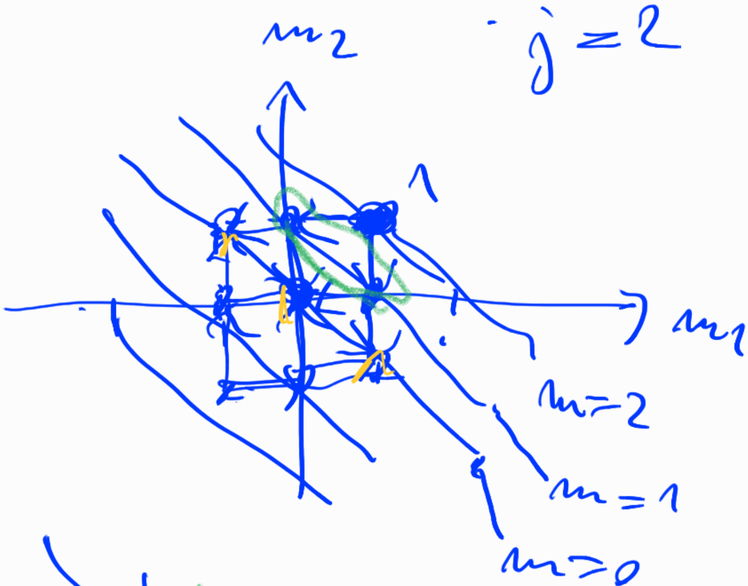
QMII-cv4

Komentář Ú1 a, b

$$|2\,{+2}\rangle = \emptyset\,|1\rangle|1\rangle$$

$$\vdots$$

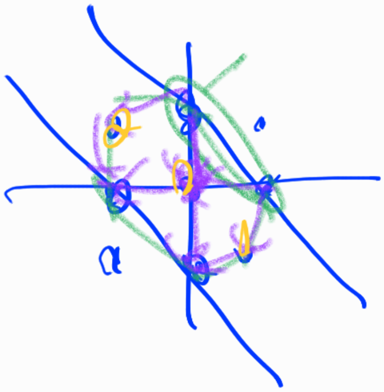
$$|2\,2\rangle = 0$$



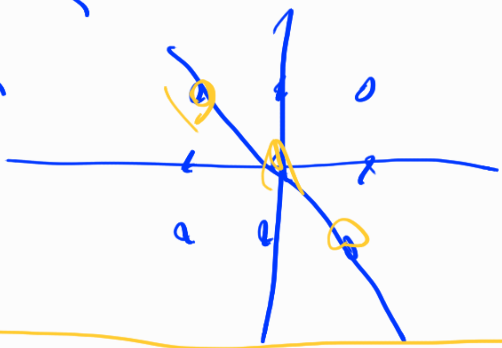
$|1\,1\rangle$

$|1\,0\rangle$

$|1\,{-1}\rangle$



$|0\,0\rangle$



ÚLOHA 2

$$\mathcal{H} = \mathcal{H}^{(1)} \otimes \mathcal{H}^{(2)} \otimes \mathcal{H}^{(3)}$$

ANALÝZA:

$$H = \frac{\omega}{\hbar}\left(2\vec{S}^{(1)}\cdot\vec{S}^{(2)} + 2\vec{S}^{(2)}\cdot\vec{S}^{(3)} - \vec{S}^{(1)}\cdot\vec{S}^{(3)}\right)$$

$$\frac{\omega}{\hbar}\left[2\vec{S}^{(2)}\cdot\left(\vec{S}^{(1)}+\vec{S}^{(3)}\right) - \vec{S}^{(1)}\cdot\vec{S}^{(3)}\right]$$

$$\vec{S}^{(2)}\cdot\vec{S}^{(13)} \to \frac{1}{2}\left(S^2 - S^{(2)2} - S^{(13)2}\right)$$

$$\vec{S}^{(1)}\cdot\vec{S}^{(3)} \to \frac{1}{2}\left(S^{(13)2} - S^{(1)2} - S^{(3)2}\right)$$

S^2, S_z : j, M

$$S = S^{(2)} + S^{(13)} = S^{(1)} + S^{(2)} + S^{(3)}$$

$S^{(13)2}, S^{(13)}_z$: j, m

$S^{(1)2}\quad S^{(2)2}\quad S^{(3)2}$

$$\left|\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, j, J, M\right\rangle$$

cílem najít spol. vl. v. $\cancel{S^{(1)2}, S^{(2)2}, S^{(3)2}}$ $\{S^{(13)2}, S^2, S_z\}$

$\mathcal{H}^{(1)} \dots S^{(1)2} \dots \hbar^2 \cdot \frac{3}{4} \mathbb{1}$

$$S^{(1)} \cdot S^{(3)} = \frac{1}{2}\left(\hbar^2 j(j+1) - 2 \cdot \hbar^2 \frac{3}{4}\right) \qquad \boxed{|j, J, M\rangle}$$

$$S^{(2)} \cdot (S^{(1)} + S^{(3)}) = \frac{1}{2}\left(\hbar^2 J(J+1) - \hbar^2 \frac{3}{4} - \hbar j(j+1)\right)$$

