Linearní U(1)-bundl

U(1)-nabitá pole

$\mathbb{C}$ $\quad \bar{\mathbb{C}}$ $\quad \mathbb{C}^{q_L\, q_R}_{\ell_L\, \ell_R}$ $\quad h_{\underline{\bar{A}B}}$

$\mathbb{C}^{q+1}_{\ell+1} \leftrightarrow \mathbb{C}^{q}_{\ell}$

$\phi^{\dots \underline{A}}_{\dots \underline{B}} \leftrightarrow \phi^{\dots \underline{M}}_{\dots \underline{M}}$ $\qquad \phi^{\dots \underline{A}}_{\dots \underline{B}} = \phi^{\dots \underline{M}}_{\dots \underline{M}}\, \mathbb{1}^{\underline{A}}_{\underline{B}}$

$\mathbb{C}^{q}_{\ell} \to \begin{cases} \mathbb{C}^{q}_{0} \\ \mathbb{C}^{0}_{\ell} \end{cases}$ $\qquad \mathbb{C}^{q\ell} \to \begin{cases} \mathbb{C}^{q0} \\ \mathbb{C}^{0\ell} \end{cases}$

$\mathbb{C}^{q+1\, \ell+1}_{pq} \leftrightarrow \mathbb{C}^{q\ell}_{pq}$

$\phi^{\dots \underline{A} \dots \underline{\bar{B}}}_{\dots\, \dots} \leftrightarrow \phi^{\dots \underline{M} \dots \underline{\bar{N}}}_{\dots\, \dots}\, h_{\underline{\bar{N}M}}$

$= \left(\phi^{\dots \underline{M} \dots \underline{\bar{N}}}_{\dots\, \dots}\, h_{\underline{\bar{N}M}}\right) h^{-1\underline{\bar{B}A}}$

$\psi^{\underline{\bar{A}}} \leftrightarrow \phi_{\underline{B}} = \psi^{\underline{\bar{A}}} h_{\underline{\bar{A}B}}$

$\psi_{\underline{\bar{A}}} \leftrightarrow \phi^{\underline{B}} = \psi_{\underline{\bar{A}}} h^{-1\underline{\bar{A}B}}$

$\mathbb{C}^{q_L\, q_R}_{\ell_L\, \ell_R} \to \begin{cases} \mathbb{C}^{q0}_{00} & \mathbb{C}^{q} \\ \mathbb{C} & \mathbb{C}^{0} \\ \mathbb{C}^{0q}_{00} & \mathbb{C}^{-q} \end{cases}$

$\mathbb{C}^{m}$ $\quad \underset{m}{\phi}\, \underset{m}{\psi} \to m+m$

nábojové sdružení

$\bar{\ }: \mathbb{C}^{m} \to \mathbb{C}^{-m}$

$\bar{\Phi} = \phi$

$(\underset{m}{\phi}, \underset{m}{\psi}) = \underbrace{\bar{\Phi}\psi}_{0} \in \mathbb{C}$

U(1)-bundl

$\mathbb{C}^{m} M$

U(1)-kalibrační transf

$U \in \mathrm{Sect}\, \mathbb{C}^{1}_{1} M = \mathrm{Sect}\, \mathbb{C}^{0} M = \mathfrak{F}M$

$U^{*}U = 1 \qquad U = \exp(iu) \qquad u \in \mathfrak{F}M$, $u(x) \in \mathbb{R}$

$\phi \in \mathbb{C}^{1}M \qquad \phi \to \tilde{\phi} = \exp(iu)\,\phi$

$\phi \in \mathbb{C}^{m}M \qquad \phi \to \tilde{\phi} = \exp(imu)\,\phi$

trivializace kovar. ohr. ste

$\mathbb{C}^{1}M \quad E \qquad \bar{E}E = 1 \qquad \phi = \varphi E \quad \varphi \in \mathfrak{F}M$

$\mathbb{C}^{m}M \quad E^{m} \qquad \psi = \varphi E^{m} \quad \varphi \in \mathfrak{F}M$

