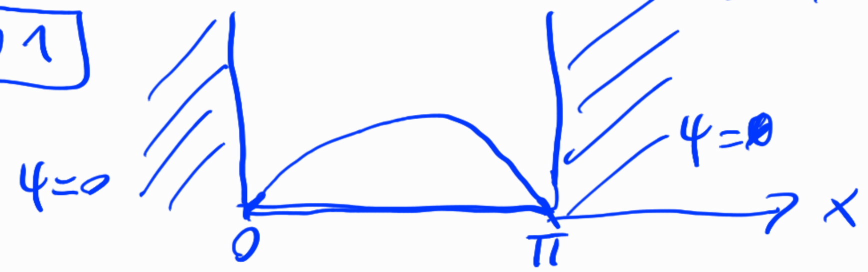
$\boxed{\text{QMII-cv9}}$

$\boxed{\text{Ú1}}$

$\hat H = \hat H^{(1)} + \hat H^{(2)}$

$\hat h \otimes I + I \otimes \hat h$



$-\frac{\hbar^2}{2m}\psi'' = E\psi$

$e^{\pm ikx}$

$E_n = \frac{n^2\hbar^2}{2m}$

$\phi_n(x) = \sqrt{\frac{2}{\pi}}\sin nx$

$n = 1, 2, 3, \dots$

$\epsilon_1 = \frac{\hbar^2}{2m}$ $\quad \epsilon_n = \epsilon_1 n^2$

Jedna částice

Energie a stac. stavy pro 2 částice

• **ROZLIŠITELNÉ:**

$\psi_{n_1 n_2}(x_1,x_2) = \phi_{n_1}(x_1)\phi_{n_2}(x_2) \longrightarrow E = \epsilon_{n_1} + \epsilon_{n_2} = \epsilon_1(n_1^2+n_2^2)$

• základní stav: $\psi^{(R)}_{11}(x_1,x_2) = \phi_1(x_1)\phi_1(x_2)$ $\quad E_{11} = 2\epsilon_1$

• Excit. stav: $\psi^{(R)}_{12}(x_1,x_2) = \phi_1(x_1)\phi_2(x_2)$ $\quad E_{12} = 5\epsilon_1$

$\psi_{21}(x_1,x_2) = \phi_2(x_1)\phi_1(x_2)$ $\quad = E_{21}$

• **BOSONY:** symetrické stavy $\psi(x_1,x_2) = \psi(x_2,x_1)$

• zákl. $E = 2\epsilon_1$ ... $\psi^{(B)}_1(x_1,x_2) = \phi_1(x_1)\phi_1(x_2)$

• excit. $E = 5\epsilon_1$

$\psi^{(B)}_2(x_1,x_2) = \frac{1}{\sqrt2}\left(\phi_1(x_1)\phi_2(x_2) + \phi_1(x_2)\phi_2(x_1)\right)$

• **Fermi:** $\frac{1}{2}$ ... $E = 2\epsilon_1$ $\quad \psi^{(F)}_1 = \phi_1(x_1)\phi_1(x_2)\,\frac{|+-\rangle - |-+\rangle}{\sqrt2}$

základní tripl. stav ≡ 1. excit. ... $E = 5\epsilon_1$ ... 4x

$\psi^{(F)}_2 = \frac{1}{\sqrt2}\left(\phi_1(x_1)\phi_2(x_2) - \phi_2(x_1)\phi_1(x_2)\right)$ $\quad |++\rangle$ $\quad |\phi_1\rangle, |\phi_2\rangle$

3x $\quad |--\rangle$

$(|+-\rangle + |-+\rangle)/\sqrt2$

sigl. $\underbrace{\frac{1}{\sqrt{2}}\left(- |\sim + -||- \right)}$ $(|+-\rangle - |-+\rangle)/\sqrt{2}$

UZDÁLENOST: $\hat{D}^2 = \langle\psi|(x_1 - x_2)^2|\psi\rangle$ 2 část.

srovn $\sigma^2 = \langle\phi|(x - \langle x\rangle)^2|\phi\rangle = \langle x^2\rangle - \langle x\rangle^2$

$$\langle\phi_n|x|\phi_m\rangle = \begin{pmatrix} \frac{\pi}{2} & -\frac{16}{9\pi} \\ & \frac{\pi}{2} \end{pmatrix} \qquad \langle x^2\rangle = \begin{pmatrix} \frac{\pi^2}{3} - \frac{1}{2} & -\frac{16}{9} \\ & \frac{\pi^2}{3} - \frac{1}{8} \end{pmatrix}$$

např.:

$\langle\phi_1|x|\phi_1\rangle = \frac{2}{\pi}\int_0^\pi \sin^2 x \cdot x \, dx$ $\qquad \langle\phi_1|x|\phi_2\rangle$

atd .. ostatní n, m

1 částice rozptyl.

ve stavu $|\phi_1\rangle$ ... $\sigma_1 = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \sqrt{\frac{\pi^2}{12} - \frac{1}{2}} = 0{,}57$

$|\phi_2\rangle$ .. $\sigma_2 = \sqrt{\frac{\pi^2}{12} - \frac{1}{8}} = 0.83$

2 částice rozlišitelné (separované stavy)

$D^2 = \langle(x_1 - x_2)^2\rangle = \boxed{\langle x_1^2\rangle + \langle x_2^2\rangle - 2\langle x_1 x_2\rangle}$

$\psi = \phi_n(1)\phi_m(2)$ $\qquad = \langle x^2\rangle_n + \langle x^2\rangle_m - 2\langle x\rangle\langle x\rangle$

$$\boxed{D^2 = \sigma_n^2 + \sigma_m^2}$$

zákl. stav $n = m = 1$ $\qquad D = \sqrt{2}\,\sigma_1 = 0{,}89$

excit stav $n = 1$ $m = 2$ $\qquad D = \sqrt{\sigma_1^2 + \sigma_2^2} = 1.01$

2 nerozl. bosony .. zákl.

$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\phi_1\rangle|\phi_2\rangle + |\phi_2\rangle|\phi_1\rangle\right)$

$D^2 = \frac{1}{2}\left[\langle x^2\rangle_1 + \langle x^2\rangle_2 + \langle x^2\rangle_1 + \langle x^2\rangle_2 - 4\langle x\rangle_1\langle x\rangle\right.$

$\left. - 4|\langle\phi_1|x|\phi_2\rangle|^2\right]$

$$D^2 = \langle x^2\rangle_1 + \langle x^2\rangle_2 - 2\langle x\rangle^2 - 2|\langle\phi_1|x|\phi_2\rangle|^2$$

$$\underbrace{D^2 = \sigma_1^2 + \sigma_2^2} - 2|\langle\phi_1|x|\phi_2\rangle|^2 \quad < \text{rozliš.}$$

$$\boxed{D \approx 0.62}$$

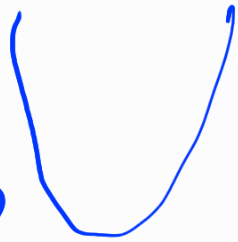
2 nerozliš. fermiony $\quad |\psi\rangle = \frac{1}{\sqrt{2}}\left(|\phi_1\rangle|\phi_2\rangle - |\phi_2\rangle|\phi_1\rangle\right)$

$$D^2 = \sigma_1^2 + \sigma_2^2 + 2|\langle\phi_1|x|\phi_2\rangle|^2$$

$$\boxed{D = 1.29}$$

ÚLOHA

2 bosony se spinem 1



$h = \hbar\omega(\hat{a}^+\hat{a} + \frac{1}{2})$

$E_n = \hbar\omega(n + \frac{1}{2})$

$n = 0, 1, 2, \dots$

$$\hat{H} = \underbrace{\hat{h}^{(1)} + \hat{h}^{(2)}} + \underbrace{\frac{\lambda}{\hbar}\vec{S}^{(1)}\cdot\vec{S}^{(2)}}$$

$\lambda \ll \omega$

Najděte 9 nejnižších energ. hladin.
(a stupně jejich degenerace)

Energie .. prostorová část .. $|n_1\rangle|n_2\rangle \leftarrow$

$$E_{n_1} + E_{n_2} = \hbar\omega + \hbar\omega(n_1 + n_2)$$

spin $\quad \frac{\lambda}{\hbar}\hat{S}^1\cdot\hat{S}^2 = \frac{1}{2}\frac{\lambda}{\hbar}\left(\hat{S}^2 - \hat{S}_1^2 - \hat{S}_2^2\right) = \frac{1}{2}\lambda\hbar\left(S(S+1) - 4\right)$

spin 1 + 1

- $|SM\rangle$ $\quad S = 0 \sim |00\rangle \leftarrow$ sym.
- $|1M\rangle$ $\quad M = 0, \pm 1 \leftarrow$ antisym P_{12}
- $|2M\rangle$ $\quad M = 0, \pm 1, \pm 2 \leftarrow$ symetrické P_{12}