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konstrukce: $\mathcal{H} = \mathcal{H}^{(1)} \otimes \mathcal{H}^{(2)} \otimes \mathcal{H}^{(3)}$

báze separ. … $|+\rangle, |-\rangle = |s_1\rangle$ … $|s_1\rangle \otimes |s_2\rangle \otimes |s_3\rangle$

$\equiv |s_1 s_2 s_3\rangle$

1) $s^1 + s^3$

$$S^{(13)}_\alpha = S^{(1)}_\alpha + S^{(3)}_\alpha \quad \dots |++\rangle_{13}$$

$|a\rangle_1 |b\rangle_2 |c\rangle_3$

$\equiv |b\rangle_2 |a_1\rangle |c\rangle_3$

$|b\rangle_2 |ac\rangle_{13}$

$|jm\rangle$: $|11\rangle = |++\rangle_{13}$

$\to |10\rangle = \frac{1}{\sqrt{2}}(|+-\rangle_{13} + |-+\rangle_{13}) \longrightarrow |00\rangle = \frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)$

$|1-1\rangle = |--\rangle_{13}$

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2) $s^{13} + s^2$ a) $j = 0$ $\qquad 0 + \frac{1}{2}$ $\qquad JM < \frac{1}{2}\ \frac{1}{2}$, $\frac{1}{2}\ -\frac{1}{2}$

$S^{(13)}\ S^{(2)}$

$$\Rightarrow J = \frac{1}{2} \dots |jJM\rangle = \boxed{|0 \tfrac{1}{2} \tfrac{1}{2}\rangle} = \frac{1}{\sqrt{2}}\left[|++-\rangle - |-++\rangle\right]$$

$$|0 \tfrac{1}{2} -\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}\left[|+--\rangle - |--+\rangle\right]$$

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b) $\boxed{j = 1}$ … $1 + \frac{1}{2}$

$\frac{1}{2} + 1$

$J = \frac{1}{2}$ … $|1 \tfrac{1}{2} +\tfrac{1}{2}\rangle$, $|1 \tfrac{1}{2} -\tfrac{1}{2}\rangle$

$J = \frac{3}{2}$ $\quad |1 \tfrac{3}{2} \pm \tfrac{3}{2}\rangle$

$|1 \tfrac{3}{2} \pm \tfrac{1}{2}\rangle$

$$|1\,\tfrac32\,\tfrac32\rangle = |\overset{1}{j}\,\overset{1}{m}\rangle_{13}\,|+\rangle_2 = |+++\rangle \qquad J_- = J_-^{(1)} + J_-^{(2)} + J_-^{(3)}$$

$$|1\,\tfrac32\,\tfrac12\rangle = \frac{1}{\sqrt3}\left(|-++\rangle + |+-+\rangle + |++-\rangle\right)$$

$$|1\,\tfrac32\,-\tfrac12\rangle = \frac{1}{\sqrt3}\left(|+--\rangle + |-+-\rangle + |---\rangle\right) \qquad J_+ = J_+^{(1)} + J_+^{(2)} + J_+^{(3)}$$

$$|1\,\tfrac32\,-\tfrac32\rangle = |---\rangle$$

$$|11\rangle_{13}\,|+\rangle_2$$

$$\frac{1}{\sqrt3}\underbrace{\left(\sqrt2\,|10\rangle_{13}|+\rangle_2 + |11\rangle_{13}|-\rangle_2\right)}_{|+-\rangle_{13}|+\rangle_2 + |++\rangle_{13}|-\rangle_2 + |++\rangle_{13}|-\rangle_2} = \left(\frac{\sqrt2}{\sqrt3}\,|\overset{++}{11}\rangle_{13}|-\rangle_2 - |10\rangle_{13}|+\rangle_2\,\frac{1}{\sqrt3}\right)$$

OC

$$\frac{1}{\sqrt2}$$

$j\ J\ M$

$$|1\,\tfrac12\,\tfrac12\rangle = \frac{1}{\sqrt6}\left(-|-++\rangle + 2|+-+\rangle - |++-\rangle\right)$$

$$|1\,\tfrac12\,-\tfrac12\rangle = \frac{1}{\sqrt6}\left(|--+\rangle - 2|-+-\rangle + |+--\rangle\right)$$

