Komplexní vekt. bundl s hermitovskou strukturou (U-bundly)

$\mathbb{E}_L = \mathbb{E}$ komplex. vekt. pr. ϕ^{A}

$\mathbb{E}_R = \bar{\mathbb{E}}$ sdružený kom. vekt. pr. $\phi^{\bar A}$

$\bar{\ } : \mathbb{E}_L \leftrightarrow \mathbb{E}_R$

$\bar{\ }$ antilineární $\overline{\alpha\phi} = \alpha^* \bar\phi$

$\bar{\bar\phi} = \phi$ $\qquad \phi^{A\ldots \bar B\ldots}_{C\ldots \bar D\ldots}$

$$\mathbb{E}^{k_L k_R}_{l_L l_R} = \underbrace{\mathbb{E}_L\otimes\cdots}_{k_L}\otimes\underbrace{\mathbb{E}^*_L\otimes\cdots}_{l_L}\otimes\underbrace{\mathbb{E}_R\otimes\cdots}_{k_R}\otimes\underbrace{\mathbb{E}^*_R\otimes\cdots}_{l_R}$$

antilin. operace

$a : \mathbb{E}M \to \mathbb{E}M$

$(a\phi)^A = a^A{}_{\bar A}\bar\phi^{\bar A}$

Skalární součin

$\langle\,,\,\rangle_L \qquad \langle\,,\,\rangle_R$

generov. pro herm. str. $h_{\bar A B}$

$\langle\phi_1,\phi_2\rangle_L = \bar\phi_1^{\bar A}\phi_2^B h_{\bar A B}$

$\langle\psi_1,\psi_2\rangle_R = \bar\psi_1^A \psi_2^{\bar B}\bar h_{A\bar B}$

1) h nedegener. $\quad h^{-1\bar A B}$

2) $\bar h_{A\bar B} = h_{\bar B A}$

3) h poz. def. $\langle\phi,\phi\rangle > 0 \quad \phi\neq 0$

$\dagger : \mathbb{E}^{10}_{00} \leftrightarrow \mathbb{E}^{00}_{10}$

$\mathbb{E}\leftrightarrow\mathbb{E}^*$

$(\phi^\dagger)_A = \bar\phi^{\bar A} h_{\bar A A}$

$\phi^{\dagger\dagger}=\phi$

$\langle\phi_1,\phi_2\rangle_L = \phi_1^\dagger\cdot\phi_2$

$\dagger : \mathbb{E}^{10}_{10}\to\mathbb{E}^{10}_{10}$

$(X^\dagger)^A{}_B = h^{-1\bar N A} h_{\bar B B}\bar X^{\bar B}{}_{\bar N}$

$\langle\phi_1, X\cdot\phi_2\rangle_L = \langle X^\dagger\cdot\phi_1,\phi_2\rangle_L$

$\dagger : \mathbb{E}^{01}_{00}\leftrightarrow\mathbb{E}^{00}_{01}$

$\bar{\mathbb{E}}\leftrightarrow\bar{\mathbb{E}}^*$

$(\psi^\dagger)_{\bar A} = \bar\psi^M \bar h_{M\bar A}$

$\psi^{\dagger\dagger}=\psi$

$\langle\psi_1,\psi_2\rangle_R = \psi_1^\dagger\cdot\psi_2$

$\dagger : \mathbb{E}^{01}_{01}\to\mathbb{E}^{01}_{01}$

$\langle\psi_1, X\cdot\psi_2\rangle_R = \langle X^\dagger\cdot\psi_1,\psi_2\rangle_R$

Komplexní vekt. bundl s hermitouskou strukturou (U-bundly)

$\mathbb{E}_L = \mathbb{E}$ komplex. vekt. pr. $\phi^{\underline{A}}$

$\mathbb{E}_R = \bar{\mathbb{E}}$ sdružený kom. vekt. pr. $\phi^{\bar{\underline{A}}}$

$\bar{\ } : \mathbb{E}_L \longleftrightarrow \mathbb{E}_R$

$\bar{\ }$ antilineární $\overline{\alpha\phi} = \alpha^* \bar{\phi}$

$\bar{\bar{\phi}} = \phi$ $\phi^{\underline{A}\ldots\bar{\underline{B}}\ldots}_{\underline{C}\ldots\bar{\underline{D}}\ldots}$

$$\mathbb{E}^{k_L\, k_R}_{l_L\, l_R} = \underbrace{\mathbb{E}_L \otimes \ldots \otimes}_{k_L} \underbrace{\mathbb{E}_L^* \otimes \ldots \otimes}_{l_L} \underbrace{\mathbb{E}_R \otimes \ldots \otimes}_{k_R} \underbrace{\mathbb{E}_R^* \otimes \ldots}_{l_R}$$

antilin. operace

$$\mathcal{A} : \mathbb{E}M \to \mathbb{E}M$$

$$(\mathcal{A}\phi)^{\underline{A}} = A^{\underline{A}}{}_{\underline{\bar{M}}}\, \bar{\phi}^{\underline{\bar{M}}}$$

