Black Hole Thermodynamics: Classical and Quantum Electromagnetic Analogues

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Conventions

- Signature: $-+++$
- Space-time indices:[1] $abcd \ldots$
- Spatial indices: $ijkl \ldots$
- Later (for cognoscenti): Boys–Post constitutive relations

[1]During my time in Prague, I might move back to Greek indices for this.

Apologies

- I called the wedge structure in the Unruh bit sometimes a "null cone"; it is null, but not a null cone.
- ~~Utter chaos on my side: These lectures were... quickly done~~
- The previous point should be ironed out by now ☺

Analogue Space-Times—Some Repetition

The Problem with Space-Times

- Special/General Relativity effects always due to an underlying space-time
- Many effects very small:
  - **Classical:** Gravitational Waves (GW), Lense–Thirring effect, micro-lensing, memory effect. . .
  - **Quantum:** Hawking effect, Unruh effect, particle creation in GW, inflation, . . .
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- Corresponding experimental/physical questions not necessarily only gravitational
- $\Longrightarrow$ Find better achievable analogies!

Two Options for Analogies

Similar effects, different theories

$$
\begin{tikzcd}
\text{Theory A} \arrow[d, "\text{provides}"] & & \text{Theory B} \arrow[d, "\text{provides}"] \\
\text{Effect A} \arrow[rr, leftrightarrow, "\text{analogy}"] & & \text{Effect B}
\end{tikzcd}
$$

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$$

The Analogue Space-Time Framework

“GR”

generalize notion of ‘relativity’

- acoustics
- optics

‘relativistic’ framework, Lorentzian geometry

- elasticity
- GR as grav. theory
- seismology

Relativity not just a “theory” — it’s a framework!

A Visual Guide

Image source: G. Rousseaux, DOI:10.1007/978-3-319-00266-8_5 in Faccio, Belgiorno, Cacciatori, Gorini, Liberati, Moschella — Analogue Gravity Phenomenology (2013), p.99

An Analogue

Some History

- EM "Analogues", 1923: [1]
- Transformation optics, 1971: [2]
- More recently: Non-linear optics, graphene, . . .

Microscopic Maxwell equations **Macroscopic Maxwell equations**

$$\nabla_{[a}F_{bc]} = 0, \qquad \nabla_{[a}F_{bc]} = 0,$$

$$\nabla_a F^{ab} = J^b \xrightarrow[\text{tutive relations}]{\text{non-trivial consti-}} \nabla_a \underbrace{\left(Z^{abcd} F_{cd}\right)}_{G^{ab}} = J^b,$$

$$\mathbf{D} = \epsilon\,\mathbf{E} + \zeta\,\mathbf{B},$$
$$\mathbf{H} = \zeta^{\dagger}\mathbf{E} + \mu^{-1}\mathbf{B}.$$

Material Properties

- For an observer of four-velocity V, we get
  - permittivity tensor ϵ
  - permeability tensor μ^{-1}
  - magneto-electric tensor ζ

  as elements of a $3+1$ decomposition of constitutive tensor Z^{abcd}

- $$\begin{aligned} \epsilon^{ab} &:= -2Z^{acbd}V_c V_d, \\ \left[\mu^{-1}\right]^{ab} &:= \frac{1}{2}\varepsilon^{ca}{}_{ef}\,\varepsilon^{db}{}_{gh}Z^{efgh}V_c V_d, \\ \zeta^{ab} &:= \varepsilon^{ca}{}_{ef}Z^{efbd}V_c V_d. \end{aligned}$$

An *Algebraic* Electromagnetic Analogue

Action for microscopic electrodynamics on a Lorentzian manifold (M, g)

$$S = -\frac{1}{4} \int \mathrm{d}^4 x \sqrt{-\det g}\, \underbrace{\frac{1}{2}\left(g^{ac} g^{bd} - g^{ad} g^{bc}\right)}_{Z_{\text{vac}}^{abcd}} F_{ab} F_{cd}$$

Find material with

$$Z^{abcd} = \frac{1}{2} \frac{\sqrt{\det g_{\text{eff}}}}{\sqrt{\det g}} \left(\left[g_{\text{eff}}^{-1}\right]^{ac} \left[g_{\text{eff}}^{-1}\right]^{bd} - \left[g_{\text{eff}}^{-1}\right]^{ad} \left[g_{\text{eff}}^{-1}\right]^{bc} \right)$$

Material *effectively* raises indices on (M, g) with g_{eff}

W. Gordon (1923) DOI:10.1002/andp.19233772202

Degrees of Freedom

- Z is a symmetric map between anti-symmetric rank-2 tensors
- Each of these rank-2 tensors has 6 d.o.f.
- So, Z has 21 d.o.f.

