

Lineární  $U(1)$ -bundl  
 $U(1)$ -nabitá pole

$$\mathbb{C} \quad \bar{\mathbb{C}} \quad \begin{matrix} \mathbb{C}^{\mathbb{Z}_L \mathbb{Z}_R} \\ \mathbb{C}^{\mathbb{L} \mathbb{L}_R} \end{matrix} \quad h_{\bar{A}B}$$

$$\mathbb{C}^{\mathbb{Z}+1} \leftrightarrow \mathbb{C}^{\mathbb{Z}}$$

$$\phi_{\dots \bar{B}}^{\dots A} \leftrightarrow \phi_{\dots \bar{M}}^{\dots M} \quad \phi_{\dots \bar{B}}^{\dots A} = \phi_{\dots \bar{M}}^{\dots M} \mathbb{1}_{\bar{B}}^A$$

$$\mathbb{C}^{\mathbb{Z}} \rightarrow \begin{cases} \mathbb{C}^{\mathbb{Z}0} \\ \mathbb{C}^0 \end{cases} \quad \mathbb{C}^{\mathbb{Z}\mathbb{Z}} \rightarrow \begin{cases} \mathbb{C}^{\mathbb{Z}0} \\ \mathbb{C}^{0\mathbb{Z}} \end{cases}$$

$$\mathbb{C}^{\mathbb{Z}+1 \mathbb{Z}+1} \leftrightarrow \mathbb{C}^{\mathbb{Z}\mathbb{Z}}$$

$$\phi_{\dots \bar{B} \dots \bar{B}}^{\dots A \dots A} \leftrightarrow \phi_{\dots \bar{M} \dots \bar{N}}^{\dots M \dots N} \quad h_{\bar{N}M}$$

$$= (\phi_{\dots \bar{M} \dots \bar{N}}^{\dots M \dots N}) h_{\bar{N}M}$$

$$\Psi_{\bar{A}} \leftrightarrow \phi_{\bar{B}} = \Psi_{\bar{A}} h_{\bar{A}B}$$

$$\Psi_{\bar{A}} \leftrightarrow \phi^{\bar{B}} = \Psi_{\bar{A}} h^{\bar{A}B}$$

$$\mathbb{C}^{\mathbb{Z}_L \mathbb{Z}_R} \rightarrow \begin{cases} \mathbb{C}^{\mathbb{Z}0} \\ \mathbb{C} \\ \mathbb{C}^{0\mathbb{Z}} \end{cases} \quad \begin{matrix} \mathbb{C}^{\mathbb{Z}} \\ \mathbb{C}^0 \\ \mathbb{C}^{-\mathbb{Z}} \end{matrix}$$

$$\mathbb{C}^m \quad \begin{matrix} \phi & \psi \\ m & m \end{matrix} \rightarrow m+m$$

μάθημα κεντρική  
 $\cdot : \mathbb{C}^m \rightarrow \mathbb{C}^{-m}$

$$\bar{\bar{\phi}} = \phi$$

$$(\phi, \psi) = \bar{\phi} \psi \in \mathbb{C}$$

$U(1)$ -bundl  
 $\mathbb{C}^m M$

$U(1)$ -kalibraien transf.

$$U \in \text{Vect } \mathbb{C}^1 M = \text{Vect } \mathbb{C}^0 M = \mathbb{F} M$$

$$U^* U = 1 \quad U = \exp(iu) \quad \begin{matrix} u \in \mathbb{F} M \\ u(x) \in \mathbb{R} \end{matrix}$$

$$\phi \in \mathbb{C}^1 M \quad \phi \rightarrow \hat{\phi} = \exp(iu) \phi$$

$$\phi \in \mathbb{C}^m M \quad \phi \rightarrow \hat{\phi} = \exp(imu) \phi$$

trivializace kous olov. str.

$$\mathbb{C}^1 M \quad E \quad \bar{E} E = 1 \quad \phi = \varphi E \quad \varphi \in \mathbb{F} M$$

$$\mathbb{C}^m M \quad E^m \quad \psi = \psi E^m \quad \psi \in \mathbb{F} M$$

# Lineární $U(1)$ -bundl

$U(1)$ -nabitá pole

$$\mathbb{C} \quad \bar{\mathbb{C}} \quad \mathbb{C} \begin{matrix} \mathbb{Z}_L & \mathbb{Z}_R \\ l_L & l_R \end{matrix} \quad h_{\bar{A}B}$$

$$\mathbb{C} \begin{matrix} \mathbb{Z}+1 \\ l+1 \end{matrix} \leftrightarrow \mathbb{C} \begin{matrix} \mathbb{Z} \\ l \end{matrix}$$

