

$G$  holomorfne  $Ad$

$\mathfrak{g}$  holomorfne  $ad$

$k$  mede gener.

$\text{Der } \mathfrak{g} = \text{ad } \mathfrak{g}$

$\rightarrow$  algebra operatoru  
Anvaru  $ad_m$   $m \in \mathfrak{g}$

prostor alg. der. tj. oper.  $\delta$   
 $\delta(m, m) = [\delta m, m] + [m, \delta m]$

vekt pr.  $A$

repr. gr.  $G$  na  $A$

repr. alg  $\mathfrak{g}$  na  $A$

$T_{A, B}^A$

$$t_m^A = m \cdot t_{\alpha}^A$$

$$A = A_0 \oplus \bigoplus_{\mathfrak{k}} A_{\mathfrak{k}}$$

$P_{\mathfrak{k}}$  projektor na  $A_{\mathfrak{k}}$

$$t_{\alpha}^A = \bigoplus_{\mathfrak{k}} t_{\mathfrak{k}, \alpha}^A$$

$$t_{\mathfrak{k}} = P_{\mathfrak{k}} \cdot t \cdot P_{\mathfrak{k}}$$

$t_{\mathfrak{k}}$  ireducibilni

$$[S, t_m] = 0$$

$\Rightarrow$

$$S = \sum_{\mathfrak{k}} A_{\mathfrak{k}} P_{\mathfrak{k}}$$

$\forall m \in \mathfrak{g}$

$G$  polynomiál  $Ad$

$\mathfrak{g}$  polynomiál  $ad$

$k$  medegener.

$\text{Der } \mathfrak{g} = \text{ad } \mathfrak{g}$

$\rightarrow$  algebra operátoru  
Annam  $ad_m$   $m \in \mathfrak{g}$

$\rightarrow$  prostor  $\text{alg. der.}$   $\dagger_j$  oper.  $\delta$   
 $\delta[m, n] = [\delta m, n] + [m, \delta n]$

vekt. pr.  $A$

repr. gr.  $G$  na  $A$

repr. alg  $\mathcal{G}$  na  $A$

$$T_{A, B}^A$$

$$t_{m, B}^A = m^\alpha t_{\alpha, B}^A$$

$$A = A_0 \oplus \bigoplus_k A_k$$

$P_k$  projektor na  $A_k$

$$t_{\alpha, B}^A = \bigoplus_k t_{k, \alpha, B}^A$$

$$t_k = P_k \cdot t \cdot P_k$$

$t_k$  irreducibilni

$$[S, t_m] = 0 \Rightarrow$$

$$S = \sum_k A_k P_k$$

$\forall m \in \mathcal{G}$

# Lokální kalibrační grupa a algebra

$GM$   $h \in \text{Vect } GM$   
 $h(x) \in G_x M$

$\mathfrak{g}M$   $m \in \text{Vect } \mathfrak{g}M$   
 $m(x) \in \mathfrak{g}_x M$

$$[m, m]^x = m^k m^l C_{kl}^x$$

$$\text{ad}_m m = [m, m]$$

$$\text{ad}_m^k = m^l C_{kl}^x$$

Kovarov. der. na bundlu Lieov alg.

Def. mějme bundl. Lieov alg  $\mathfrak{g}M$   
kov. der.  $\mathcal{D}$  je kovarov. Lieov str.

$$\mathcal{D}c = 0$$

$$\mathcal{D}[m, m] = [\mathcal{D}m, m] + [m, \mathcal{D}m]$$

Lema

$$\mathcal{D}k = 0$$

Def. Trivializace kovarov. Lieov str.

$e_\alpha$  báze  $\mathfrak{g}M$

$C_{\alpha\beta}^x = \text{konst}$  (konst.  $C$  vůči  $e_\alpha$ )

Lema

$\partial$  kovarov. der. trivial. Lieov str.

$$\partial e_\alpha = 0$$

$$\Downarrow \partial c = 0 \quad \partial k = 0$$

# Lokální kalibrační grupa a algebra

$$GM \quad h \in \text{Sect } GM$$

$$h(x) \in G_x M$$

$$\mathfrak{g}M \quad m \in \text{Sect } \mathfrak{g}M$$

$$m(x) \in \mathfrak{g}_x M$$

$$[m, m]^\alpha = m^k m^l C_{kl}^\alpha$$

$$\text{ad}_m m = [m, m]$$

$$\text{ad}_m^\alpha \mathfrak{f} = m^k C_{kl}^\alpha \mathfrak{f}$$

Konv. der. ma. bundle Liouville alg.

