline{A} \dots \overline{B} \dots}$$

$$E_{\lambda_L \lambda_R} = \underbrace{E_L \otimes \dots \otimes E_L}_{\lambda_L} \otimes \underbrace{E_L^* \otimes \dots \otimes E_L^*}_{\lambda_L} \otimes \underbrace{E_R \otimes \dots \otimes E_R}_{\lambda_R} \otimes \underbrace{E_R^* \otimes \dots \otimes E_R^*}_{\lambda_R}$$

antilineární operace

$$a : EM \rightarrow EM$$

$$(a\phi)^{\overline{A}} = A^{\overline{A}}_{\overline{B}} \overline{\phi}^{\overline{B}}$$

Skalární součin

$$\langle \cdot, \cdot \rangle_L \quad \langle \cdot, \cdot \rangle_R$$

generování pro hermit. str. $h_{\overline{A}\overline{B}}$

$$\langle \phi_1, \phi_2 \rangle_L = \overline{\phi}_1^{\overline{A}} \phi_2^{\overline{B}} h_{\overline{A}\overline{B}}$$

$$\langle \psi_1, \psi_2 \rangle_R = \overline{\psi}_1^{\overline{A}} \psi_2^{\overline{B}} \overline{h}_{\overline{A}\overline{B}}$$

1) h nedegeener $h^{-1\overline{A}\overline{B}}$

$$2) \overline{h}_{\overline{A}\overline{B}} = h_{\overline{B}\overline{A}}$$

3) h pos. def. $\langle \phi, \phi \rangle_L > 0 \quad \phi \neq 0$

$$+ : E_{00}^{10} \leftrightarrow E_{10}^{00}$$

$$E \leftrightarrow E^*$$

$$(\phi^{\dagger})_{\overline{A}} = \overline{\phi}^{\overline{B}} h_{\overline{B}\overline{A}}$$

$$\phi^{\dagger\dagger} = \phi$$

$$\langle \phi_1, \phi_2 \rangle_L = \phi_1^{\dagger} \cdot \phi_2$$

$$+ : E_{10}^{10} \rightarrow E_{10}^{10}$$

$$(\chi^{\dagger})^{\overline{A}}_{\overline{B}} = \overline{h}^{\overline{A}\overline{C}} h_{\overline{C}\overline{B}} \overline{\chi}^{\overline{D}}_{\overline{D}}$$

$$\langle \phi_1, \chi\phi_2 \rangle_L = \langle \chi^{\dagger}\phi_1, \phi_2 \rangle_L$$

$$+ : E_{00}^{01} \leftrightarrow E_{01}^{00}$$

$$\overline{E} \leftrightarrow \overline{E}^*$$

$$(\psi^{\dagger})_{\overline{B}} = \overline{\psi}^{\overline{A}} \overline{h}_{\overline{A}\overline{B}}$$

$$\psi^{\dagger\dagger} = \psi$$

$$\langle \psi_1, \psi_2 \rangle_R = \psi_1^{\dagger} \cdot \psi_2$$

$$+ : E_{01}^{01} \rightarrow E_{01}^{01}$$

$$\langle \psi_1, \chi\psi_2 \rangle_R = \langle \chi^{\dagger}\psi_1, \psi_2 \rangle_R$$

Komplexní vekt. bundl s hermitovskou strukturou (U-bundly)

$E_L = E$ komplex. vekt. \mathbb{R} . ϕ^A
 $E_R = \bar{E}$ sdružený kom. vekt. \mathbb{R} . $\phi^{\bar{A}}$

$$\bar{\quad} : E_L \leftrightarrow E_R$$

$\bar{\quad}$ antilineární $\overline{\alpha\phi} = \alpha^* \bar{\phi}$

$$\overline{\overline{\phi}} = \phi$$

$$\phi_{\substack{A \dots B \dots \\ C \dots \bar{D} \dots}}$$

$$E_{\substack{\lambda_L \lambda_R \\ \lambda_L \lambda_R}} = \underbrace{E_L \otimes \dots \otimes E_L}_{\lambda_L} \otimes \underbrace{E_L^* \otimes \dots \otimes E_L^*}_{\lambda_L} \otimes \underbrace{E_R \otimes \dots \otimes E_R}_{\lambda_R} \otimes \underbrace{E_R^* \otimes \dots \otimes E_R^*}_{\lambda_R}$$