Skalární součin

$$\langle\ ,\ \rangle_L \qquad \langle\ ,\ \rangle_R$$

generov. pro herm. str. $h_{\underline{\bar{A}B}}$

$$\langle \phi_1, \phi_2 \rangle_L = \bar{\phi}_1^{\underline{\bar{A}}}\, \phi_2^{\underline{B}}\, h_{\underline{\bar{A}B}}$$

$$\langle \psi_1, \psi_2 \rangle_R = \bar{\psi}_1^{\underline{A}}\, \psi_2^{\underline{\bar{B}}}\, \bar{h}_{\underline{A\bar{B}}}$$

1) h nedegener. $h^{-1\underline{\bar{A}B}}$

2) $\bar{h}_{\underline{A\bar{B}}} = h_{\underline{\bar{B}A}}$

3) h poz. def. $\langle \phi, \phi \rangle_L > 0 \quad \phi \neq 0$

$$\dagger : \mathbb{E}^{10}_{00} \leftrightarrow \mathbb{E}^{00}_{10}$$

$$\mathbb{E} \leftrightarrow \mathbb{E}^*$$

$$(\phi^\dagger)_{\underline{A}} = \bar{\phi}^{\bar{\underline{M}}} h_{\bar{\underline{M}}\underline{A}}$$

$$\phi^{\dagger\dagger} = \phi$$

$$\langle \phi_1, \phi_2 \rangle_L = \phi_1^\dagger \cdot \phi_2$$

$$\dagger : \mathbb{E}^{10}_{10} \to \mathbb{E}^{10}_{10}$$

$$(X^\dagger)^{\underline{A}}{}_{\underline{B}} = h^{-1\bar{\underline{N}}\underline{A}} h_{\bar{\underline{M}}\underline{B}} \bar{X}^{\bar{\underline{M}}}{}_{\bar{\underline{N}}}$$

$$\langle \phi_1, X \cdot \phi_2 \rangle_L = \langle X^\dagger \phi_1, \phi_2 \rangle_L$$

$$\dagger : \mathbb{E}^{01}_{00} \leftrightarrow \mathbb{E}^{00}_{01}$$

$$\bar{\mathbb{E}} \leftrightarrow \bar{\mathbb{E}}^*$$

$$(\psi^\dagger)_{\bar{\underline{A}}} = \bar{\psi}^{\underline{M}} \bar{h}_{\underline{M}\bar{\underline{A}}}$$

$$\psi^{\dagger\dagger} = \psi$$

$$\langle \psi_1, \psi_2 \rangle_R = \psi_1^\dagger \cdot \psi_2$$

$$\dagger \; \mathbb{E}^{01}_{01} \to \mathbb{E}^{01}_{01}$$

$$\langle \psi_1, X \cdot \psi_2 \rangle_R = \langle X^\dagger \psi_1, \psi_2 \rangle$$

Komplexní vekt bundl s hermitovskou strukturou (U- bundy)

$\mathbb{E}_L M \updownarrow \mathbb{E}_R M$ — $h_{\bar{A}B}$ hladká herm. struktura

$\mathbb{E}^{k_L k_R}_{l_L l_R} M$

Unitární kalibr. transf

ultralok. lin operace

$U : \mathrm{Sect}\, \mathbb{E}_L M \to \mathrm{Sect}\, \mathbb{E}_L M$

$U \in \mathrm{Sect}\, \mathbb{E}^{10}_{10} M$

$\phi \to \tilde{\phi} = U \cdot \phi \qquad \phi \in \mathrm{Sect}\, \mathbb{E}_L M$

$\bar{U} \in \mathrm{Sect}\, \mathbb{E}^{01}_{01} M$

$\psi \to \tilde{\psi} = \bar{U} \cdot \psi \qquad \psi \in \mathrm{Sect}\, \mathbb{E}_R M$

$\langle \tilde{\phi}_1, \tilde{\phi}_2 \rangle_L = \langle \phi_1, \phi_2 \rangle_L \qquad \langle \tilde{\psi}_1, \tilde{\psi}_2 \rangle_R = \langle \psi_1, \psi_2 \rangle_R$