Def  
miejm bundle. Lie alg  $\mathfrak{g}^M$   
konv. der.  $D$  je konv. s alg. str.

$$Dc = 0$$

$$\Downarrow D[m, m] = [Dm, m] + [m, Dm]$$

Lema

$$Dk = 0$$

Def Anisotropy konv. s alg. str.

$e_\alpha$  baze  $\mathfrak{g}^M$

$C_{\alpha\beta}^{\gamma} = \text{konst}$  (konj. C mezi  $e_\alpha$ )

Lemma

① sowie der trivial  $e_x$   
konver  $\rightarrow$  alg. str.

$$\partial e_x = 0$$

$\Downarrow$

$$\partial c = 0$$

$$\partial k = 0$$

② konver  $\rightarrow$  alg str

Viete:

$\tilde{D}, \tilde{D}$  kov. der me  $\mathfrak{g}M$

$$D = \tilde{D} + A \quad A_{\mu\nu}^{\alpha} \in \mathbb{R}$$

$$A \cdot [m, m] = [A \cdot m, m] + [m, A \cdot m]$$

$$\Rightarrow \exists \mathcal{R} \quad A = \text{ad}_{\mathcal{R}}$$

$$\mathcal{R}_{\mu\nu}^{\alpha} \quad A_{\mu\nu}^{\alpha} = \mathcal{R}_{\mu\nu}^{\beta} C_{\beta\gamma}^{\alpha}$$

Lemma

$\partial$  sowie der.

$$\equiv A = \text{ad}_{\mathcal{R}}$$

$$D = \partial + A \quad \text{je kov. u. abg. str.}$$

Viete

me  $\mathfrak{g}M$

$$\begin{aligned} D_{\mu} m^{\nu} &= \partial_{\mu} m^{\nu} + A_{\mu\lambda}^{\nu} m^{\lambda} \\ &= \partial_{\mu} m^{\nu} + [\mathcal{R}_{\mu}, m]^{\nu} \end{aligned}$$

$$B \in \mathfrak{g}M \quad B_{\mu}^{\alpha} = \text{ad}_{B^{\alpha}} = B^{\beta} C_{\beta\gamma}^{\alpha}$$

$$D_{\mu} B_{\nu}^{\alpha} = \partial_{\mu} B_{\nu}^{\alpha} + [A_{\mu}, B]^{\alpha}_{\nu}$$

$$(D_{\mu} B^{\alpha}) C_{\beta\gamma}^{\alpha} = (\partial_{\mu} B^{\alpha} + [\mathcal{R}_{\mu}, B]^{\alpha}) C_{\beta\gamma}^{\alpha}$$

$$\begin{aligned} \hat{F}_{\mu\nu}^{\alpha} &= \partial_{\mu} \mathcal{R}_{\nu}^{\alpha} - \partial_{\nu} \mathcal{R}_{\mu}^{\alpha} + [\mathcal{R}_{\mu}, \mathcal{R}_{\nu}]^{\alpha} \\ &= \partial_{\mu} \mathcal{R}_{\nu}^{\beta} - \partial_{\nu} \mathcal{R}_{\mu}^{\beta} + \mathcal{R}_{\mu}^{\gamma} \mathcal{R}_{\nu}^{\delta} C_{\gamma\delta}^{\beta} \\ T &= 0 \end{aligned}$$

Viete:

Viete  $F_{\mu\nu}^{\alpha}$  tensor für der  $D_{\mu}$

$$F_{\mu\nu} c = 0$$

$$F_{\mu\nu} \cdot [m, m] = [F_{\mu\nu} \cdot m, m] + [m, F_{\mu\nu} \cdot m]$$

$$\Downarrow \quad F = \text{ad}_{\mathcal{F}}$$

$$F_{\mu\nu}^{\alpha} = F_{\mu\nu}^{\beta} C_{\beta\gamma}^{\alpha}$$

$$F_{\mu\nu} B^{\alpha} = F_{\mu\nu}^{\beta} B^{\alpha} = [F, B]^{\alpha}$$

$$F_{\mu\nu} B_{\beta}^{\alpha} = [F_{\mu\nu}, B]_{\beta}^{\alpha} = [F, B]_{\beta}^{\alpha} C_{\beta\gamma}^{\alpha}$$