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Here's a problem.

- Use Moore–Penrose pseudo-inverse $A^{\#}$ (see [5, 6]), and the pseudo-determinant

$$\mathrm{pdet}(A) = \prod_{\substack{i=1 \\ \lambda_i \neq 0}}^{\mathrm{rank}(A)} \lambda_i$$

- $\mu^{ab} := \left[\left[\mu^{-1}_{\bullet\bullet} \right]^{\#} \right]^{ab}$ and $\beta^{e} := \sqrt{\dfrac{\mathrm{pdet}(\mu^{\bullet\bullet})}{-\det(g^{\bullet\bullet})}}\, \varepsilon^{ecad}\, \mu^{-1}_{bc}\, \zeta_{a}{}^{b} V_{d}$

- Then, the consistency condition is:

$$\epsilon^{ab} = \mu^{ab}(1 - \mu^{-1}_{cd}\beta^c\beta^d) + \beta^a\beta^b$$

- This yields 10 equations, so d.o.f. match[2]

[2] As long as we in/exclude conformal freedom equally.

The Consistency Conditions

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- Fun fact: It's possible to show there will always be a frame in which $\beta = 0$ and $\zeta = 0$ by varying

$$L = [g_{\text{eff}}]_{ab} U^a U^b - \lambda(g_{ab} U^a U^b + 1),$$

  and going to the rest frame of the local extremum found this way

[2] As long as we in/exclude conformal freedom equally.

Fresnel–Fizeau, Part 1

- Why I was told there "is no covariant, macroscopic EM"
  - Permittivity and permeability couple only electric and magnetic fields to themselves (respectively)
  - Lorentz trafos mix them
  - ...

Fresnel–Fizeau, Part 1

- Why I was told there "is no covariant, macroscopic EM"
  - Permittivity and permeability couple only electric and magnetic fields to themselves (respectively)
  - Lorentz trafos mix them
  - ...
- Why it still works: Gets absorbed by "magneto-electric tensor"
- This is old news: 1851, experiments by Fizeau showed that in flowing water *they mix* [8, 9, 3, 10, 11]

Fresnel–Fizeau Part II

- The Kaluza–Klein decomposition:[3]

$$[g_{\text{eff}}^{-1}]^{ab} = \begin{pmatrix} -\sqrt{\det(\mu^{\circ\circ})}\,(1-\mu^{-1}_{kl}\beta^k\beta^l) & \beta^j \\ \beta^i & \dfrac{\mu^{ij}}{\sqrt{\det(\mu^{\circ\circ})}} \end{pmatrix}.$$

- Make (again) clever use of pseudo-determinants, Penrose pseudo-inverse $A^\#$ & consistence cond's
- After **WORK**:

$$[g_{\text{eff}}]_{ab} = -\sqrt{\frac{-\det g^{\bullet\bullet}}{\mathrm{pdet}\epsilon^{\bullet\bullet}}}\left(1-\epsilon^\#_{cd}\beta^c\beta^d\right)V_aV_b - V_a\epsilon^\#_{bd}\beta^d - V_b\epsilon^\#_{ad}\beta^d + \sqrt{\frac{\mathrm{pdet}\epsilon^{\bullet\bullet}}{-\det g^{\bullet\bullet}}}\epsilon^\#_{ab}$$

where

$$\beta^e = \sqrt{\frac{\mathrm{pdet}(\epsilon^{\bullet\bullet})}{-\det(g^{\bullet\bullet})}}\,\varepsilon^{ecad}\,\epsilon^\#_{bc}\,\zeta_a{}^b V_d.$$