Linearní U(1)-bundl

U(1)-nabitá pole

$$\mathbb{C} \qquad \overline{\mathbb{C}} \qquad \mathbb{C}^{k_L\,k_R}_{l_L\,l_R} \qquad h_{\underline{\bar{A}B}}$$

$$\mathbb{C}^{k+1}_{l+1} \longleftrightarrow \mathbb{C}^{k}_{l}$$

$$\phi^{\dots\underline{A}}_{\dots\underline{B}} \longleftrightarrow \phi^{\dots\underline{M}}_{\dots\underline{M}} \qquad \phi^{\dots\underline{A}}_{\dots\underline{B}} = \phi^{\dots\underline{M}}_{\dots\underline{M}}\, \mathbb{1}^{\underline{A}}_{\underline{B}}$$

$$\mathbb{C}^{k}_{l} \longrightarrow \left\{ \begin{array}{l} \mathbb{C}^{k}_{0} \\ \mathbb{C}^{0}_{l} \end{array} \right. \qquad \mathbb{C}^{kl} \longrightarrow \left\{ \begin{array}{l} \mathbb{C}^{k0} \\ \mathbb{C}^{0l} \end{array} \right.$$

$$\mathbb{C}^{k+1\,l+1}_{pq} \longleftrightarrow \mathbb{C}^{kl}_{pq}$$

$$\phi^{\dots\underline{A}\dots\underline{\bar{B}}}_{\dots\ \dots} \longleftrightarrow \phi^{\dots\underline{M}\dots\underline{\bar{N}}}_{\dots\ \dots}\, h_{\underline{\bar{N}M}}$$

$$= \left(\phi^{\dots\underline{M}\dots\underline{\bar{N}}}_{\dots\ \dots}\, h_{\underline{\bar{N}M}}\right) h^{-1\,\underline{\bar{B}A}}$$

$$\psi^{\bar{A}} \leftrightarrow \phi_{\underline{B}} = \psi^{\bar{A}} h_{\bar{A}\underline{B}}$$

$$\psi_{\bar{A}} \leftrightarrow \phi^{\underline{B}} = \psi_{\bar{A}} h^{-1\bar{A}\underline{B}}$$

$$\mathbb{C}^{q_L q_R}_{l_L l_R} \longrightarrow \begin{cases} \mathbb{C}^{q0}_{00} & \mathbb{C}^{q} \\ \mathbb{C} & \mathbb{C}^{0} \\ \mathbb{C}^{0q}_{00} & \mathbb{C}^{-q} \end{cases}$$

$$\mathbb{C}^m \qquad \underset{m}{\phi}\,\underset{m}{\psi} \to m+m$$

nábojové sdružení

$$\bar{\ } : \mathbb{C}^m \to \mathbb{C}^{-m}$$

$$\bar{\bar{\phi}} = \phi$$

$$(\underset{m}{\phi}, \underset{m}{\psi}) = \underbrace{\bar{\phi}\psi}_{0} \in \mathbb{C}$$

U(1)-bundl

$\mathbb{C}^m M$

U(1)-kalibrační transf.

$$U \in \mathrm{Sect}\, \mathbb{C}^1_1 M = \mathrm{Sect}\, \mathbb{C}^0 M = \mathcal{F}M$$

$$U^* U = 1 \qquad U = \exp(iu) \qquad u \in \mathcal{F}M, \; u(x) \in \mathbb{R}$$

$$\phi \in \mathbb{C}^1 M \qquad \phi \to \tilde{\phi} = \exp(iu)\,\phi$$

$$\phi \in \mathbb{C}^m M \qquad \phi \to \tilde{\phi} = \exp(imu)\,\phi$$

trivializace kom. ohr. sta

$$\mathbb{C}^1 M \qquad E \qquad \bar{E} E = 1 \qquad \phi = \varphi E \qquad \varphi \in \mathcal{F}M$$

$$\mathbb{C}^m M \qquad E^m \qquad \psi = \varphi E^m \qquad \varphi \in \mathcal{F}M$$

U(1)-Kovar. derivace

$\mathbb{C}^n M \quad \bar{\phi} \quad \phi\,\psi$

$$D(\phi\psi) = (D\phi)\psi + \phi(D\psi)$$

$$D\bar{\phi} = \overline{D\phi}$$

$D, \tilde{D} \qquad \phi \in \mathbb{C}^1 M$

$$D_m\phi - \tilde{D}_m\phi = iA_m\phi \qquad A_m \in \mathbb{C}^0 M$$

$\psi \in \mathbb{C}^n M \qquad D_m\psi - \tilde{D}_m\psi = imA_m\psi$

triviál $E \qquad \partial E = 0$

$D\phi = \partial\phi + iA_m\phi \qquad A_m$ vekt. potenciál

$$A_m^* = A_m$$

tenzor křivosti

$\phi \in \mathbb{C}^1 M \qquad iF_{mn}\phi = [D_m D_n - D_n D_m + T^k_{mn} D_k]\phi = {}^D d_m {}^D d_n \phi$