J_-

Sodian prostor : $|s_1 s_2 s_3\rangle$

$\longrightarrow |j\,J\,M\rangle$

$$|0\,\tfrac12\,\tfrac12\rangle \qquad |1\,\tfrac32\,\tfrac32\rangle$$

$$|0\,\tfrac12\,-\tfrac12\rangle \qquad |1\,\tfrac32\,\tfrac12\rangle \qquad |1\,\tfrac12\,\tfrac12\rangle$$

$$|1\,\tfrac32\,-\tfrac12\rangle \qquad |1\,\tfrac12\,-\tfrac12\rangle$$

$$|1\,\tfrac32\,-\tfrac32\rangle$$

$J\,j$: $(0, \tfrac12)$ 2x

$(1, \tfrac32)$ 4x

$(1\,\tfrac12)$ 2x

$$H = \frac{\omega}{\hbar}\left[2\,\vec{S}^{(2)}\cdot\vec{S}^{(13)} - S^{(1)}\cdot S^{(3)}\right]$$

$$= \frac{1}{2}\hbar\omega\left[\left(J(J+1) - \cancel{\tfrac{3}{4}} - j(j+1)\right)2 - j(j+1) + \cancel{\tfrac{6}{4}}\right]$$

$$= \hbar\omega\left[J(J+1) - \frac{3}{2}j(j+1)\right]$$

$\boxed{\text{vl. } H}$ ... $H|jJM\rangle = E(jJ)|jJM\rangle$

$$E(0,\tfrac{1}{2}) = \tfrac{3}{4}\hbar\omega \;(2\times)$$

$$E(1,\tfrac{3}{2}) = \hbar\omega\left(\tfrac{3}{2}\cdot\tfrac{5}{2} - \tfrac{3}{2}\cdot 2\right) = \hbar\omega\tfrac{3}{4}\hbar\omega \;(4\times)$$

$6\times$

$$E(1,\tfrac{1}{2}) = \hbar\omega\left(\tfrac{1}{2}\cdot\tfrac{3}{2} - \tfrac{3}{2}\cdot 2\right) = -\hbar\omega\tfrac{9}{4} \quad 2\times$$

$\boxed{\text{Dynamika:}}$ $|\psi\rangle = |-++\rangle$ ... $t=0$

$S_z = \frac{\hbar}{2}(-1+1+1)$

$\boxed{S_z = \frac{\hbar}{2}}$

$\boxed{?\; p\left(S_z^{(3)} = -\frac{\hbar}{2}\right)(t)\; ?}$

$[S_z, H] = 0$

$$|\psi(t)\rangle = \mathcal{L}\left(|-++\rangle, |+-+\rangle, |++-\rangle\right)$$

$$= \mathcal{L}\left(|0\tfrac{1}{2}\tfrac{1}{2}\rangle, |1\tfrac{3}{2}\tfrac{1}{2}\rangle, |1\tfrac{1}{2}\tfrac{1}{2}\rangle\right)$$

$M \quad j \quad M \quad M$

$\frac{1}{\sqrt{2}}\left(|++-\rangle - |-++\rangle\right)$, $\frac{1}{\sqrt{3}}\left(|++-\rangle + |+-+\rangle + |-++\rangle\right)$ $\frac{3}{4}\hbar\omega$

$\frac{3}{4}\hbar\omega$

$\frac{1}{\sqrt{6}}\left(-|++-\rangle + 2|+-+\rangle - |-++\rangle\right) \leftarrow -\frac{9}{4}\hbar\omega$

$\underline{-3(1,0,-1)}$

$$\begin{array}{r} 2\times(1,\;1,\;1) \\ \hline -(-1,\;2,\;-1) \\ \hline (3,\;0,\;3) \end{array}$$

$t=0$

$(0,0,1)$

$?$

$$6\times|\psi\rangle = \{2(1,1,1) + (1,-2,1) + 3(-1,0,1)\}$$

$$e\{i\tfrac{3}{4}\omega t\} + \exp\{i\tfrac{9}{4}\omega t) \quad \tfrac{9}{4}-\tfrac{3}{4} \quad |e^{\frac{3}{4}i\omega t}$$

$$6\times|\psi\rangle = \text{fáze}\{2(1,1,1) + (1,-2,1)e^{i3\omega t} + 3(-1,0,1)\}$$

$$|\psi\rangle = \text{fáze}\,\frac{1}{6}\left\{\begin{array}{l} 2(|++-\rangle + |+-+\rangle + |-++\rangle) \\ +(|\quad\rangle - 2|\quad\rangle + |\quad\rangle)e^{i3\omega t} \\ 3(-|++-\rangle + 0 + |\quad\rangle) \end{array}\right\}$$

$$p(S_z^{(3)} = -1\} \sim P_{S_3=-1} = \sum_{s_1 s_2} |s_1 s_2 -\rangle\langle s_1 s_2 -|$$

$$P_{S_z^{(3)}=-1}|\psi\rangle = |++-\rangle\underbrace{\frac{1}{6}\left(e^{i3\omega t} - 1\right)}_{a}$$

$$p = |a|^2 = \frac{1}{36}\left|e^{3i\omega t} - 1\right|^2$$

$$(e^{3i\omega t} - 1)(e^{-3i\omega t} - 1)$$

$$2 - \underbrace{e^{3i\omega t} - e^{-3i\omega t}}_{2\cos 3\omega t}$$

výsledek:

$$\boxed{p = \frac{1}{18} - \frac{1}{18}\cos 3\omega t}$$

$(t=0 \qquad p=0)$

$S_z^{(3)} = +$