$D$  kov. der. dann wohl pot  $\mathcal{R}$  u.  $\partial$

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + [A_{\mu}, A_{\nu}]$$

$$\hat{F}_{\mu\nu} = \partial_{\mu} \mathcal{R}_{\nu} - \partial_{\nu} \mathcal{R}_{\mu} + [\mathcal{R}_{\mu}, \mathcal{R}_{\nu}]$$



Viele:

$D, \tilde{D}$  kov. der ma  $\mathfrak{g}M$

$$D = \tilde{D} + A \quad A_{m \times \mathbb{R}}$$

$$A \cdot [m, m] = [A \cdot m, m] + [m, A \cdot m]$$

$$\Rightarrow \exists \mathfrak{X} \quad A = \text{ad}_{\mathfrak{X}}$$
$$\mathfrak{X}_{m \times \mathbb{R}} \quad A_{m \times \mathbb{R}} = \mathfrak{X}_{m \times \mathbb{R}} C_{\mathbb{R} \times \mathbb{R}}$$

Lemma

$\partial$  sowie der.

$$\Leftrightarrow A = \text{ad}_{\mathfrak{X}}$$

$$D = \partial + A \quad \text{je kov. u. alg. str.}$$

Let  $e$

$m \in \text{Lie } \mathfrak{g}_1 M$

$$\begin{aligned} D_m m^{\text{R}} &= \partial_m m^{\text{R}} + A_{\text{B}^{\text{R}}} m^{\text{R}} \\ &= \partial_m m^{\text{R}} + [\mathcal{F}_m, m]^{\text{R}} \end{aligned}$$

$$B \in \text{Lie } \mathfrak{g}_1 M \quad B^{\text{R}}_{\text{F}} = \text{ad}_{B^{\text{R}}} = B^{\text{K}} C_{\text{K}^{\text{R}}}^{\text{R}}$$

$$D_m B^{\text{R}}_{\text{F}} = \partial_m B^{\text{R}}_{\text{F}} + [A_m, B]^{\text{R}}_{\text{F}}$$

$$(D_m B^{\text{K}}) C_{\text{K}^{\text{R}}}^{\text{R}} = (\partial_m B^{\text{K}} + [\mathcal{F}_m, B]^{\text{K}}) C_{\text{K}^{\text{R}}}^{\text{R}}$$

Wieder  $F_{mn}^\alpha$  tensor für der  $D_m$

$$F_{mn} C = 0$$

$$F_{mn} \cdot [m, m] = [F_{mn} \cdot m, m] + [m, F_{mn} \cdot m]$$

$$\Downarrow \exists F_{mn} \quad F = \text{ad}_F$$

$$F_{mn}^\alpha = F_{mn}^k C_{k\beta}^\alpha$$

$$F_{mn} B^k = F_{mn}^k B^\alpha = [F, B]^k$$

$$F_{mn} B^\alpha = [F_{mn}, B]^\alpha = [F, B]^k C_{k\beta}^\alpha$$

Wieder:  
D kov. der, dann nicht pot  $\mathcal{R}$  nicht  $\circ$

$$F_{mn} = \partial_m A_n - \partial_n A_m + [A_m, A_n]$$

$$F_{mn} = \partial_m \mathcal{R}_n - \partial_n \mathcal{R}_m + [\mathcal{R}_m, \mathcal{R}_n]$$

$$\hat{F}_{mn}^k = \partial_m \mathcal{R}_n^k - \partial_n \mathcal{R}_m^k + [\mathcal{R}_m, \mathcal{R}_n]^k$$

$$= \partial_m \mathcal{R}_n^k - \partial_n \mathcal{R}_m^k + \mathcal{R}_m^l \mathcal{R}_n^z C_{lz}^k$$

$$T=0$$