(1.) Najděte operátor $R_x(\alpha) = \exp\left(-i\frac{\alpha}{2}\sigma_x\right)$. Jak působí na stavy $|y:\pm\rangle, |z:\pm\rangle$, pokud zvolíme $\alpha = \frac{\pi}{2}$. $\hat{S}_\mu$ operátor složek spinového momentu hybnosti: $S_\mu \mapsto \hat{R}_x\left(\frac{\pi}{2}\right)\hat{S}_\mu\hat{R}_x^{+}\left(\frac{\pi}{2}\right)$

Maticová exponenciála: diagonalizujeme $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

→ diagonalizaci známe, protože známe vl. v. i vl. č. ze spinové teorie

$|+\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ vl. č. +1 $\quad |-\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ vl. č. −1 $\quad \Rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}$

$$\rightarrow \hat{R}_x = \frac{1}{2}\underbrace{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} e^{-i\frac{\alpha}{2}} & 0 \\ 0 & e^{i\frac{\alpha}{2}} \end{pmatrix}}_{\begin{pmatrix} e^- & e^+ \\ e^- & -e^+ \end{pmatrix}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} e^- + e^+ & \frac{i}{i}(e^- - e^+) \\ (e^- - e^+)\frac{i}{i} & e^- + e^+ \end{pmatrix} = \begin{pmatrix} \cos\frac{\alpha}{2} & -i\sin\left(\frac{\alpha}{2}\right) \\ -i\sin\left(\frac{\alpha}{2}\right) & \cos\frac{\alpha}{2} \end{pmatrix}$$

$$\hat{R}_x(\alpha) = \begin{pmatrix} \cos\frac{\alpha}{2} & -i\sin\frac{\alpha}{2} \\ -i\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} \end{pmatrix}$$

$$\hat{R}_x\left(\frac{\pi}{2}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \qquad \hat{R}_x^{+}\left(\frac{\pi}{2}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$\hat{R}_x\left(\frac{\pi}{2}\right)|+\rangle_z = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} = |-\rangle_y$$

$$\hat{R}_x\left(\frac{\pi}{2}\right)|-\rangle_z = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix} = \frac{-i}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} = e^{-i\frac{\pi}{2}}|+\rangle_y$$

$$\hat{R}_x\left(\frac{\pi}{2}\right)|+\rangle_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |+\rangle_z$$

$$\hat{R}_x\left(\frac{\pi}{2}\right)|-\rangle_y = \frac{1}{2}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \begin{pmatrix} 0 \\ -i \end{pmatrix} = e^{-i\frac{\pi}{2}}|-\rangle_z$$

$\mu=1$: $$\hat{R}_x\left(\frac{\pi}{2}\right)\hat{S}_x\hat{R}_x^{+}\left(\frac{\pi}{2}\right) = \frac{1}{2}\underbrace{\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\begin{pmatrix} -i & 1 \\ 1 & -i \end{pmatrix}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \hat{S}_x$$

$\mu=2$: $$\hat{R}_x\left(\frac{\pi}{2}\right)\hat{S}_y\hat{R}_x^{+}\left(\frac{\pi}{2}\right) = \frac{1}{2}\underbrace{\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}_{\begin{pmatrix} 1 & -i \\ i & -1 \end{pmatrix}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \hat{S}_z$$

$\mu=3$: $$\hat{R}_x\left(\frac{\pi}{2}\right)\hat{S}_z\hat{R}_x^{+}\left(\frac{\pi}{2}\right) = \frac{1}{2}\underbrace{\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{\begin{pmatrix} 1 & i \\ -i & -1 \end{pmatrix}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = e^{-i\pi}\hat{S}_y$$