antilinear operation

$$a : EM \rightarrow EM$$

$$(a\phi)^A = A^A_{\bar{B}} \bar{\phi}^{\bar{B}}$$

Skalarinė daugyba

$$\langle \cdot, \cdot \rangle_L \quad \langle \cdot, \cdot \rangle_R$$

generuoti. nuo hermit. str. $h_{\bar{A}\bar{B}}$

$$\langle \phi_1, \phi_2 \rangle_L = \bar{\phi}_1^{\bar{A}} \phi_2^{\bar{B}} h_{\bar{A}\bar{B}}$$

$$\langle \psi_1, \psi_2 \rangle_R = \bar{\psi}_1^A \psi_2^{\bar{B}} \bar{h}_{A\bar{B}}$$

1) h nedegeneruoti $h^{-1\bar{A}\bar{B}}$

2) $\bar{h}_{\bar{A}\bar{B}} = h_{\bar{B}\bar{A}}$

3) h pos def. $\langle \phi, \phi \rangle_L > 0 \quad \phi \neq 0$

$$+ : E_{00}^{10} \leftrightarrow E_{10}^{00}$$

$$E \leftrightarrow E^*$$

$$(\phi^\dagger)_{\bar{A}} = \bar{\phi}^{\bar{B}} h_{\bar{B}A}$$

$$\phi^{\dagger\dagger} = \phi$$

$$\langle \phi_1, \phi_2 \rangle_L = \phi_1^\dagger \cdot \phi_2$$

$$+ : E_{10}^{10} \rightarrow E_{10}^{10}$$

$$(\chi^\dagger)_{\bar{B}} = \bar{h}^{\bar{N}A} h_{\bar{N}B} \bar{\chi}^{\bar{N}}$$

$$\langle \phi_1, \chi \cdot \phi_2 \rangle_L = \langle \chi^\dagger \cdot \phi_1, \phi_2 \rangle_L$$

$$+ : E_{00}^{01} \leftrightarrow E_{01}^{00}$$

$$\bar{E} \leftrightarrow \bar{E}^*$$

$$(\psi^\dagger)_{\bar{A}} = \bar{\psi}^{\bar{B}} \bar{h}_{\bar{B}\bar{A}}$$

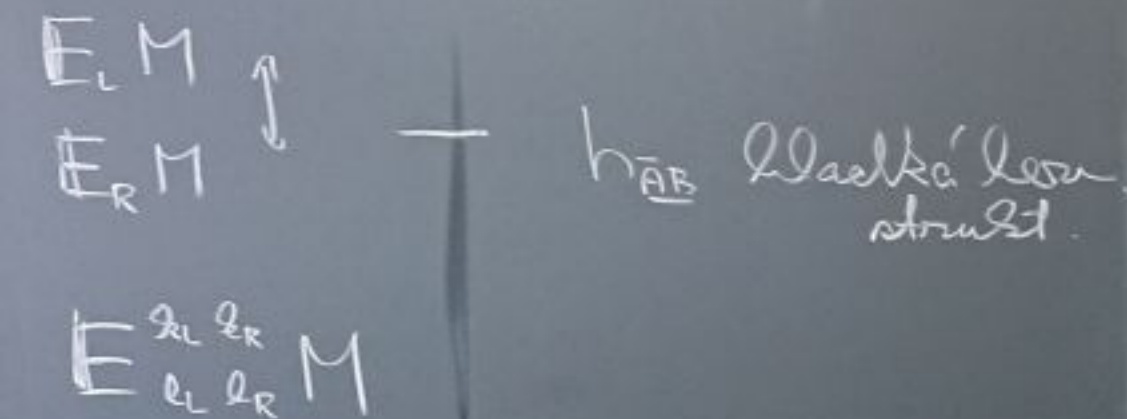
$$\psi^{\dagger\dagger} = \psi$$

$$\langle \psi_1, \psi_2 \rangle_R = \psi_1^\dagger \cdot \psi_2$$

$$+ : E_{01}^{01} \rightarrow E_{01}^{01}$$

$$\langle \psi_1, \chi \cdot \psi_2 \rangle_R = \langle \chi^\dagger \cdot \psi_1, \psi_2 \rangle_R$$