$$\phi \begin{matrix} \dots A \\ \dots B \end{matrix} \leftrightarrow \phi \begin{matrix} \dots M \\ \dots \bar{M} \end{matrix} \quad \phi \begin{matrix} \dots A \\ \dots B \end{matrix} = \phi \begin{matrix} \dots M \\ \dots \bar{M} \end{matrix} \begin{matrix} \uparrow A \\ \downarrow B \end{matrix}$$

$$\mathbb{C} \begin{matrix} \mathbb{Z} \\ l \end{matrix} \rightarrow \left\{ \begin{matrix} \mathbb{C} \begin{matrix} \mathbb{Z} \\ 0 \end{matrix} \\ \mathbb{C} \begin{matrix} \mathbb{Z} \\ l \end{matrix} \end{matrix} \right. \quad \mathbb{C} \begin{matrix} \mathbb{Z} \\ l \end{matrix} \rightarrow \left\{ \begin{matrix} \mathbb{C} \begin{matrix} \mathbb{Z} \\ 0 \end{matrix} \\ \mathbb{C} \begin{matrix} \mathbb{Z} \\ l \end{matrix} \end{matrix} \right.$$

$$\mathbb{C} \begin{matrix} \mathbb{Z}+1 & l+1 \\ p & q \end{matrix} \leftrightarrow \mathbb{C} \begin{matrix} \mathbb{Z} & l \\ p & q \end{matrix}$$

$$\phi \begin{matrix} \dots A \dots \bar{B} \\ \dots \end{matrix} \mapsto \phi \begin{matrix} \dots M \dots \bar{N} \\ \dots \end{matrix} \quad h_{\bar{N}M}$$

$$= \left( \phi \begin{matrix} \dots M \dots \bar{N} \\ \dots \end{matrix} h_{\bar{N}M} \right) h_{\bar{A}B}$$

$$\psi^{\bar{A}} \leftrightarrow \phi_{\underline{B}} = \psi^{\bar{A}} h_{\underline{A}\underline{B}}$$

$$\psi_{\bar{A}} \leftrightarrow \phi^{\underline{B}} = \psi_{\bar{A}} h^{\bar{A}\underline{B}}$$

$$\begin{matrix} \mathbb{C}^{z_L z_R} \\ \mathbb{C}^{l_L l_R} \end{matrix} \rightarrow \left\{ \begin{matrix} \mathbb{C}^{z 0} \\ \mathbb{C}^{0 0} \\ \mathbb{C} \\ \mathbb{C}^{0 z} \\ \mathbb{C}^{0 0} \end{matrix} \right. \quad \begin{matrix} \mathbb{C}^{z} \\ \mathbb{C}^0 \\ \mathbb{C}^{-z} \end{matrix}$$

$$\begin{matrix} \mathbb{C}^m & \phi & \psi & \rightarrow & m+m \\ & m & m & & \end{matrix}$$

μάθημα κεντρική

$$- : \mathbb{C}^m \rightarrow \mathbb{C}^{-m}$$

$$\bar{\phi} = \phi$$

$$\underbrace{(\phi, \psi)}_{m \quad m} = \underbrace{\bar{\phi}}_0 \psi \in \mathbb{C}$$



$U(1)$ -bundl

$C^m M$

$U(1)$ -Kalibriertransf.

$$U \in \text{Vect } C^1 M = \text{Vect } C^0 M = \mathcal{F}M$$

$$U^* U = 1 \quad U = \exp(iu) \quad \begin{array}{l} u \in \mathcal{F}M \\ u(x) \in \mathbb{R} \end{array}$$

$$\phi \in C^1 M \quad \phi \rightarrow \hat{\phi} = \exp(iu) \phi$$

$$\phi \in C^m M \quad \phi \rightarrow \hat{\phi} = \exp(imu) \phi$$

trivialisierung  $\text{Kov}$  &  $\text{obv. Ste}$

$$C^1 M \quad E \quad \bar{E} E = 1 \quad \phi = \varphi E \quad \varphi \in \mathcal{F}M$$

$$C^m M \quad E^m \quad \psi = \psi E^m \quad \psi \in \mathcal{F}M$$

U(1)-Kovar. derivace

$$\mathbb{C}^m M \quad \bar{\phi} \quad \phi \psi$$

$$D(\phi \psi) = (D\phi) \psi + \phi (D\psi)$$

$$D\bar{\phi} = \overline{D\phi}$$

$$D, \tilde{D} \quad \phi \in \mathbb{C}^m M$$

$$D_m \phi - \tilde{D}_m \phi = i A_m \phi \quad A_m \in \mathbb{C}^m M$$

$$\psi \in \mathbb{C}^m M \quad D_m \psi - \tilde{D}_m \psi = im A_m \psi$$

Arnold E  $\partial E = 0$

$$D\phi = \partial\phi + iA_m \phi \quad A_m \text{ vekt. potenciál}$$

$$A_m^x = A_m$$

tenzor křivosti

$$\phi \in \mathbb{C}^m M \quad i F_{mn} \phi = [D_m D_n - D_n D_m + T_{mn}^k D_k] \phi = \partial_m \partial_n \phi$$

