Seznam vzorců pro zápočtovou písemku II.

Důležité komutátory

$$[x_\alpha, p_\beta] = i\hbar\delta_{\alpha,\beta}$$
$$[J_\alpha, V_\beta] = i\hbar\varepsilon_{\alpha,\beta,\gamma}V_\gamma$$

Moment hybnosti a sférické harmoniky

$$J^2|jm\rangle = \hbar^2 j(j+1)|jm\rangle$$
$$J_z|jm\rangle = \hbar m|jm\rangle$$
$$J_\pm = J_x \pm iJ_y$$
$$J_\pm|jm\rangle = \hbar\sqrt{(j\mp m)(j\pm m+1)}|jm\pm 1\rangle$$

Několik prvních sférických harmonik:

$$Y_{00} = \frac{1}{\sqrt{4\pi}}$$
$$Y_{11} = -\sqrt{\frac{3}{8\pi}}\frac{x+iy}{r}$$
$$Y_{10} = \sqrt{\frac{3}{4\pi}}\frac{z}{r}$$
$$Y_{1-1} = \sqrt{\frac{3}{8\pi}}\frac{x-iy}{r}$$
$$Y_{22} = \sqrt{\frac{15}{32\pi}}\frac{(x+iy)^2}{r^2}$$
$$Y_{21} = -\sqrt{\frac{15}{8\pi}}\frac{(x+iy)z}{r^2}$$
$$Y_{20} = \sqrt{\frac{5}{16\pi}}\frac{3z^2-r^2}{r^2}$$
$$Y_{2-1} = \sqrt{\frac{15}{8\pi}}\frac{(x-iy)z}{r^2}$$
$$Y_{2-2} = \sqrt{\frac{15}{32\pi}}\frac{(x-iy)^2}{r^2}$$

Harmonický oscilátor

$$H = \frac{p^2}{2m} + \frac{m\omega^2}{2}x^2 = \hbar\omega(a^+a + 1/2)$$

$$x_0 = \sqrt{\frac{\hbar}{m\omega}} \qquad p_0 = \frac{\hbar}{x_0} \qquad a = \frac{x/x_0 + ip/p_0}{\sqrt{2}}$$

$$\left[a, a^+\right] = 1 \qquad a|n\rangle = \sqrt{n}|n-1\rangle \qquad a^+|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\langle x|n\rangle = \sqrt{\frac{1}{\sqrt{\pi}x_0 n! 2^n}} H_n(x/x_0)\exp(-(x/x_0)^2/2)$$

Hermitovy polynomy

$$H_0(x) = 1 \qquad H_2(x) = 4x^2 - 2$$
$$H_1(x) = 2x \qquad H_3(x) = 8x^3 - 12x$$

Coulombický potenciál

$$V(x) = \gamma/r$$
$$E_n = -\frac{m\gamma^2}{2\hbar^2 n^2}$$
$$a = \frac{\hbar^2}{m|\gamma|}$$
$$R_{10}(r) = 2\sqrt{\frac{1}{a^3}}\exp(-r/a)$$
$$R_{20}(r) = \sqrt{\frac{1}{2a^3}}(1 - r/2a)\exp(-r/2a)$$
$$R_{21}(r) = \frac{1}{2}\sqrt{\frac{1}{6a^3}}r/a\exp(-r/2a)$$

Wignerova-Eckartova věta

$$\langle \alpha j m | T^{(J)}_M | \alpha' j' m' \rangle = \langle J j' M m' | j m \rangle \frac{(\alpha j || T^{(J)} || \alpha' j')}{\sqrt{2j+1}}$$

Clebsch-Gordanovy koeficienty $\langle j_1 j_2 m_1 m_2 | j m \rangle$

$\boxed{j_2 = 0}$ $\langle j 0 m 0 | j m \rangle = 1$

$\boxed{j_2 = 1/2}$ $\langle j_1 \frac{1}{2} m_1 \pm \frac{1}{2} | j m \rangle$

	$m = m_1 + \frac{1}{2}$	$m = m_1 - \frac{1}{2}$
$j = j_1 - \frac{1}{2}$	$-\sqrt{\frac{j+1-m}{2j+2}}$	$\sqrt{\frac{j+1+m}{2j+2}}$
$j = j_1 + \frac{1}{2}$	$\sqrt{\frac{j+m}{2j}}$	$\sqrt{\frac{j-m}{2j}}$

$\boxed{j_2 = 1}$ $\langle j_1 1 m_1 m_2 | j m \rangle$

	$m = m_1 + 1$	$m = m_1$	$m = m_1 - 1$
$j = j_1 - 1$	$\sqrt{\frac{(j-m+2)(j-m+1)}{(2j+2)(2j+3)}}$	$-\sqrt{\frac{(j+m+1)(j-m+1)}{(j+1)(2j+3)}}$	$\sqrt{\frac{(j+m+2)(j+m+1)}{(2j+2)(2j+3)}}$
$j = j_1$	$-\sqrt{\frac{(j-m+1)(j+m)}{2j(j+1)}}$	$\sqrt{\frac{m}{j(j+1)}}$	$\sqrt{\frac{(j+m+1)(j-m)}{2j(j+1)}}$
$j = j_1 + 1$	$\sqrt{\frac{(j+m)(j+m-1)}{2j(2j-1)}}$	$\sqrt{\frac{(j+m)(j-m)}{j(2j-1)}}$	$\sqrt{\frac{(j-m)(j-m-1)}{2j(2j-1)}}$

Gauntova formule

$$\int Y_{lm}(\theta,\varphi)^* Y_{l_1 m_1}(\theta,\varphi) Y_{l_2 m_2}(\theta,\varphi) \mathrm{d}\Omega = \sqrt{\frac{(2l_1+1)(2l_2+1)}{4\pi(2l+1)}} \langle l_1 l_2 0 0 | l 0 \rangle \langle l_1 l_2 m_1 m_2 | l m \rangle$$

Table of Clebsch–Gordan coefficients - Wikipedia

$j_1 = 2$, $j_2 = 1$

$m = 3$

m_1, m_2 \ j	3
2, 1	1

$m = 2$

m_1, m_2 \ j	3	2
2, 0	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$
1, 1	$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$

$m = 1$

m_1, m_2 \ j	3	2	1
2, −1	$\sqrt{\frac{1}{15}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{3}{5}}$
1, 0	$\sqrt{\frac{8}{15}}$	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{3}{10}}$
0, 1	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{10}}$

$m = 0$

m_1, m_2 \ j	3	2	1
1, −1	$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$
0, 0	$\sqrt{\frac{3}{5}}$	0	$-\sqrt{\frac{2}{5}}$
−1, 1	$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$

For brevity, solutions with $M < 0$ and $j_1 < j_2$ are omitted. They may be calculated using the simple relations

$$\langle j_1, j_2; m_1, m_2 \mid j_1, j_2; J, M \rangle = (-1)^{J-j_1-j_2} \langle j_1, j_2; -m_1, -m_2 \mid j_1, j_2; J, -M \rangle.$$

and

$$\langle j_1, j_2; m_1, m_2 \mid j_1, j_2; J, M \rangle = (-1)^{J-j_1-j_2} \langle j_2, j_1; m_2, m_1 \mid j_2, j_1; J, M \rangle.$$