Komplexní vekt. bundl s
hermitovskou strukturou
(U-bundly)



Unitární Galilej. transf.
ultralok. lin. operace

$$U: \text{Vect } E_L M \rightarrow \text{Vect } E_L M$$

$$U \in \text{Vect } E_{\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}} M$$

$$\phi \rightarrow \tilde{\phi} = U \cdot \phi \quad \phi \in \text{Vect } E_L M$$

$$\bar{U} \in \text{Vect } E_{\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}} M$$

$$\psi \rightarrow \tilde{\psi} = \bar{U} \cdot \psi \quad \psi \in \text{Vect } E_R M$$

$$\langle \tilde{\phi}_1, \tilde{\phi}_2 \rangle_L = \langle \phi_1, \phi_2 \rangle_L \quad \langle \tilde{\psi}_1, \tilde{\psi}_2 \rangle_R = \langle \psi_1, \psi_2 \rangle_R$$

$$h_{\bar{B}A} = \tilde{h}_{\bar{B}A} = \bar{U}^{-1\bar{N}}_{\bar{B}} U^N_A h_{\bar{N}M}$$

$$U^\dagger \cdot U = \mathbb{1} = U \cdot U^\dagger$$

Ložní Gal. Lieova algebra

$$U_\alpha = \mathbb{1} + i\alpha u + O(\alpha^2)$$

$$iu - iu^\dagger = 0$$

$$u^\dagger = u$$

$$U_\alpha = \exp(i\alpha u)$$

Trivializ. konzist. s hermit. str.

$$E_A \quad \bar{E}_A$$

$$\langle E_A, E_B \rangle_L = \text{konst} \quad \langle \bar{E}_A, \bar{E}_B \rangle_R = \text{konst}$$

Pr. orthonom. báze

$$\langle E_A, E_B \rangle_L = \begin{bmatrix} 1 & & \\ & 1 & \\ & & \ddots \\ & & & 1 \end{bmatrix}$$

Komplexní vekt. bundl s
hermitovskou strukturou
(U-bundly)

$$\begin{array}{c} E_L M \\ E_R M \end{array} \begin{array}{c} \updownarrow \end{array}$$

$h_{\bar{A}B}$ hermitovská
struktura

$$\begin{array}{c} E \\ E \end{array} \begin{array}{cc} g_L & g_R \\ g_L & g_R \end{array} M$$

Unitary Galilean transf.

ultralocal lin. operator

$$U: \text{Sect } E_L M \rightarrow \text{Sect } E_L M$$

$$U \in \text{Sect } E_{10}^{10} M$$

$$\phi \rightarrow \tilde{\phi} = U \cdot \phi \quad \phi \in \text{Sect } E_L M$$

$$\bar{U} \in \text{Sect } E_{01}^{01} M$$

$$\psi \rightarrow \tilde{\psi} = \bar{U} \cdot \psi \quad \psi \in \text{Sect } E_R M$$

$$\langle \tilde{\phi}_1, \tilde{\phi}_2 \rangle_L = \langle \phi_1, \phi_2 \rangle_L \quad \langle \tilde{\psi}_1, \tilde{\psi}_2 \rangle_R = \langle \psi_1, \psi_2 \rangle_R$$

$$h_{\underline{B}\underline{A}} = \tilde{h}_{\underline{B}\underline{A}} = \bar{U}^{-1 \underline{N}}_{\underline{B}} U^{\underline{M}}_{\underline{A}} h_{\underline{N}\underline{M}}$$

$$U^\dagger \cdot U = \mathbb{1} = U \cdot U^\dagger$$

Lokaler Gal. Lieova algebra

$$U_\alpha = \mathbb{1} + i\alpha u + \mathcal{O}(\alpha^2)$$

$$iu - iu^\dagger = 0$$

$$u^\dagger = u$$

$$U_\alpha = \exp(i\alpha u)$$

Trivializ. konzist. s herm. str

$$E_A \quad \bar{E}_A$$

$$\langle E_A, E_B \rangle_L = \text{konst} \quad \langle \bar{E}_A, \bar{E}_B \rangle_R = \text{konst}$$

Pr. ortonom. baze

$$\langle E_A, E_B \rangle_L = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$$

