$$R_n(\alpha) = \exp\left(-i\frac{\alpha}{2}\vec{n}\cdot\vec{\sigma}\right) = \exp\left(-i\frac{\alpha}{2}C\right)$$

$$C = (n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)$$

$$\sigma_\alpha\sigma_\beta = \delta_{\alpha\beta}I + i\varepsilon_{\alpha\beta\gamma}\sigma_\gamma$$

$$C^2 = (n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)^2$$

$$= n_x^2 I + i n_x n_y \sigma_z - i n_x n_z \sigma_y$$

$$+ n_y^2 I - i n_y n_x \sigma_z + i n_y n_z \sigma_x$$

$$+ n_z^2 I + i n_z n_x \sigma_y - i n_z n_y \sigma_x$$

$$= (n_x^2 + n_y^2 + n_z^2) I = n^2 I = I$$

$$C^3 = I\cdot C = C$$

$$C^4 = C\cdot C^3 = C^2 = I$$

$$\vdots$$

$$R_n(\alpha) = \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{-i\alpha}{2}\right)^k C^k$$

$$= I + \frac{\alpha}{2}(-i)C + \frac{1}{2!}\left(\frac{\alpha}{2}\right)^2(-i)^2 I + \frac{1}{3!}\left(\frac{\alpha}{2}\right)^3(-i)^2(-i)C + \dots$$

$$= \left(1 - \frac{1}{2!}\left(\frac{\alpha}{2}\right)^2 + \frac{1}{4!}\left(\frac{\alpha}{2}\right)^4 + \dots\right) I$$

$$+ \left(\frac{\alpha}{2} - \frac{1}{3!}\left(\frac{\alpha}{2}\right)^3 + \frac{1}{5!}\left(\frac{\alpha}{2}\right)^5 + \dots\right)(-i)C$$

$$= \cos\left(\frac{\alpha}{2}\right) I + \sin\left(\frac{\alpha}{2}\right)(-i)C$$

$$C = \vec{n}\cdot\vec{\sigma}$$

$$= \sin\theta\cos\varphi\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} + \sin\theta\sin\varphi\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} + \cos\theta\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$= \begin{pmatrix}\cos\theta & \sin\theta\, e^{-i\varphi}\\ \sin\theta(\cos\varphi + i\sin\varphi) & -\cos\theta\end{pmatrix}$$

$$R_n(\alpha) = \begin{pmatrix}\cos\left(\frac{\alpha}{2}\right) - i\sin\left(\frac{\alpha}{2}\right)\cos\theta & -i\sin\left(\frac{\alpha}{2}\right)\sin\theta\, e^{-i\varphi}\\ -i\sin\left(\frac{\alpha}{2}\right)\sin\theta\, e^{i\varphi} & +i\sin\left(\frac{\alpha}{2}\right)\cos\theta + \cos\left(\frac{\alpha}{2}\right)\end{pmatrix}$$