@M11-8 **Kvantová teorie rozptylu**

OPAKOVÁNÍ: $H = H_0 + V$ ⇝ $|\phi_{out}\rangle = \hat{S}|\phi_{in}\rangle$ $\quad (E + i\varepsilon - H_0)^{-1}$

$$\langle \vec{p}|\hat{S}|\vec{p}'\rangle = \delta_3(\vec{p}-\vec{p}') - 2\pi i\,\delta(E_p - E_{p'})\, t_{p\leftarrow p'}$$

ČASOVĚ NEZÁVISLÁ FORMULACE: $|\psi_p^{(+)}\rangle = \hat{\Omega}_+|\vec{p}\rangle$ ⇝ $\hat{H}|\psi_p^{(+)}\rangle = E\psi_p^{(+)}$

$$\boxed{|\psi_p^{(+)}\rangle = |\vec{p}\rangle + \hat{G}_0^{(+)}(E_p)\hat{V}|\psi_p^{(+)}\rangle} \leftarrow \text{Lippmann-Schwingerova (LS}$$

1D: $\langle x|G_0^{(+)}(E)|x'\rangle = \frac{2m}{\hbar^2}\frac{1}{2ik}e^{\pm ik|x-x'|}$ $\quad k = \frac{p}{\hbar} \quad E_p = \frac{p^2}{2m}$

3D: $\langle \vec{x}|G_0^{(+)}(E)|\vec{x}'\rangle = -\frac{2m}{\hbar^2}\frac{1}{4\pi}\frac{1}{|\vec{x}-\vec{x}'|}\exp\{\pm\frac{i}{\hbar}p_E|\vec{x}-\vec{x}'|\}$ $\quad p_E = \sqrt{2mE}$

(LS)
$$\psi(\vec{x}) = \frac{1}{\sqrt{(2\pi\hbar)^3}}e^{i\vec{k}\cdot\vec{x}} - \int d^3x' \frac{e^{ik_E|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|}\underbrace{\frac{2m}{\hbar^2}V(\vec{x}')}_{U}\psi(\vec{x}')$$

$r \equiv |\vec{x}| \to \infty \qquad |\vec{x}-\vec{x}'| = \sqrt{(r\vec{n}-\vec{x}')^2} \simeq r - \vec{n}\cdot\vec{x}' + O\left(\left(\frac{x'}{r}\right)^2\right)$

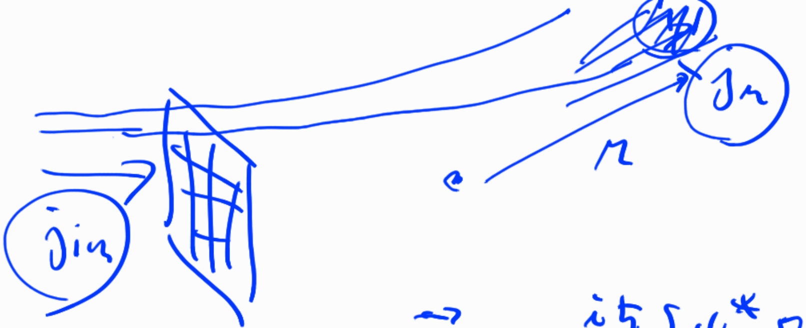
$$\psi(\vec{x}) \xrightarrow{r\to\infty} \frac{1}{\sqrt{(2\pi\hbar)^3}}\Big(e^{i\vec{k}\cdot\vec{x}} - \underbrace{\frac{(2\pi\hbar)^3}{\sqrt{(2\pi\hbar)^3}}\frac{2m}{4\pi\hbar^2}\int e^{-\frac{i}{\hbar}p\vec{n}\cdot\vec{x}'}V(x')\psi(x')\,d^3x'}_{f}\frac{e^{ikr}}{r}\Big)$$

$\vec{p}$ $\qquad f \quad |-m\hbar(2\pi)^2 \quad \langle p|V|\psi\rangle\,($

$$\dashrightarrow \frac{1}{\sqrt{(2\pi\hbar)^3}}\left(e^{i\vec{k}\cdot\vec{x}} + f(\vec{n})\frac{e^{ikr}}{r}\right) \leftarrow \partial_r + ik \quad \vec{n} \quad \bar{e}^{ikr} \quad \vec{\partial}_r$$

$$f(\vec{n}) = m\hbar(2\pi i)^2\langle \vec{p}'|\hat{V}|\psi_{\vec{p}}^{(+)}\rangle \quad \dots \quad \vec{p}' = p_E\vec{n}$$

$\Rightarrow \frac{d\sigma}{d\Omega} = |f|^2 \qquad \vec{p}, \vec{p}' \qquad = -(2\pi)^2 m\, t_{p'\leftarrow p}$



$$\Delta S = r^2 \Delta\Omega$$

$$\frac{d\sigma}{d\Omega} = \frac{r^2 |\vec{j}_r|}{|j_{in}|} \leftarrow \text{def. dif. úč. průř.}$$

$$\vec{j}_{in} = -\frac{i\hbar}{2m}\left[\psi_{in}^* \nabla \psi_{in} - \psi_{in} \nabla \psi_{in}^*\right] = \frac{1}{(2\pi\hbar)^3}\frac{\vec{p}}{m}$$

$$\vec{j}_r = -\frac{i\hbar}{2m}\left[\psi_{out}^* \partial_r \psi_{out} - \text{c.c.}\right] = \underbrace{\frac{|f|^2}{(2\pi\hbar)^3}\frac{p}{m}\frac{1}{r^2}} + \mathcal{O}\left(\frac{1}{r^2}\right) \quad r \to \infty$$