[3] ADM for *inverse* metric

Fresnel–Fizeau Part II

- As there will be a system, where the magneto-electric effects, and thus β, in the effective metric vanishes. . .
- . . . take that. In this system, decompose Z as

$$Z^{abcd} = \frac{1}{2}\left(E_V + (*M_V*) + (*A_V) + (A_V^T*)\right)^{abcd}.$$

- In a different system:

$$\begin{aligned}
\epsilon_W^{ab} &= -2Z^{dacb}W_d W_c, \\
\zeta_W^{ab} &= 2(*Z)^{dacb}W_d W_c, \\
[\mu_W^{-1}]^{ab} &= 2(*Z*)^{dacb}W_d W_c, \\
\left[\zeta^\dagger\right]^{ab} &= 2(Z*)^{dacb}W_d W_c.
\end{aligned}$$

- This already qualitatively describes Fresnel–Fizeau

Fresnel–Fizeau Part III

- Take moving isotropic media:

$$\epsilon^{ab} = \epsilon(g^{ab} + V^a V^b) = \epsilon h^{ab}$$

and

$$[\mu^{-1}]^{ab} = \mu^{-1}(g^{ab} + V^a V^b) = \mu^{-1} h^{ab}.$$

- Then:

$$\begin{aligned}
\epsilon_W^{bd} =& \epsilon\, h_W^{bd} + (\epsilon - \mu^{-1}) \left[(V \cdot W)^2 h_W^{bd} - h_W^{be} h_{ef} h_W^{fd}\right], \\
[\mu_W^{-1}]^{bd} =& \frac{h_W^{bd}}{\mu} + (\mu^{-1} - \epsilon) \left((V \cdot W)^2 h_W^{bd} - h_W^{be} h_{ef} h_W^{fd}\right), \\
\zeta_W^{ac} =& (\epsilon - \mu^{-1})(V \cdot W) \left(\epsilon^{acef} W_e V_f\right)
\end{aligned}$$

Fresnel–Fizeau Part IV: Summary

- Pull out a factor ϵ, remainder contains a factor of $1 - {}^1/_{\epsilon\mu} = 1 - \frac{1}{n^2}$—the Fresnel–Fizeau effect in flat space.
- Flat space, both observer and natural reference frame inertial frames: $(V \cdot W)^2 = \gamma^2$—the Lorentz factor we expect second-rank tensors to have.
- This is more general than the original Fresnel–Fizeau (and less limited to linear regimes as in [3])
- Observer change: Loss of isotropy! Even for inertial observers in Minkowski space!

Cartography!

- $q_{bca}^{\text{lab/eff}} := \nabla_a^{\text{lab/eff}} g_{bc}^{\text{eff/lab}} \neq 0.$
- Non-metricity can appear in inhomogeneous Maxwell equation

$$\nabla_a^{\text{lab}} \left(Z^{abcd} F_{cd} \right) = J^b$$

- Complicates comparison of laboratory measurements with *effective* space-time electrodynamics
- What is $x_{\text{eff}}(x_{\text{lab}})$?
- What does it do?

Source: `https://en.wikipedia.org/wiki/List_of_map_projections`

Details

With:

$$\left[\Gamma^{\text{lab}}\right]^{c}{}_{ab} = \left[\Gamma^{\text{eff}}\right]^{c}{}_{ab} + \underbrace{\left[g_{\text{eff}}^{-1}\right]^{cm}\left(\frac{1}{2}\left[q_{abm} - 2q_{m(ab)}\right] + \left[2T_{(ab)m} - T_{mba}\right]\right)}_{=:\tilde{\Delta}^{c}{}_{ab}}.$$

Inhomogeneous Maxwell equation:

$$\begin{aligned}\nabla_a^{\text{lab}}(Z^{abcd}F_{cd}^{\text{lab}}) = J^b &+ \text{terms dependent on } \Gamma_{\text{eff}} \text{ and its coupling to } F^{\text{eff}} \\ &+ \tilde{\Delta}^{a}{}_{ma}Z^{mbcd}F_{cd}^{\text{lab}} + \tilde{\Delta}^{b}{}_{ma}Z^{amcd}F_{cd}^{\text{lab}} + \tilde{\Delta}^{c}{}_{ma}Z^{abmd}F_{cd}^{\text{lab}} \\ &+ \tilde{\Delta}^{d}{}_{ma}Z^{abcm}F_{cd}^{\text{lab}} - \tilde{\Delta}^{m}{}_{ca}Z^{abcd}F_{md}^{\text{lab}} - \tilde{\Delta}^{m}{}_{da}Z^{abcd}F_{cm}^{\text{lab}}.\end{aligned}$$

