$\boxed{\text{ÚLOHA 1}}$ $\hat{U}_k, \hat{V}_\ell$ $[\hat{J}_k, \hat{V}_\ell] = i\hbar \varepsilon_{k\ell n} \hat{V}_n$

- ukažte, že $\hat{S} = \hat{\vec{U}} \cdot \hat{\vec{V}}$ je skalár:

tj. $\hat{R}^+ \hat{S} \hat{R} = \hat{S} \iff [\hat{R}, \hat{S}] = 0 \iff [J_k, S] = 0$

$$[\hat{J}_k, \hat{S}] = [\hat{J}_k, \hat{U}_\ell \hat{V}_\ell] = \hat{U}_\ell \underbrace{[\hat{J}_k, \hat{V}_\ell]} + \underbrace{[\hat{J}_k, \hat{U}_\ell]} \hat{V}_\ell$$

$$= i\hbar \varepsilon_{k\ell n} \left(\hat{U}_\ell \hat{V}_n + \hat{U}_n \hat{V}_\ell \right) = i\hbar \varepsilon_{k\ell n} \left(\hat{U}_\ell \hat{V}_n - \hat{U}_\ell \hat{V}_n \right) = 0$$

- pokud $[\hat{A}, \hat{J}_x] = [\hat{A}, \hat{J}_y] = 0 \Rightarrow [\hat{A}, \hat{J}_z] = 0$

$$[J_x, J_y] = i\hbar J_z \longrightarrow J_z = \frac{1}{i\hbar}(J_x J_y - J_y J_x)$$

$$[\hat{A}, \hat{J}_z] = \frac{1}{i\hbar}\left([\hat{A}, J_x J_y] - [A, J_y J_x] \right) = 0$$

$J[A,J] + [A,J]J$

- ověřte, že $\hat{V}_0^{(1)} = \hat{V}_z$, $\hat{V}_{\pm 1}^{(1)} = \mp \frac{1}{\sqrt{2}}(\hat{V}_x \pm i \hat{V}_y)$

jsou IR. složky:

$[\hat{J}_z, \hat{V}_m^{(1)}] = \hbar m \hat{V}_m^{(1)}$; $[\hat{J}_\pm, \hat{V}_m^{(1)}] = \hbar\sqrt{2}\, \hat{V}_{m\pm 1}^{(1)}$

$m = 0, 1, -1$ (nebo 0)

$\pm$ 0

$+$ $\pm$ $-$ $\pm$

$$[\hat{J}_z, \hat{V}_0^{(1)}] = [\hat{J}_z, \hat{V}_z] = 0$$

$$[\hat{J}_z, \hat{V}_m^{(1)}] = \mp \frac{1}{\sqrt{2}} [\hat{J}_z, \hat{V}_x \pm i \hat{V}_y] = \mp \frac{i\hbar}{\sqrt{2}} (V_y \mp i V_x)$$

$$= \frac{\hbar}{\sqrt{2}} (-V_x \mp i V_y)$$