SKLÁDÁNÍ SPINU 1+1

DOMINIK STARÝ

a) značení separované báze

$$|j_1 m_1\rangle |j_2 m_2\rangle = |s_1 m_1\rangle|s_2 m_2\rangle = |1 m_1\rangle|1 m_2\rangle = |m_1\rangle|m_2\rangle$$

konvence $\hbar = 1$

$$S^{(i)2}|1 m_i\rangle = 1(1+1)|-{''}-\rangle$$

$$S_z^{(i)}|-{''}-\rangle = m_i|-{''}-\rangle$$

$$S_\pm^{(i)}|-{''}-\rangle = \sqrt{(1\mp m_i)(1\pm m_i+1)}\,|1\, m_i\pm 1\rangle$$

$$S^2 = S^{(1)2} + S^{(2)2} + 2S_z^{(1)}S_z^{(2)} + S_+^{(1)}S_-^{(2)} + S_-^{(1)}S_+^{(2)}$$

$$S^2|1\rangle|1\rangle = 2\cdot 2|1\rangle|1\rangle + 2\cdot 1\cdot 1|1\rangle|1\rangle + 0 + 0 = 2(2+1)|1\rangle|1\rangle$$

$$\Rightarrow |s\, m_s\rangle = |2\,2\rangle = |1\rangle|1\rangle$$

$$S^2|-1\rangle|-1\rangle = 4|-1\rangle|-1\rangle + 2(-1)(-1)|-1\rangle|-1\rangle = 2(2+1)|-1\rangle|-1\rangle$$

$$|2\,2\rangle = |1\rangle|1\rangle$$

$$S_-\downarrow \quad = S_-^{(1)} + S_-^{(2)}, \quad S_\pm|s\,m\rangle = \sqrt{(s\mp m)(s\pm m+1)}\,|s\; m\pm 1\rangle$$

$$|2\,1\rangle = \frac{\sqrt{2}}{2}\left(|0\rangle|1\rangle + |1\rangle|0\rangle\right)$$

$$\xrightarrow{OG} |1\,1\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle|0\rangle - |0\rangle|1\rangle\right)$$

$$S_-\downarrow$$

$$|2\,0\rangle = \frac{1}{\sqrt{6}}\frac{1}{\sqrt{2}}\left(\sqrt{2}|-1\rangle|1\rangle + 2\sqrt{2}|0\rangle|0\rangle + \sqrt{2}|1\rangle|-1\rangle\right)$$

$$= \frac{1}{\sqrt{6}}\left(2|0\rangle|0\rangle + |-1\rangle|1\rangle + |1\rangle|-1\rangle\right)$$

$$S_+\uparrow$$

$$|2\,{-1}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|-1\rangle + |-1\rangle|0\rangle\right)$$

$$S_+\uparrow$$

$$|2\,{-2}\rangle = |-1\rangle|-1\rangle$$

$$|1\,1\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle|0\rangle - |0\rangle|1\rangle\right)$$

$$S_-\downarrow$$

$$|1\,0\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left(\sqrt{2}|0\rangle|0\rangle + \sqrt{2}|1\rangle|-1\rangle - \sqrt{2}|-1\rangle|1\rangle - \sqrt{2}|0\rangle|0\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|1\rangle|-1\rangle - |-1\rangle|1\rangle\right)$$

$$\downarrow$$

$$|1\,{-1}\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left(\sqrt{2}|0\rangle|-1\rangle - \sqrt{2}|-1\rangle|0\rangle\right) = \frac{1}{\sqrt{2}}\left(|0\rangle|-1\rangle - |-1\rangle|0\rangle\right)$$

$|0\underline{0}\rangle$ ... OG doplněk k $|1\underline{0}\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle|-1\rangle - |-1\rangle|1\rangle\right)$

a $|2\underline{0}\rangle = \frac{1}{\sqrt{6}}\left(2|0\rangle|0\rangle + |-1\rangle|1\rangle + |1\rangle|-1\rangle\right)$

$$\begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \longrightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\frac{1}{\sqrt{3}}$$

$$|00\rangle = \frac{1}{\sqrt{3}}\left(-|0\rangle|0\rangle + |1\rangle|-1\rangle + |-1\rangle|1\rangle\right)$$

b) $C^{sm}_{1m_1\,1m_2}$; $s \in \{0,1,2\}$, $m = -s, -s+1, \dots, s$

- $C^{00}_{1m_1\,1m_2}$: $C^{00}_{111-1} = C^{00}_{1-111} = -C^{00}_{1010} = \frac{1}{\sqrt{3}}$

- $C^{1-1}_{1m_1\,1m_2}$: $C^{1-1}_{101-1} = -C^{1-1}_{1-110} = \frac{1}{\sqrt{2}}$

  $C^{10}_{1m_1\,1m_2}$: $C^{10}_{111-1} = -C^{10}_{1-111} = \frac{1}{\sqrt{2}}$, $C^{10}_{1010} = 0$

  $C^{11}_{1m_1\,1m_2}$: $C^{11}_{1110} = -C^{11}_{1011} = \frac{1}{\sqrt{2}}$

- $C^{2-2}_{1-11-1} = 1$

  $C^{2-1}_{101-1} = C^{2-1}_{1-110} = \frac{1}{\sqrt{2}}$

  $C^{20}_{1010} = \frac{2}{\sqrt{6}}$, $C^{20}_{111-1} = C^{20}_{1-111} = \frac{1}{\sqrt{6}}$

  $C^{21}_{1011} = C^{21}_{1110} = \frac{1}{\sqrt{2}}$

  $C^{22}_{1111} = 1$