$\psi \in \mathbb{C}^n M \qquad imF_{mn}\psi = [DD - DD + TD]\psi = {}^D d_m {}^D d_n \psi$

$$F_{mn} = {}^\partial d_m A_n + i\underbrace{[A_m, A_n]}_{0} = d_m A_n \qquad \mathbb{C}^0 M$$

$$D = \partial + iA$$

$$g^{mn} D_m F_{an} = J_a \qquad \nabla_m F^{am} = J^a$$

$$-g^{mn} D_m D_n \phi + M^2\phi = 0$$

$$J_a = mi(\bar{\phi} D_a \phi - \phi D_a \bar{\phi}) \qquad \phi \in \mathbb{C}^n M$$

$\phi \in \mathbb{C}^n M \qquad \phi \to \tilde{\phi} = U^m \phi$

$$U = \exp(iu)$$

$$D \to \tilde{D} = D + \Lambda$$

$$\Lambda = -(DU)\cdot U^{-1} = -i\,du$$

$$\tilde{D}\phi = D\phi - im\,du\,\phi$$

$$A \to \tilde{A} = U\cdot A\cdot U^{-1} + i(\partial U)\cdot U^{-1}$$

$$= A - du$$

$$F \to \tilde{F} = U\cdot F\cdot U^{-1} = F$$

U(1)-Kovar. derivace

$\mathbb{C}^m M \quad \overline{\phi} \quad \phi \psi$

$$D(\phi\psi) = (D\phi)\psi + \phi(D\psi)$$

$$D\overline{\phi} = \overline{D\phi}$$

$D, \tilde{D} \qquad \phi \in \mathbb{C}^1 M$

$$D_{\underline{m}}\phi - \tilde{D}_{\underline{m}}\phi = iA_{\underline{m}}\phi \qquad A_{\underline{m}} \in \mathbb{C}^0 M$$

$\psi \in \mathbb{C}^m M \quad D_{\underline{m}}\psi - \tilde{D}_{\underline{m}}\psi = imA_{\underline{m}}\psi$

triviál $E \quad \partial E = 0$

$D\phi = \partial\phi + iA_{\underline{m}}\phi \quad A_{\underline{m}}$ vekt. potenciál

$$A_{\underline{m}}^* = A_{\underline{m}}$$

tenzor křivosti

$\phi \in C^1 M \qquad iF_{\underline{mn}}\phi = [D_{\underline{m}}D_{\underline{n}} - D_{\underline{n}}D_{\underline{m}} + T^{\underline{k}}_{\underline{mn}}D_{\underline{k}}]\phi = {}^{D}d_{\underline{m}}{}^{D}d_{\underline{n}}\phi$

$\psi \in C^n M \qquad i\,m\,F_{\underline{mn}}\psi = [DD - DD + TD]\psi = {}^{D}d_{\underline{m}}{}^{D}d_{\underline{n}}\psi$

$$F_{\underline{mn}} = {}^{\partial}d_{\underline{m}}A_{\underline{n}} + i\underbrace{[A_{\underline{m}}, A_{\underline{n}}]}_{0} = d_{\underline{m}}A_{\underline{n}}$$

$D = \partial + iA$

$$\phi \in C^n M \qquad \phi \to \tilde{\phi} = U^m \phi$$

$$U = \exp(iu)$$

$$D \to \tilde{D} = D + \Lambda$$

$$\Lambda = -(DU)\cdot U^{-1} = -i\,du$$

$$\tilde{D}\phi = D\phi - im\,du\,\phi$$

$$A \to \tilde{A} = U\cdot A\cdot U^{-1} + i(\partial U)\cdot U^{-1}$$

$$= A - du$$

$$F \to \tilde{F} = U\cdot F\cdot U^{-1} = F$$

$$F_{\underline{mn}} = \overset{\partial}{d}_{\underline{m}} A_{\underline{n}} + i\underbrace{[A_{\underline{m}}, A_{\underline{n}}]}_{0} = d_{\underline{m}} A_{\underline{n}} \qquad C^0 M$$

$$D = \partial + iA$$

$$g^{\underline{mn}} D_{\underline{m}} F_{\underline{an}} = J_{\underline{a}} \qquad \nabla_{\underline{m}} F^{\underline{am}} = J^{\underline{a}}$$

$$-g^{\underline{mn}} D_{\underline{m}} D_{\underline{n}} \phi + M^2 \phi = 0$$

$$J_{\underline{a}} = mi(\bar{\phi} D_{\underline{a}} \phi - \phi D_{\underline{a}} \bar{\phi}) \qquad \phi \in \mathbb{C}^n M$$