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Inhomogeneous Maxwell equation:

$$\begin{aligned}\nabla^{\text{lab}}_{a}(Z^{abcd}F^{\text{lab}}_{cd}) = J^{b} &+ \text{terms dependent on } \Gamma_{\text{eff}} \text{ and its coupling to } F^{\text{eff}} \\ &+ \tilde{\Delta}^{a}{}_{ma}Z^{mbcd}F^{\text{lab}}_{cd} + \tilde{\Delta}^{b}{}_{ma}Z^{amcd}F^{\text{lab}}_{cd} + \tilde{\Delta}^{c}{}_{ma}Z^{abmd}F^{\text{lab}}_{cd} \\ &+ \tilde{\Delta}^{d}{}_{ma}Z^{abcm}F^{\text{lab}}_{cd} - \tilde{\Delta}^{m}{}_{ca}Z^{abcd}F^{\text{lab}}_{md} - \tilde{\Delta}^{m}{}_{da}Z^{abcd}F^{\text{lab}}_{cm}.\end{aligned}$$

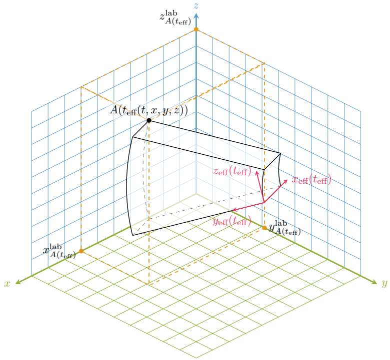
I would like to see what can be done with this—analogues “beyond GR”, *i.e.*, for modified gravity?

Complicating My Life...

- Algebraic extension to general analogues of modified theories of gravity straightforward
- *Minimal* coupling of torsion to EM will break gauge invariance[4]
- $\Longrightarrow$ Issue when building analogue
- *Non-minimal* couplings: Vast plethora of options

[4] Hehl, Obukhov: arXiv:gr-qc/0001010

Issues of Coordinates



Coordinates on (M, g)

$\not\Downarrow$

Coordinates on (M, g_{eff})!

Example: Scaled lab coordinates

Hawking temperature of a Schwarzschild BH

- Hawking temperature can be rephrased in terms of either M or ϵ, $\left[\mu^{-1}\right], \zeta$
- $T_{\text{H,eff}} = \left(\frac{\hbar}{4\pi\sqrt{\det g \ \det \epsilon}} \right)_{,r_{\text{eff}}} \Bigg|_{r_{\text{eff}}=r_+}$
- Assume $r_{\text{eff}} = a\, r_{\text{lab}}$
- $T_{\text{H, lab}} = \frac{\hbar}{8\pi a^{9/2} M} = a^{-9/2}\ T_{\text{H, eff}}$

The Numbers

$M[\mathrm{kg}]$	$r_{\mathrm{H}}[\mathrm{m}]$	$T_{\mathrm{H}}[\mathrm{K}]$	$T_{\mathrm{lab}}[\mathrm{K}]$
$M_p = 1.6726 \times 10^{-27}$	2.4841×10^{-54}	7.3355×10^{49}	3.8651×10^{286}
1	1.4852×10^{-27}	1.2269×10^{23}	2.0693×10^{139}
$M_{☾} = 7.342 \times 10^{22}$	1.0904×10^{-4}	1.6711	3.5796×10^{13}
$M_{♂} = 6.4171 \times 10^{23}$	9.5306×10^{-4}	0.1912	2.3738×10^{8}
$M_{♀} = 4.8685 \times 10^{24}$	7.2306×10^{-3}	2.5202×10^{-2}	3428.8
$M_{\oplus} = 5.9736 \times 10^{24}$	8.8719×10^{-3}	2.0539×10^{-2}	1113.1
$M_{♅} = 8.6832 \times 10^{25}$	0.1290	1.4130×10^{-3}	4.4986×10^{-4}
$M_{\odot} = 1.9886 \times 10^{30}$	2953.4	6.1700×10^{-8}	4.7191×10^{-28}
$M_{\mathrm{Sgr\ A^*}} = 7.9542 \times 10^{36}$	1.1813×10^{10}	1.5425×10^{-14}	2.3043×10^{-64}