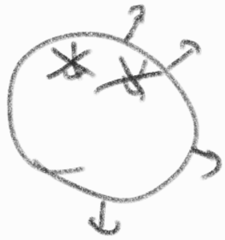
$$\frac{d\sigma}{d\Omega} = |f|^2$$

pozn.: $\boxed{\text{Ekvival (LS) rovnice}} \Longleftrightarrow \boxed{\text{(SR) + okr. podm.}}$

$$(E - H)\psi_p^{(+)} = 0 \qquad (E - H_0)\, p(x) = 0$$

rozptýlenou vlnu -- $\psi_p^{(+)} - p(x) \equiv \boxed{\psi_S(\vec{x})}$

-- jednoduchá okraj podm. -- $\boxed{\partial_r \psi_S = ik\, \psi_S}$



$$(E - H_0 - V)\,\psi_p = 0$$

$$(-1) \quad (E - H_0 - V)\, p(x) = -V p(x)$$

$$\boxed{(E - H)\,\psi_S = \underbrace{V p(x)}_{\text{nehomog}}} \Leftarrow \quad \psi - p = \psi_S$$

- Transition operator (T matice), Bornova řada

$$|\psi_p^{(+)}\rangle = |p\rangle + G_0^{(+)}(E_p)\, V\, |\psi_p^{(+)}\rangle \qquad H = H_0 + V$$

$$|\psi\rangle = |p\rangle + G_0 V|p\rangle + (G_0V)^2|p\rangle + (G_0V)^3|p\rangle \dots \text{ — rozpt. řeš.}$$

$$\left(I + G_0V + (G_0V)^2 + \dots\right)|p\rangle = (I - G_0V)^{-1}|p\rangle$$

$$G^{(+)} = G_0 + G_0 V G \dots$$

$$\to G = G_0 + (G_0V)G + (G_0V)^2 G + \dots \text{ — G-fce}$$

T-operator: $\hat{T}(E) = \hat{V} + \hat{V}\,\underline{\hat{G}(E)}\,\hat{V}$

− LS: $T(E) = V + V G_0 T(E)$

$$T(E) = V + V G_0 V + (V G_0)^2 V + \cdots$$

pro ampl. rozp.

z Born. řad.:

$$G_0 T = G V \qquad T G_0 = V G$$

$$V|\psi_p^{(+)}\rangle = T|p\rangle$$

$$|\psi\rangle = \Omega|p\rangle = |p\rangle + \underbrace{GV}|p\rangle = |p\rangle + G_0 V|\psi\rangle$$

$t_{p'p}$

Souvislost T-operátoru a S-matice:

$$\langle \vec{p}\,'|S|\vec{p}\rangle = \lim_{\substack{t\to+\infty \\ t'\to-\infty}} \langle \vec{p}\,'| e^{iH_0 t} e^{-iHt} e^{iHt'} e^{-iH_0 t'}|p\rangle$$

$$t' = -t$$

$$= \lim_{t\to\infty} \langle \vec{p}\,'| \underbrace{e^{iH_0 t} e^{-2iHt} e^{iH_0 t}}|p\rangle$$

$$F(t) = F(0) + \int_0^\infty \frac{dF}{d\tau}\, d\tau$$

$$e^{iH_0 t} e^{-iHt} \qquad e^{-iHt} e^{iH_0 t}$$

$$(iH_0 - iH) \qquad -iH + iH_0$$

$$-iV \qquad -iV$$

$$\langle p'|S|p\rangle = \delta(\vec{p}-\vec{p}\,') - i\int_0^\infty d\tau \langle p'| V \frac{e^{i(E_{p'}+E_p-2H)\tau} + e^{i(E_{p'}+E_p-2H)\tau}}{} V|p\rangle$$

$$V \to V e^{-\varepsilon\tau}$$

$$\left(\frac{E_{p'}+E_p}{2} - H\right)^{-1}$$

$$= \delta(\vec{p}-\vec{p}\,') + \frac{1}{2}\langle p'| \left\{ V \hat{G}^{(+)}\left(\frac{E_{p'}+E_p}{2}\right) + \hat{G}^{(+)}\left(\frac{E_p+E_{p'}}{2}\right) V \right\} |p\rangle$$

$$\underbrace{T\left(\frac{E_{p'}+E_p}{2}\right) G_0^{(+)}\left(\frac{E_{p'}+E_p}{2}\right)} \qquad G_0 T$$

$$\bar{E} = \frac{E_p + E_{p'}}{2}$$

$$\langle p'| T \left[\frac{E_{p'}+E_p}{2} - H_0\right]^{-1} |p\rangle \qquad \frac{E_p - E_{p'}}{2}$$

$$E_p$$

$$\frac{E_{p'} - E_p}{2}$$

$$= \delta(\vec{p}-\vec{p}\,') + \frac{1}{2}\langle p'|T(\bar{E})|p\rangle \left\{ \frac{2}{E_{p'}-E_p+2i\varepsilon} + \frac{2}{E_p - E_{p'} + 2i\varepsilon} \right\}$$

$$\frac{1}{x+i\varepsilon} = \frac{\text{v.p.}}{x} - i\pi\delta(x)$$

$$= \delta(\vec{p}-\vec{p}\,') - 2\pi i \underbrace{\langle p'|\hat{T}(E_p)|p\rangle}_{t_{p'\leftarrow p}} \delta(E_p - E_{p'})$$

$$\boxed{t_{p' \leftarrow p} = \langle \vec{p}' | \hat{T}(E) | \vec{p} \rangle = \langle \vec{p}' | \hat{V} | \psi_p^{(+)} \rangle}$$