G poloprostá Ad .

$\mathfrak{g}$ poloprostá ad

k nedegener.

$\mathrm{Der}\,\mathfrak{g} = \mathrm{ad}_{\mathfrak{g}}$

↳ algebra operátorů tvaru ad_m, $m \in \mathfrak{g}$

↳ prostor alg. der. tj. oper. δ

$$\delta[m,n] = [\delta m, n] + [m, \delta n]$$

vekt. pr. A

repr. gr. G na A $\quad T^{A}{}_{B}$

repr. alg. $\mathfrak{g}$ na A $\quad t_m{}^{A}{}_{B} = m^{\alpha}\, t_{\alpha}{}^{A}{}_{B}$

$$A = A_0 \oplus \bigoplus_{r} A_r$$

P_r projektor na A_r

$$t_{\alpha}{}^{A}{}_{B} = \bigoplus t_{r\alpha}{}^{A}{}_{B}$$

$$t_r = P_r \cdot t \cdot P_r$$

t_r ireducibilní

$$[S, t_m] = 0 \quad \forall m \in \mathfrak{g} \quad \Rightarrow \quad S = \sum_{r} \lambda_r P_r$$

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→ algebra operátorů tvaru ad_m $m \in \mathfrak{g}$

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vekt. pr. $\mathbb{A}$

repr. gr. G na $\mathbb{A}$ $\qquad T_{a}{}^{\underline{A}}{}_{\underline{B}}$

repr. alg. $\mathfrak{g}$ na $\mathbb{A}$ $\qquad t_{m}{}^{\underline{A}}{}_{\underline{B}} = m^{\alpha}\, t_{\underline{\alpha}}{}^{\underline{A}}{}_{\underline{B}}$

$$\mathbb{A} = \mathbb{A}_0 \oplus \bigoplus_r \mathbb{A}_r$$

P_r projektor na $\mathbb{A}_r$

$$t_{\underline{\alpha}}{}^{\underline{A}}{}_{\underline{B}} = \bigoplus t_{r\underline{\alpha}}{}^{\underline{A}}{}_{\underline{B}} \qquad t_r = P_r \cdot t \cdot P_r$$

t_r ireducibilní

$$[S, t_m] = 0 \implies S = \sum_r \lambda_r P_r$$

$\forall m \in \mathfrak{g}$

Lokální kalibrační grupa a algebra

GM $\qquad h \in \mathrm{Sect}\, GM$

$\qquad\qquad h(x) \in G_x M$

gM $\qquad m \in \mathrm{Sect}\, gM$

$\qquad\qquad m(x) \in g_x M$

$$[m,n]^\alpha = m^\mu n^\nu c_{\mu\nu}{}^\alpha$$

$$\mathrm{ad}_m n = [m,n]$$

$$\mathrm{ad}_m{}^\alpha{}_\beta = m^\kappa c_{\kappa\beta}{}^\alpha$$

Kovar. der. na bundlu Lieovy alg.

Def
mějme bundl. L. alg gM
kov. der. D je kons. s alg. str.

$$Dc = 0$$
$$\Updownarrow$$
$$D[m,n] = [Dm,n] + [m,Dn]$$

Lema
$$Dk = 0$$

Def trivializace kons. s alg. str.
e_α báze v gM
$c_{\alpha\beta}{}^\gamma = \mathrm{konst}$ (komp. c vůči e_α)

Lema
∂ souř. der. trivial e_α
kons. s alg. str.

$$\partial e_\alpha = 0$$
$$\Downarrow$$
$$\partial c = 0 \qquad \partial \text{ kons. s alg. str.}$$
$$\partial k = 0$$

Lokální kalibrační grupa a algebra

$\mathbb{G}M \qquad h \in \mathrm{Sect}\,\mathbb{G}M$

$\qquad\qquad h(x) \in \mathbb{G}_x M$

$\mathfrak{g}M \qquad m \in \mathrm{Sect}\,\mathfrak{g}M$

$\qquad\qquad m(x) \in \mathfrak{g}_x M$

$$[m,n]^\alpha = m^\mu n^\nu c_{\mu\nu}{}^\alpha$$

$$\mathrm{ad}_m n = [m,n]$$

$$\mathrm{ad}_m{}^\alpha{}_\beta = m^\kappa c_{\kappa\beta}{}^\alpha$$

Kovar. der. na bundle Lieovy alg.

Def
mějme bundl. L. alg gM
kov. der. D je kons. s alg. str.

$$D c = 0$$
$$\Updownarrow$$
$$D[m,n] = [Dm,n] + [m,Dn]$$

Kon.
$$D k = 0$$

Def trivializace kons. s alg. str.
e_α báze v gM
$c_{\alpha\beta}{}^{\gamma} = \text{konst}$ (komp. c vůči e_α)

Lema

∂ souř. der. triviál e_α
komut. s alg. str.