$$\psi \in \mathbb{C}^m M \quad im F_{mn} \psi = [D_m D_n - D_n D_m + T D] \psi = \partial_m \partial_n \psi$$

$$F_{mn} = \partial_m A_n - \partial_n A_m + i \underbrace{[A_m, A_n]}_0 = \partial_m A_n \quad \mathbb{C}^0 M$$

$$D = \partial + iA$$

$$g^{mn} D_m F_{an} = J_a \quad \nabla_m F^{am} = J^a$$

$$-g^{mn} D_m D_n \phi + m^2 \phi = 0$$

$$J_a = mi (\bar{\phi} D_a \phi - \phi D_a \bar{\phi}) \quad \phi \in \mathbb{C}^m M$$

$$\phi \in \mathbb{C}^m M \quad \phi \rightarrow \tilde{\phi} = U^m \phi$$

$$U = \exp(iu)$$

$$D \rightarrow \tilde{D} = D + \Lambda$$

$$\Lambda = -(DU) \cdot U^{-1} = -idu$$

$$\tilde{D}\phi = D\phi - im du \phi$$

$$A \rightarrow \tilde{A} = U \cdot A \cdot U^{-1} + i(\partial U) \cdot U^{-1} = A - du$$

$$F \rightarrow \tilde{F} = U \cdot F \cdot U^{-1} = F$$



# $U(1)$ -Kovar. derivace

$$\mathbb{C}^m M \quad \bar{\phi} \quad \phi \psi$$

$$D(\phi \psi) = (D\phi) \psi + \phi(D\psi)$$

$$D\bar{\phi} = \overline{D\phi}$$

$$D, \tilde{D} \quad \phi \in \mathbb{C}^m M$$

$$D_m \phi - \tilde{D}_m \phi = i A_m \phi \quad A_m \in \mathbb{C}^m M$$

$$\psi \in \mathbb{C}^m M \quad D_m \psi - \tilde{D}_m \psi = i m A_m \psi$$

$$\text{Anivial } E \quad \partial E = 0$$

$$D\phi = \partial\phi + i A_m \phi \quad A_m \text{ vekt. potenciál}$$

$$A_m^* = A_m$$

tenzor krivosti

$$\phi \in \mathbb{C}^1 M \quad i F_{\underline{mn}} \phi = [D_{\underline{m}} D_{\underline{n}} - D_{\underline{n}} D_{\underline{m}} + T_{\underline{mn}}^k D_k] \phi = \overset{D}{\partial}_{\underline{m}} \overset{D}{\partial}_{\underline{n}} \phi$$

$$\psi \in \mathbb{C}^n M \quad i_m F_{\underline{mn}} \psi = [D D - D D + T D] \psi = \overset{D}{\partial}_{\underline{m}} \overset{D}{\partial}_{\underline{n}} \psi$$

$$F_{\underline{mn}} = \overset{\partial}{\partial}_{\underline{m}} A_{\underline{n}} + i \underbrace{[A_{\underline{m}}, A_{\underline{n}}]}_0 = \overset{\partial}{\partial}_{\underline{m}} A_{\underline{n}}$$

$$D = \partial + iA$$

$$\phi \in \mathbb{C}^n M \quad \phi \rightarrow \tilde{\phi} = U^m \phi$$

$$U = \exp(iu)$$

$$D \rightarrow \tilde{D} = D + \Lambda$$

$$\Lambda = -(DU) \cdot U^{-1} = -i du$$

$$\tilde{D} \phi = D\phi - im du \phi$$

$$\begin{aligned} A \rightarrow \tilde{A} &= U \cdot A \cdot U^{-1} + i(DU) \cdot U^{-1} \\ &= A - du \end{aligned}$$

$$F \rightarrow \tilde{F} = U \cdot F \cdot U^{-1} = F$$



$$F_{mn} = \partial_m A_n - \partial_n A_m + i \underbrace{[A_m, A_n]}_0 = d_m A_n \quad \mathbb{C}^0 M$$

$$D = \partial + iA$$

$$g^{mn} D_m F_{an} = J_a \quad \nabla_m F^{am} = J^a$$

$$- g^{mn} D_m D_n \phi + M^2 \phi = 0$$

$$J_a = mi (\bar{\phi} D_a \phi - \phi D_a \bar{\phi}) \quad \phi \in \mathbb{C}^n M$$