$\boxed{\text{QM-II-cv7}}$ Wignerova - Eckartova věta

$$\langle j_1 m_1 \alpha_1 | \hat{T}^{(j)}_m | \underline{j_2 m_2} \alpha_2 \rangle = \langle j m\, j_2 m_2 | j_1 m_1 \rangle \frac{(j_1 \alpha_1 \| T^{(j)} \| j_2 \alpha_2)}{\sqrt{2j_1+1}}$$

$\boxed{\text{Rozcvička}}$ .. $\hat{L}_1 \quad \hat{L}_2$ .. $\mathcal{H}_1 \otimes \mathcal{H}_2$ dvě částice

$\hat{L} = \hat{L}_1 + \hat{L}_2$

$LM \ldots L^2, L_z$

$$\langle l_1 l_2 L M | \underbrace{\vec{L}_1 \cdot \vec{L}_2}_{\text{skalár } S_0^{(0)}} | l_1 l_2 L' M' \rangle$$

$$\langle 0 0 L' M' | L M \rangle = \delta_{LL'} \delta_{MM'}$$

$0 + L' \to L' \qquad L = L'$

$$\langle l_1 l_2 L M | \vec{L}_1 \cdot \vec{L}_2 | l_1 l_2 L' M' \rangle = \delta_{LL'} \delta_{MM'} \frac{(L \| S^{(0)} \| L')}{\sqrt{2L+1}} \; ?$$

$$\vec{L}_1 \cdot \vec{L}_2 = \frac{1}{2}\left(L^2 - L_1^2 - L_2^2\right) \to \frac{\hbar^2}{2}\left[L(L+1) - l_1(l_1+1) - l_2(l_2+1)\right]$$

$$(L \| S^{(0)} \| L') = \delta_{LL'} \sqrt{2L+1}\, \frac{\hbar^2}{2}\left[L(L+1) - l_1(l_1+1) - l_2(l_2+1)\right]$$

$\boxed{\text{ÚLOHA 1}}$ atom vodíku -- $\hat{H} |\underbrace{nlm}_{Y_{lm}}\rangle = E_n |nlm\rangle$

$\langle nlm | r_i | n' l' m' \rangle = M \leftarrow$ optické přechody

$\vec{r} = (x, y, z)$ $\qquad n = n' = 2$ .. α

$r_i \in \{x, y, z\}$

a) jaký je počet M

b) kolik $M \neq 0$

c) kolik M musíme spočíst sami

a) $|nlm\rangle = R_{nl}(r)\, Y_{lm}(\theta, \varphi)$ $\qquad l = 0, 1, \ldots, n-1$

$n = 2 \quad l = 0, 1$

$m = 0, \; 1, 0, -1$ } 4 stavy

dohromady $M = \langle nlm | r_i | n' l' m' \rangle$

$\boxed{48} = 4 \quad 3 \quad 4$

b) nenulové $\langle n l m | r_i | n' l' m' \rangle$

$$\left.\begin{matrix} X \\ Y \\ Z \end{matrix}\right\} \to \left\{\begin{matrix} r_0 = z \\ r_\pm = \mp \frac{x \pm iy}{\sqrt{2}} \end{matrix}\right\} \text{ 3 IR komponenty } R^{(1)}_\mu$$

$l = 0$ $\cancel{3}$ $\langle 200 | R^{(1)}_\mu | 200 \rangle = 0$ $\quad 1+0 \quad \cancel{3}$

$l = 1$
$\boxed{3} + \cancel{8}$ $\langle 200 | R^{(1)}_\mu | 21m' \rangle$
$\boxed{3} + \cancel{6}$ $\langle 21m | R^{(1)}_\mu | 200 \rangle$ ze samosdruž.

$\mu = -m'$

$3 \times 3 = 9$

nenul. 3

$m = 0$ $\quad 3 + \cancel{6}$ $\langle 21m | R^{(1)}_\mu | 21m' \rangle$
1 $\quad 2 + \cancel{7}$
-1 $\quad 2 + \cancel{7}$

$13 + \cancel{35} = 48$ ✓

nenulové 13 ze 48

c) kolik musí vypočíst?

$l = 0$ .. 0

$l = 1, l' = 0 \Leftrightarrow l = 0, l' = 1$ $\quad 3 + \cancel{3} \longrightarrow 1$

$(02 \| R^{(1)} \| 12)$

$\langle 200 | R^{(1)}_\mu | 21m' \rangle = \langle 1\mu 1 m' | 00 \rangle$

$\mu = -m'$ $\quad m' = 0, \pm 1$ $\quad r\cos\theta$ $\quad m' = \mu = 0$ $\sin\theta$

$$(0 2 \| R^{(1)} \| 1 2) = \int R_{20} Y_{00}(\vec{n}) \, z \, R_{21} Y_{10}(\vec{n}) \, r^2 dr \, d\theta \, d\varphi$$