Kov. der. konzistentní s
hermitovskou strukturou

Def pozitivní $D \in \text{EM}$ na $\bar{E}M$

$$\bar{D}\bar{\phi} = \overline{D\phi}$$

společná der. na $E_{\ell_1 \ell_2}^{\ell_1 \ell_2} M$

$$D = D \oplus \bar{D}$$

celková der. je reálná

$$\bar{D} = D$$

Def Kov. der. D je konz. s
hermit. strukturou

$$Dh = 0$$

$$D_{\xi} \langle \phi_1, \phi_2 \rangle_L = \langle D_{\xi} \phi_1, \phi_2 \rangle_L + \langle \phi_1, D_{\xi} \phi_2 \rangle_L$$

Lem
trivial. konz. s her. str.

$$\partial h = 0$$

$$\begin{matrix} M \\ \bar{M} \end{matrix}$$

$$\begin{matrix} \rightarrow \\ \rightarrow \end{matrix}$$

$$\begin{aligned} M \chi_{\underline{C}}^{\underline{A} \bar{B}} &= M_{\underline{K}}^{\underline{A}} \chi_{\underline{C} \bar{D}}^{\underline{K} \bar{B}} + \dots - M_{\underline{C}}^{\underline{K}} \chi_{\underline{K} \bar{D}}^{\underline{A} \bar{B}} \\ &+ \bar{M}_{\underline{K}}^{\bar{B}} \chi_{\underline{C} \bar{D}}^{\underline{A} \underline{K}} + \dots - \bar{M}_{\bar{D}}^{\underline{K}} \chi_{\underline{C} \bar{K}}^{\underline{A} \bar{B}} \end{aligned}$$

Konzistentní pseudoder. na $E_{\ell_1 \ell_2}^{\ell_1 \ell_2} M$

$$\bar{M} \bar{\chi} = \overline{M \chi}$$

společné pseudoder.

$$M = M \oplus \bar{M}$$

$$M = \bar{M}$$

$$M \phi^{\underline{A}} = M_{\underline{B}}^{\underline{A}} \phi^{\underline{B}} \quad \bar{M} \psi^{\bar{A}} = \bar{M}_{\bar{B}}^{\bar{A}} \psi^{\bar{B}}$$

Kov. der. konzistentní s

hermitovskou strukturou

Def. rozšíření $D \supseteq \mathbb{E} \Gamma_{mc} \bar{\mathbb{E}} M$

$$\bar{D} \bar{\phi} = \overline{D \phi}$$

společná der. na $\mathbb{E} \begin{matrix} \mathbb{R}_L & \mathbb{R}_R \\ \mathbb{R}_L & \mathbb{R}_R \end{matrix} M$

$$D = D \oplus \bar{D}$$

celková der. je reálná

$$\bar{D} = D$$

Def Kov. der D je Kov. σ
hermitetrisch

$$Dh = 0$$

$$D_{\xi} \langle \phi_1, \phi_2 \rangle_L = \langle D_{\xi} \phi_1, \phi_2 \rangle_L + \langle \phi_1, D_{\xi} \phi_2 \rangle_L$$

Lem

trivial. Kov. σ hermitetrisch

$$\partial h = 0$$

Rozložení - pseudosym. na $E_{\mathbb{R}}^{\mathbb{R}} M$

$$\overline{M} \overline{X} = \overline{M} X$$

symetrické pseudo

$$M = M \oplus \overline{M}$$

$$M = \overline{M}$$

$$M \phi^A = M^A_{\underline{B}} \phi^B \quad \overline{M} \psi^{\overline{A}} = \overline{M}^{\overline{A}}_{\overline{B}} \psi^{\overline{B}}$$

$$M \chi_{\underline{C} \overline{D}}^{\overline{A} \underline{B}} = M^{\overline{A}}_{\underline{K}} \chi_{\underline{C} \overline{D}}^{\underline{K} \overline{B}} + \dots - M^{\underline{K}}_{\underline{C}} \chi_{\underline{K} \overline{D}}^{\overline{A} \overline{B}} - \dots \\ + \overline{M}^{\overline{B}}_{\overline{K}} \chi_{\underline{C} \overline{D}}^{\overline{A} \underline{K}} + \dots - \overline{M}^{\overline{K}}_{\overline{D}} \chi_{\underline{C} \overline{K}}^{\overline{A} \underline{B}} - \dots$$