$h_{\underline{\bar{B}A}} = \tilde{h}_{\underline{\bar{B}A}} = \bar{U}^{-1\bar{N}}{}_{\bar{B}}\, U^{-1M}{}_{A}\, h_{\bar{N}M}$

$U^{\dagger} \cdot U = \mathbb{1} = U \cdot U^{\dagger}$

Lokální kal. Lieova algebra

$U_\alpha = \mathbb{1} + i\alpha u + \mathcal{O}(\alpha^2)$

$iu - iu^{\dagger} = 0$

$u^{\dagger} = u$

$U_\alpha = \exp(i\alpha u)$

Trivializ. konzist. s herm. str

$E_A \qquad \bar{E}_A$

$\langle E_A, E_B \rangle_L = \text{konst} \quad \langle \bar{E}_A, \bar{E}_D \rangle_R = \text{konst}$

Př. ortonorm. báze

$$\langle E_A, E_B \rangle_L = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

Komplexní vekt. bundl s hermitovskou strukturou

(U-bundy)

$\mathbb{E}_L M$

$\updownarrow$

$\mathbb{E}_R M$

$h_{\bar{A}B}$ hladká herm. strukt.

$\mathbb{E}^{\ell_L \ell_R}{}_{\ell_L \ell_R} M$

Unitární kalibr. transf

ultralok. lin operace

$$U : \mathrm{Sect}\, \mathbb{E}_L M \to \mathrm{Sect}\, \mathbb{E}_L M$$

$$U \in \mathrm{Sect}\, \mathbb{E}^{1\,0}_{1\,0} M$$

$$\phi \to \tilde{\phi} = U \cdot \phi \qquad \phi \in \mathrm{Sect}\, \mathbb{E}_L M$$

$$\bar{U} \in \mathrm{Sect}\, \mathbb{E}^{0\,1}_{0\,1} M$$

$$\psi \to \tilde{\psi} = \bar{U} \cdot \psi \qquad \psi \in \mathrm{Sect}\, \mathbb{E}_R M$$

$$\langle \tilde{\phi}_1, \tilde{\phi}_2 \rangle_L = \langle \phi_1, \phi_2 \rangle_L \qquad \langle \tilde{\psi}_1, \tilde{\psi}_2 \rangle_R = \langle \psi_1, \psi_2 \rangle_R$$

$$h_{\underline{\bar{B}A}} = \tilde{h}_{\underline{\bar{B}A}} = \bar{U}^{-1\underline{\bar{N}}}{}_{\underline{\bar{B}}}\, U^{-1\underline{M}}{}_{\underline{A}}\, h_{\underline{\bar{N}M}}$$

$$U^{\dagger} \cdot U = \mathbb{1} = U \cdot U^{\dagger}$$

Lokální zal. Lieova algebra

$$U_\alpha = \mathbb{1} + i\alpha u + \mathcal{O}(\alpha^2)$$

$$iu - iu^\dagger = 0$$

$$u^\dagger = u$$

$$U_\alpha = \exp(i\alpha u)$$

Trivializ. konzist. s herm. str

$$E_A \qquad \bar{E}_A$$

$$\langle E_A, E_B \rangle_L = \text{konst} \qquad \langle \bar{E}_A, \bar{E}_B \rangle_R = \text{konst}$$

Př. ortonorm. báze

$$\langle E_A, E_B \rangle_L = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

Kov. der. konzistentní s hermitovskou strukturou

Def rozšíření D z EM na $\bar{E}M$

$$\bar{D}\bar{\phi} = \overline{D\phi}$$

společná der na $E^{\mathfrak{k}_1 \mathfrak{k}_2}_{\mathfrak{l}_1 \mathfrak{l}_2} M$

$$D = D \oplus \bar{D}$$

celkové der je reálná

$$\bar{D} = D$$

Def Kov. der D je konz s hermit strukt.

$$D h = 0$$

$$D_\xi \langle \phi_1, \phi_2 \rangle_h = \langle D_\xi \phi_1, \phi_2 \rangle_h + \langle \phi_1, D_\xi \phi_2 \rangle_h$$