$l \quad \alpha = n \quad\quad l' \quad \alpha' = n'$

$\langle 10 \, 10 | 00 \rangle \to \frac{2}{\sqrt{6}} = \sqrt{\frac{2}{3}}$

$|11\rangle |11\rangle$

$|10\rangle |11\rangle + |11\rangle |10\rangle$

$|00\rangle = \frac{1}{\sqrt{6}} \left(|1-1\rangle |11\rangle + 2 |10\rangle |10\rangle + |11\rangle |1-1\rangle \right)$

$6 \to 1$

$$(02 \| R^{(1)} \| 12) = \sqrt{\frac{3}{2}} \, 2\pi \int R_{20}(r) \, r \, R_{21}(r) \, r^2 dr \int_0^\pi Y_{00} Y_{10} \cos\theta \sin\theta d\theta$$

$\to \int_0^\infty x^n e^{-x} dx = n!$

$l = 1; \, l' = 1$ $\quad 7 \longrightarrow 1$

$$\langle 2 1 m | R^{(1)}_{\mu} | 2 1 m' \rangle = \langle 1 \mu 1 m' | 1 m \rangle \frac{\langle 1 2 \| R^{(1)} \| 1 2 \rangle}{\sqrt{3}}.$$

$$\langle 1 2 \| R^{(1)} \| 1 2 \rangle = \frac{\sqrt{3}}{\langle 1 0 1 0 | 1 0 \rangle} \int R_{21} Y_{10}^{*} \, z \, R_{21} Y_{10} \, d\Omega \, dr$$

13 nenulový $\longrightarrow$ 2 (dle WE věty)

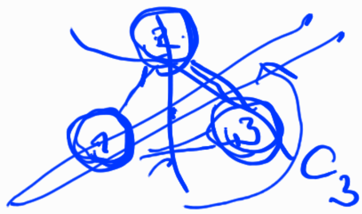
parita: $\langle \uparrow | R | \uparrow \rangle$ = parita $(-1)^3 = -1$

1m .. $(-1)^{\ell} = -1$ (-1) -1 $= 0$

48 maticových elementů $\xrightarrow{WE}$ 13 nenulových $\xrightarrow{WE}$ 2 nezávislé (+CG m, μ, m') $\xrightarrow{\text{parita}}$ 1 nezávislý

$\boxed{\text{ÚLOHA 2}}$ – rozumný sled. Bloch teorém

$\boxed{\text{ÚLOHA 3}}$ $\mathcal{H} = \mathcal{L}\{|1\rangle, |2\rangle, |3\rangle\}$



Symetrie $S_3 \longleftrightarrow \underline{C_{3v}}$

C_3 .. $\frac{2\pi}{3}$ rotace C_3 $3\sigma_v$ .. zrcadlení

1 2 3 $\longrightarrow$ 2 3 1 $\longrightarrow$ 3 1 2   cyklické permutace

Grupa symetrie S_3 $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \longleftrightarrow C_3$ fyzická operace

$\begin{pmatrix} 1 & 2 & 3 \\ \pi_1 & \pi_2 & \pi_3 \end{pmatrix}$

na $\mathcal{H}$: $\underbrace{C_3 |n\rangle = |\pi_n\rangle}$

- invariantní podprostory ?

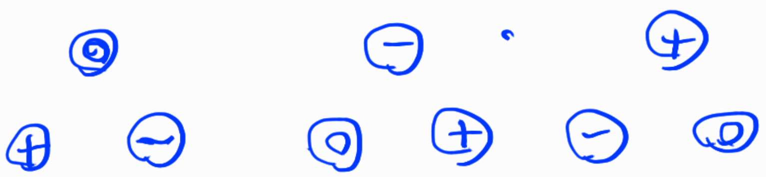
totálně symetrický stav $|\psi_0\rangle = \frac{1}{\sqrt{3}}(|1\rangle + |2\rangle + |3\rangle)$

└ totál sym. invar podprostor $\mathcal{L}\{|\psi_0\rangle\}$

reprezentace S_3 $\quad U_{C_3} = \begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix} \quad U_{\sigma_v} = \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$

$$U\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix} \leftarrow$$

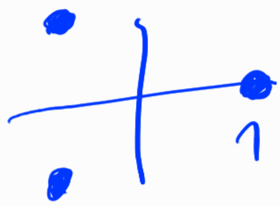
v reálné vektor. prostoru



} lin obal libovol dvojice

v komplexní $\quad |1\rangle + e^{i\phi}|2\rangle + e^{2i\phi}|3\rangle$

$e^{3i\phi} = 1 \qquad \phi = \pm\frac{2\pi}{3}$



$\phi = \frac{2\pi}{3}$

dva komplexní 1D invar prostory

$$|\psi_\pm\rangle = \frac{1}{\sqrt{3}}\left(|1\rangle + e^{\pm i\phi}|2\rangle + e^{\pm 2i\phi}|3\rangle\right)$$

pokud $[H, U] = 0$ invar. podpr. $\equiv$ vl. podpr.

$$H = \begin{pmatrix}\alpha&\beta&\beta\\\beta&\alpha&\beta\\\beta&\beta&\alpha\end{pmatrix} \qquad |\psi_0\rangle, |\psi_\pm\rangle$$