← pseudo M

← pseudo \overline{M}

$D \tilde{D}$ komutativ hermitična

$D - \tilde{D}$ pseudodiferencialni gener.

rozdiel tenz. iA_m^k

$$D_m \phi^A - \tilde{D}_m \phi^A = iA_m^A{}^B \phi^B$$

$$D_m \psi^{\bar{A}} - \tilde{D}_m \psi^{\bar{A}} = -i\bar{A}_m^{\bar{A}}{}^{\bar{B}} \psi^{\bar{B}}$$

plund $\tilde{D} = \partial$ A_m vekt. potencial

Vektor

$$A_m^+ = A_m$$

$$\text{div } Dh = \tilde{D}h = 0$$

tenzor křivost: D

$$iF_{mn}^A \phi^B = [D_m D_n - D_n D_m + T_{mn}^k D_k] \phi^A = \overset{D}{d}_m \overset{D}{d}_n \phi^A$$

$$-i\bar{F}_{mn}^{\bar{A}} \psi^{\bar{B}} = [D_m D_n - D_n D_m + T_{mn}^k D_k] \psi^{\bar{A}} = \overset{D}{d}_m \overset{D}{d}_n \psi^{\bar{A}}$$

Vektor

$$F^+ = F$$

$$Fh = 0$$

kalibrační transform

$$\phi \rightarrow \tilde{\phi} = U \cdot \phi$$

$$\psi \rightarrow \tilde{\psi} = \bar{U} \cdot \psi$$

$$D \rightarrow \tilde{D} = D + \Lambda$$

$$\Lambda = -(DU) \cdot U^\dagger$$

$$D = \partial + iA \quad \tilde{D} = \partial + i\tilde{A}$$

$$\tilde{A} = U \cdot A \cdot U^\dagger + i(\partial U) \cdot U^{-1}$$

$$= A + i(DU) \cdot U^{-1}$$

$$= A + U^{-1} \cdot (\tilde{D}U)$$

$$\tilde{F} = U \cdot F \cdot U^{-1} \quad \bar{U}^{-1} = U^\dagger$$

$D \tilde{D}$ komutativ hermitisch

$D - \tilde{D}$ pseudodifferenzial gener.

rozdiel tenz iA_m^k

$$D_m \phi^A - \tilde{D}_m \phi^A = iA_m^A{}_B \phi^B$$

$$D_m \psi^{\bar{B}} - \tilde{D}_m \psi^{\bar{B}} = -i\bar{A}_m^{\bar{B}}{}_{\bar{A}} \psi^{\bar{A}}$$

plund $\tilde{D} = 0$ A_m vekt. potenciál

Väta

$$A_m^+ = A_m$$

die $Dh = \tilde{D}h = 0$

tenzor křivosti D

$$i F_{\underline{mn}B}^A \phi^B = [D_{\underline{m}D_{\underline{n}}} - D_{\underline{n}D_{\underline{m}}} + T_{\underline{mn}}^{\underline{k}} D_{\underline{k}}] \phi^A = \overset{D}{d_{\underline{m}}} \overset{D}{d_{\underline{n}}} \phi^A$$

$$-i \bar{F}_{\underline{mn} \bar{B}}^{\bar{A}} \psi^{\bar{B}} = [D D - D D + T D] \psi^{\bar{A}} = \overset{D}{d_{\underline{m}}} \overset{D}{d_{\underline{n}}} \psi^{\bar{A}}$$

Věta

$$F^{\dagger} = F$$

$$F h = 0$$

Kalibrāem' transformācijs

$$\phi \rightarrow \tilde{\phi} = U \cdot \phi$$

$$\psi \rightarrow \tilde{\psi} = \bar{U} \cdot \psi$$

$$D \rightarrow \tilde{D} = D + \Lambda$$

$$\Lambda = - (DU) \cdot U^\dagger$$

$$D = \partial + iA \quad \tilde{D} = \partial + i\tilde{A}$$

$$\tilde{A} = U \cdot A \cdot U^\dagger + i(\partial U) \cdot U^{-1}$$

$$= A + i(DU) \cdot U^{-1}$$

$$= A + U^{-1} \cdot (\tilde{D}U)$$

$$\tilde{F} = U \cdot F \cdot U^{-1} \quad \bar{U}^{-1} = U^\dagger$$