Lem

triviál. konz s herm str

$$\partial h = 0$$

Rozšíření pseudoder. na $E^{\mathfrak{k}_1 \mathfrak{k}_2}_{\mathfrak{l}_1 \mathfrak{l}_2} M$

$$\bar{\mathbb{M}}\bar{\chi} = \overline{\mathbb{M}\chi}$$

společné působení

$$\mathbb{M} = \mathbb{M} \oplus \bar{\mathbb{M}}$$

$$\mathbb{M} = \bar{\mathbb{M}}$$

$$\mathbb{M}\phi^{\underline{A}} = M^{\underline{A}}{}_{\underline{B}}\phi^{\underline{B}} \qquad \bar{\mathbb{M}}\psi^{\bar{\underline{A}}} = \bar{M}^{\bar{\underline{A}}}{}_{\bar{\underline{B}}}\psi^{\bar{\underline{B}}}$$

$$\mathbb{M} \longrightarrow \quad \mathbb{M}\chi^{\underline{A}\ldots\bar{\underline{B}}\ldots}_{\underline{C}\ldots\bar{\underline{D}}\ldots} = M^{\underline{A}}{}_{\underline{K}}\chi^{\underline{K}\ldots\bar{\underline{B}}\ldots}_{\underline{C}\ldots\bar{\underline{D}}\ldots} + \ldots - M^{\underline{K}}{}_{\underline{C}}\chi^{\underline{A}\ldots\bar{\underline{B}}\ldots}_{\underline{K}\ldots\bar{\underline{D}}\ldots} - \ldots$$

$$\bar{\mathbb{M}} \longrightarrow \quad + \bar{M}^{\bar{\underline{B}}}{}_{\bar{\underline{K}}}\chi^{\underline{A}\ldots\bar{\underline{K}}\ldots}_{\underline{C}\ldots\bar{\underline{D}}\ldots} + \ldots - \bar{M}^{\bar{\underline{K}}}{}_{\bar{\underline{D}}}\chi^{\underline{A}\ldots\bar{\underline{B}}\ldots}_{\underline{C}\ldots\bar{\underline{K}}\ldots} - \ldots$$

Kov. der. konzistentní s hermitovskou strukturou

Def rozšíření D z EM na $\bar{E}M$

$$\bar{D}\bar{\phi} = \overline{D\phi}$$

spojená der na $\mathbb{E}^{\bar{l}_1 \bar{l}_2}_{l_1 l_2} M$

$$\mathbb{D} = D \oplus \bar{D}$$

celková der je reálná

$$\bar{\mathbb{D}} = \mathbb{D}$$

Def Kov. der. D je [illegible] s hermit. [illegible]

$$Dh = 0$$

$$D_\xi \langle \phi_1, \phi_2 \rangle_L = \langle D_\xi \phi_1, \phi_2 \rangle_L + \langle \phi_1, D_\xi \phi_2 \rangle_L$$

Lem

trivial. [illegible] s hem. str.

$$\partial h = 0$$

Rozšíření pseudoder. na $\mathbb{E}^{k_1 k_2}_{l_1 l_2} M$

$$\bar{\mathbb{M}}\,\bar{\chi} = \overline{\mathbb{M}\chi}$$

společné působen

$$\mathbb{M} = \mathbb{M} \oplus \bar{\mathbb{M}}$$

$$\mathbb{M} = \bar{\mathbb{M}}$$

$$\mathbb{M}\phi^{A} = M^{A}{}_{B}\phi^{B} \qquad \bar{\mathbb{M}}\psi^{\bar{A}} = \bar{M}^{\bar{A}}{}_{\bar{B}}\psi^{\bar{B}}$$

$$\mathbb{M}\chi^{A\ldots\bar{B}\ldots}_{C\ldots\bar{D}\ldots} = M^{A}{}_{K}\chi^{K\ldots\bar{B}\ldots}_{C\ldots\bar{D}\ldots} + \ldots - M^{K}{}_{C}\chi^{A\ldots\bar{B}\ldots}_{K\ldots\bar{D}\ldots} - \ldots \quad \longleftarrow \text{působení } \mathbb{M}$$

$$+ \bar{M}^{\bar{B}}{}_{\bar{K}}\chi^{A\ldots\bar{K}\ldots}_{C\ldots\bar{D}\ldots} + \ldots - \bar{M}^{\bar{K}}{}_{\bar{D}}\chi^{A\ldots\bar{B}\ldots}_{C\ldots\bar{K}\ldots} - \ldots \quad \longleftarrow \text{působení } \bar{\mathbb{M}}$$

$D \quad \tilde D$ konz s koren. str

$D - \tilde D$ pseudoder gener.