$\partial e_\alpha = 0$

$\Downarrow$ $\partial c = 0$ $\quad$ ∂ komut. s alg. str.

$\partial k = 0$

Věta:

$\mathcal{D}, \tilde{\mathcal{D}}$ kov. der na $\mathfrak{g}M$

$$\mathcal{D} = \tilde{\mathcal{D}} + \mathsf{A} \qquad A_{m}{}^{\alpha}{}_{\beta}$$

$$\mathsf{A}\cdot[m,n] = [\mathsf{A}\cdot m, n] + [m, \mathsf{A}\cdot n]$$

$$\Rightarrow \exists\, \mathfrak{A} \qquad \mathsf{A} = \mathrm{ad}_{\mathfrak{A}}$$

$$\mathfrak{A}_{m}{}^{\kappa} \qquad A_{m}{}^{\alpha}{}_{\beta} = \mathfrak{A}_{m}{}^{\lambda} c_{\lambda\beta}{}^{\alpha}$$

Lemma

∂ souř. der.

$$\mathsf{A} = \mathrm{ad}_{\mathfrak{A}}$$

$$\Updownarrow$$

$\mathcal{D} = \partial + \mathsf{A}$ je kovar. s alg. str.

Věta

$m \in \mathrm{Sect}\, \mathfrak{g}M$

$$\mathcal{D}_m m^{\alpha} = \partial_m m^{\alpha} + A_{m}{}^{\alpha}{}_{\beta}\, m^{\beta}$$
$$= \partial_m m^{\alpha} + [\mathfrak{A}_m, m]^{\alpha}$$

$\mathsf{B} \in \mathrm{Sect}\, \mathfrak{g}^1_1 M \qquad B^{\alpha}{}_{\beta} = \mathrm{ad}_{B}{}^{\alpha}{}_{\beta} = B^{\kappa} c_{\kappa\beta}{}^{\alpha}$

$$\mathcal{D}_m B^{\alpha}{}_{\beta} = \partial_m B^{\alpha}{}_{\beta} + [A_m, B]^{\alpha}{}_{\beta}$$

$$(\mathcal{D}_m B^{\kappa})\, c_{\kappa\beta}{}^{\alpha} = (\partial_m B^{\kappa} + [\mathfrak{A}_m, B]^{\kappa})\, c_{\kappa\beta}{}^{\alpha}$$

$$\mathfrak{F}_{mn}{}^{\kappa} = \partial_m \wedge \mathfrak{A}_n{}^{\kappa} + [\mathfrak{A}_m, \mathfrak{A}_n]^{\kappa}$$
$$= \partial_m \mathfrak{A}_n{}^{\kappa} - \partial_n \mathfrak{A}_m{}^{\kappa} + \mathfrak{A}_m{}^{\mu} \mathfrak{A}_n{}^{\nu} c_{\mu\nu}{}^{\kappa}$$
$$T = 0$$

Věta $F_{mn}{}^{\alpha}{}_{\beta}$ tenz. kř. der $\mathcal{D}_m$

$$F_{mn}\, c = 0$$

$$F_{mn}\cdot[m,n] = [F_{mn}\cdot m, n] + [m, F_{mn}\cdot n]$$

$$\Downarrow$$

$$\exists\, \mathfrak{F}_{mn} \qquad F = \mathrm{ad}_{\mathfrak{F}}$$

$$F_{mn}{}^{\alpha}{}_{\beta} = \mathfrak{F}_{mn}{}^{\kappa} c_{\kappa\beta}{}^{\alpha}$$

$$F_{mn} B^{\kappa} = F_{mn}{}^{\kappa}{}_{\lambda} B^{\lambda} = [\mathfrak{F}, B]^{\kappa}$$

$$F_{mn} B^{\alpha}{}_{\beta} = [F_{mn}, B]^{\alpha}{}_{\beta} = [\mathfrak{F}, B]^{\kappa} c_{\kappa\beta}{}^{\alpha}$$

Věta: $\mathcal{D}$ kov. der. dané vekt. pot. $\mathfrak{A}$ vůči ∂

$$F_{mn} = \partial_m A_n + [A_m, A_n]$$

$$\mathfrak{F}_{mn} = \partial_m \mathfrak{A}_n + [\mathfrak{A}_m, \mathfrak{A}_n]$$

Věta:

$D, \tilde{D}$ kov. der. na $\mathfrak{g}M$

$$D = \tilde{D} + \mathbb{A} \qquad A_{\underline{m}\,\beta}{}^{\alpha}$$

$$A \cdot [m, n] = [A \cdot m, n] + [m, A \cdot n]$$

$$\Rightarrow \exists\, \mathfrak{k} \qquad A = \mathrm{ad}_{\mathfrak{k}}$$

$$\mathfrak{k}_{\underline{m}}{}^{k} \qquad A_{\underline{m}\beta}{}^{\alpha} = \mathfrak{k}_{\underline{m}}{}^{\delta} C_{\delta\beta}{}^{\alpha}$$

Lemma

∂ souř. der.