- T_{H} as "observed" within the effective (Schwarzschild) space-time itself
- Actually observed temperature T_{lab} of laboratory
- Scale factor a such that $r_{\mathrm{lab}} = 10\,\mathrm{cm}$, *i.e.*, $a = 10 \times r_{\mathrm{H}}[\mathrm{m}]$

An Analogy

An Electromagnetic Analogy

- In stratified media and with the right coordinates, the inhomogeneous Maxwell equation

$$\nabla_a^{\text{lab}} \left(Z^{abcd} \, F_{cd} \right) = J^b$$

can be separated to a Helmholtz equation:

$$\frac{\mathrm{d}^2 f(r)}{\mathrm{d}\, r^2} + \left(\frac{\omega^2}{c^2} n^2(r) - \frac{D}{r^2} \right) f(r) = 0,$$

where D is a separation constant.

An Electromagnetic Analogy

- In certain space-times and with the right coordinates, the wave equation for EM fields

$$\nabla_a^{\mathrm{eff}} \left(Z_{\mathrm{eff}}^{abcd} \, F_{cd} \right) = J^b$$

can be separated to the following equation:

$$\frac{\mathrm{d}^2 f(r)}{\mathrm{d}\, r^2} + \qquad\qquad V(r,\omega)\; f(r) = 0,$$

where $V_{\mathrm{Kerr}}(r,\omega) := \frac{[am - \omega(r^2+a^2)]^2}{\Delta^2} - \lambda_{-1}(\ell) - \frac{[am-\omega(r^2+a^2)]p_\ell'' - 2r\omega p_\ell'}{(am-\omega[r^2+a^2])p_\ell}$

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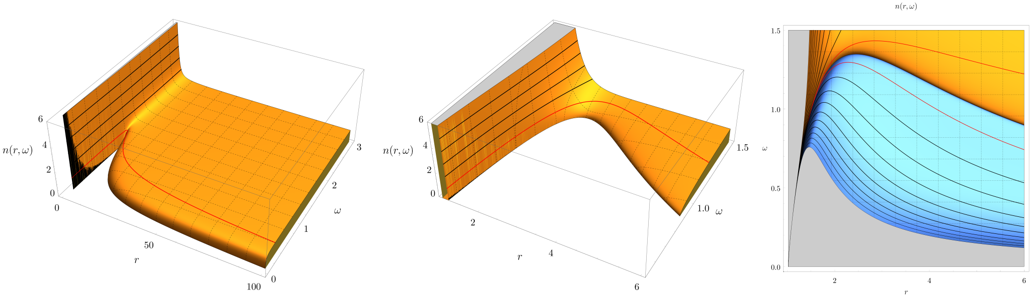
- Compare with Helmholtz eqn., get index of refraction:

$$n(r,\omega) = \frac{c}{\omega}\sqrt{V(r,\omega) + \frac{D}{r^2}}.$$

Refractive Index Profiles for Kerr



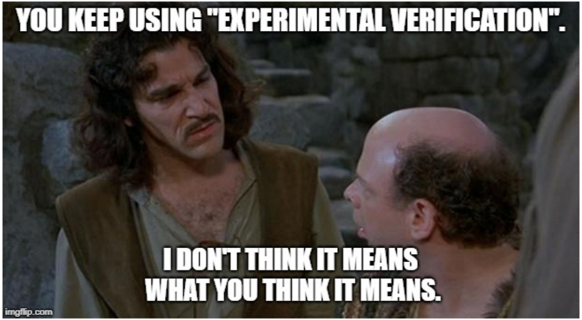
Orange: Real part; **Blue:** Imaginary part.

Here: $M = 1,\ a = 0.99975,\ \ell = 6,\ m = 3,\ D = 0$

Concluding Remarks

What We Have and Haven't

- **Analogues/analogies show *falsifiably*:**
  Relativity/curved space-time quantum field theory *in the analogous model*
- **They show *only non-falsifiably*:**
  Confidence in the *astrophysical* relativity/curved space-time quantum field theory



Thank you! Questions?

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7. R. H. Jonsson *et al.* Communication through quantum fields near a black hole. *Physical Review D* **101,** 125005. doi:10.1103/PhysRevD.101.125005. arXiv: 2002.05482 [gr-qc] (June 2020)

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