rozdíl tenz $iA_{m}{}^{k}{}_{l}$

$$D_{m}\phi^{A} - \tilde D_{m}\phi^{A} = iA_{m}{}^{A}{}_{B}\phi^{B}$$

$$D_{m}\psi^{\bar A} - \tilde D_{m}\psi^{\bar A} = -i\bar A_{m}{}^{\bar A}{}_{\bar B}\psi^{\bar B}$$

pokud $\tilde D = \partial$ $\quad A_{m}$ vekt. potenciál

Věta

$$A_{m}^{\dagger} = A_{m}$$

důk $Dh = \tilde D h = 0$

tenzor křivosti: D

$$iF_{mn}{}^{A}{}_{B}\phi^{B} = [D_{m}D_{n} - D_{n}D_{m} + T_{mn}{}^{k}D_{k}]\phi^{A} = {}^{D}d_{m}{}^{D}d_{n}\phi^{A}$$

$$-i\bar F_{mn}{}^{\bar A}{}_{\bar B}\psi^{\bar B} = [DD - DD + TD]\psi^{\bar A} = {}^{D}d_{m}{}^{D}d_{n}\psi^{\bar A}$$

Věta

$$F^{\dagger} = F$$

$$Fh = 0$$

kalibrační transf

$$\phi \to \tilde\phi = U\cdot\phi$$

$$\psi \to \tilde\psi = \bar U\cdot\psi$$

$$D \to \tilde D = D + \Lambda$$

$$\Lambda = -(DU)\cdot U^{\dagger}$$

$$D = \partial + iA \qquad \tilde D = \partial + i\tilde A$$

$$\tilde A = U\cdot A\cdot U^{\dagger} + i(\partial U)\cdot U^{-1}$$
$$= A + i(DU)\cdot U^{-1}$$
$$= A + U^{-1}\cdot(\tilde D U)$$

$$\tilde F = U\cdot F\cdot U^{-1} \qquad U^{-1} = U^{\dagger}$$

D $\tilde{D}$ kovar. a herm. str.

$D - \tilde{D}$ pseudoder. gener.

rozdíl tenz $i A_{\underline{m}}{}^{\underline{K}}{}_{\underline{L}}$

$$D_{\underline{m}} \phi^{\underline{A}} - \tilde{D}_{\underline{m}} \phi^{\underline{A}} = i A_{\underline{m}}{}^{\underline{A}}{}_{\underline{B}} \phi^{\underline{B}}$$

$$D_{\underline{m}} \psi^{\bar{\underline{A}}} - \tilde{D}_{\underline{m}} \psi^{\bar{\underline{A}}} = -i \bar{A}_{\underline{m}}{}^{\bar{\underline{A}}}{}_{\bar{\underline{B}}} \psi^{\bar{\underline{B}}}$$

příkl. $\tilde{D} = \partial$ $\quad A_{\underline{m}}$ vekt. potenciál

Věta

$$A_{\underline{m}}^{\dagger} = A_{\underline{m}}$$

důk. $Dh = \tilde{D}h = 0$

tenzor křivosti: D

$$i F_{\underline{mn}}{}^{\underline{A}}{}_{\underline{B}}\, \phi^{\underline{B}} = \left[D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}} + T_{\underline{mn}}^{\underline{k}} D_{\underline{k}} \right] \phi^{\underline{A}} = {}^{D}d_{\underline{m}}\, {}^{D}d_{\underline{n}}\, \phi^{\underline{A}}$$

$$-i \bar{F}_{\underline{mn}}{}^{\bar{\underline{A}}}{}_{\bar{\underline{B}}}\, \psi^{\bar{\underline{B}}} = \left[DD - DD + TD \right] \psi^{\bar{\underline{A}}} = {}^{D}d_{\underline{m}}\, {}^{D}d_{\underline{n}}\, \psi^{\bar{\underline{A}}}$$

Věta

$$F^{+} = F$$

$$F h = 0$$

Kalibrační transf

$$\phi \to \tilde{\phi} = U \cdot \phi$$

$$\psi \to \tilde{\psi} = \bar{U} \cdot \psi$$

$$D \to \tilde{D} = D + \Lambda$$

$$\Lambda = -(DU) \cdot U^{\dagger}$$

$$D = \partial + iA \qquad \tilde{D} = \partial + i\tilde{A}$$

$$\tilde{A} = U \cdot A \cdot U^{\dagger} + i(\partial U) \cdot U^{-1}$$

$$= A + i(DU) \cdot U^{-1}$$

$$= A + U^{-1} \cdot (\tilde{D}U)$$

$$\tilde{F} = U \cdot F \cdot U^{-1} \qquad U^{-1} = U^{\dagger}$$