$A = \mathrm{ad}_{\mathfrak{k}}$

$\Downarrow$

$D = \partial + \mathbb{A}$ je kovar. s alg. str.

Vièta

$m \in \mathrm{Sect}\,\mathfrak{g}M$

$$D_{\underline{m}} m^{\underline{\alpha}} = \partial_{\underline{m}} m^{\underline{\alpha}} + A_{\underline{m}}{}^{\underline{\alpha}}{}_{\underline{\beta}}\, m^{\underline{\beta}}$$

$$= \partial_{\underline{m}} m^{\underline{\alpha}} + [\mathfrak{A}_{\underline{m}}, m]^{\underline{\alpha}}$$

$B \in \mathrm{Sect}\,\mathfrak{g}^1 M \qquad B^{\underline{\alpha}}{}_{\underline{\beta}} = \mathrm{ad}_B{}^{\underline{\alpha}}{}_{\underline{\beta}} = B^{\underline{\kappa}} c_{\underline{\kappa\beta}}{}^{\underline{\alpha}}$

$$D_{\underline{m}} B^{\underline{\alpha}}{}_{\underline{\beta}} = \partial_{\underline{m}} B^{\underline{\alpha}}{}_{\underline{\beta}} + [A_{\underline{m}}, B]^{\underline{\alpha}}{}_{\underline{\beta}}$$

$$(D_{\underline{m}} B^{\underline{\kappa}})\, c_{\underline{\kappa\beta}}{}^{\underline{\alpha}} = \left(\partial_{\underline{m}} B^{\underline{\kappa}} + [\mathfrak{A}_{\underline{m}}, B]^{\underline{\kappa}}\right) c_{\underline{\kappa\beta}}{}^{\underline{\alpha}}$$

Věta $F_{\underline{mn}}{}^{\alpha}{}_{\beta}$ tenz. kov. der. $D_{\underline{m}}$

$$F_{\underline{mn}}\, c = 0$$

$$F_{\underline{mn}} \cdot [m, m] = [F_{\underline{mn}} \cdot m, m] + [m, F_{\underline{mn}} \cdot m]$$

$$\Downarrow$$

$$\exists\, \mathcal{F}_{\underline{mn}} \qquad F = \mathrm{ad}_{\mathcal{F}}$$

$$F_{\underline{mn}}{}^{\alpha}{}_{\beta} = \mathcal{F}_{\underline{mn}}{}^{\kappa}\, c_{\kappa\beta}{}^{\alpha}$$

$$F_{\underline{mn}} B^{\kappa} = F_{\underline{mn}}{}^{\kappa}{}_{\alpha} B^{\alpha} = [\mathcal{F}, B]^{\kappa}$$

$$F_{\underline{mn}} B^{\alpha}{}_{\beta} = [F_{\underline{mn}}, B]^{\alpha}{}_{\beta} = [\mathcal{F}, B]^{\kappa}\, c_{\kappa\beta}{}^{\alpha}$$

Věta: D kov. der. daná vekt. pot. $\mathcal{A}$ vůči ∂

$$F_{\underline{mn}} = \partial_{\underline{m}} A_{\underline{n}} + [A_{\underline{m}}, A_{\underline{n}}]$$

$$\mathcal{F}_{\underline{mn}} = \partial_{\underline{m}} \mathcal{A}_{\underline{n}} + [\mathcal{A}_{\underline{m}}, \mathcal{A}_{\underline{n}}]$$

$$\mathcal{F}_{\underline{mn}}{}^{\kappa} = \partial_{\underline{m}} \wedge \mathcal{A}_{\underline{n}}{}^{\kappa} + [\mathcal{A}_{\underline{m}}, \mathcal{A}_{\underline{n}}]^{\kappa}$$

$$= \partial_{\underline{m}} \mathcal{A}_{\underline{n}}{}^{\kappa} - \partial_{\underline{n}} \mathcal{A}_{\underline{m}}{}^{\kappa} + \mathcal{A}_{\underline{m}}{}^{\mu} \mathcal{A}_{\underline{n}}{}^{\nu}\, c_{\mu\nu}{}^{\kappa}$$

$$T = 0$$