Klasická elektrodynamika v relativistickém formalismu

Slovník veličin

metrika	$\overset{\leftrightarrow}{I}$	$\eta_{\mu\nu} \equiv \begin{bmatrix} -1 & 0 \\ 0 & \overset{\leftrightarrow}{I} \end{bmatrix}$	$\eta^{\mu}{}_{\nu} = \delta^{\mu}_{\nu} \equiv \begin{bmatrix} 1 & 0 \\ 0 & \overset{\leftrightarrow}{I} \end{bmatrix}$
poloha	$t,\ \vec{r}$	$x^{\mu} \equiv \begin{bmatrix} ct \\ \vec{r} \end{bmatrix}$	
gradient	$\dfrac{\partial}{\partial t},\ \vec{\nabla}$	$\nabla_{\mu} \equiv \begin{bmatrix} \frac{1}{c}\frac{\partial}{\partial t} & \vec{\nabla} \end{bmatrix}$	$\nabla^{\mu} \equiv \begin{bmatrix} -\frac{1}{c}\frac{\partial}{\partial t} \\ \vec{\nabla} \end{bmatrix}$
zdroje	$\rho,\ \vec{j}$	$j^{\mu} \equiv \begin{bmatrix} c\rho \\ \vec{j} \end{bmatrix}$	
potenciály	$\phi,\ \vec{A}$	$A^{\mu} \equiv \begin{bmatrix} \frac{1}{c}\phi \\ \vec{A} \end{bmatrix}$	
EM pole	$\vec{E},\ \vec{B}$	$F_{\mu\nu} \equiv \begin{bmatrix} 0 & -\frac{1}{c}\vec{E} \\ \frac{1}{c}\vec{E} & \overset{\leftrightarrow}{B} \end{bmatrix}$	$F^{\mu}{}_{\nu} \equiv \begin{bmatrix} 0 & \frac{1}{c}\vec{E} \\ \frac{1}{c}\vec{E} & \overset{\leftrightarrow}{B} \end{bmatrix}$
hustota 4-síly	$w,\ \vec{f}$	$\mathit{\Phi}^{\mu} \equiv \begin{bmatrix} \frac{1}{c}w \\ \vec{f} \end{bmatrix}$	
hustoty energie, hybnosti a jejich toky	$u,\ \vec{g},\ \vec{\mathcal{S}},\ \overset{\leftrightarrow}{T}$	$T^{\mu\nu}_{\text{EM}} \equiv \begin{bmatrix} u & c\vec{g} \\ \frac{1}{c}\vec{\mathcal{S}} & \overset{\leftrightarrow}{T} \end{bmatrix}$	$T^{\mu}_{\text{EM}\,\nu} \equiv \begin{bmatrix} -u & c\vec{g} \\ -\frac{1}{c}\vec{\mathcal{S}} & \overset{\leftrightarrow}{T} \end{bmatrix}$

Důležité vztahy

Maxwellovy rovnice

$$\nabla_{\nu}F^{\mu\nu} = \frac{1}{\varepsilon_{\text{o}}c^2}\, j^{\mu} \qquad \vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_{\text{o}}}\,\rho \qquad \vec{\nabla}\times\vec{B} - \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} = \frac{1}{\varepsilon_{\text{o}}c^2}\,\vec{j}$$

$$\nabla_{[\alpha}F_{\beta\gamma]} = 0 \qquad \vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0$$

zákon zachování náboje

$$\nabla_{\mu}j^{\mu} = 0 \qquad \frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0$$

EM pole a potenciály

$$F_{\mu\nu} = \nabla_{\mu}A_{\nu} - \nabla_{\nu}A_{\mu} \qquad \vec{E} = -\vec{\nabla}\phi - \frac{\partial}{\partial t}\vec{A} \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

kalibrační volnost

$$A_{\mu} \to \tilde{A}_{\mu} = A_{\mu} + \nabla_{\mu}\psi \qquad \phi \to \tilde{\phi} = \phi - \frac{\partial\psi}{\partial t} \qquad \vec{A} \to \tilde{\vec{A}} = \vec{A} + \vec{\psi}$$

Lorenzova podmínka

$$\nabla_{\mu}A^{\mu} = 0 \qquad \frac{1}{c^2}\frac{\partial\phi}{\partial t} + \vec{\nabla}\cdot\vec{A} = 0$$

vlnová rovnice pro potenciály

$$\Box A^{\mu} = \eta^{\kappa\lambda}\nabla_{\kappa}\nabla_{\lambda}A^{\mu} = -\frac{1}{\varepsilon_{\text{o}}c^2}\, j^{\mu} \qquad \Box\phi = \Big[-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \Delta\Big]\phi = -\frac{1}{\varepsilon_{\text{o}}}\,\rho \qquad \Box\vec{A} = \Big[-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \Delta\Big]\vec{A} = -\frac{1}{\varepsilon_{\text{o}}c^2}\,\vec{j}$$

hustota Lorentzovy síly

$$\mathit{\Phi}^{\mu} = F^{\mu}{}_{\nu}j^{\nu} \qquad w = \vec{j}\cdot\vec{E} \qquad \vec{f} = \rho\vec{E} + \vec{j}\times\vec{B}$$

tenzor energie-hybnosti EM pole

$$T^{\mu\nu}_{\text{EM}} = \varepsilon_{\text{o}}c^2\Big(F^{\mu\kappa}F^{\nu\lambda}\eta_{\kappa\lambda} - \frac{1}{4}F_{\kappa\lambda}F^{\kappa\lambda}\eta^{\mu\nu}\Big)$$

$$u = \frac{\varepsilon_{\text{o}}}{2}E^2 + \frac{\varepsilon_{\text{o}}c^2}{2}B^2 \qquad c\vec{g} = \frac{1}{c}\vec{\mathcal{S}} = \varepsilon_{\text{o}}c\,\vec{E}\times\vec{B} \qquad \overset{\leftrightarrow}{T} = -\varepsilon_{\text{o}}\Big(\vec{E}\vec{E} + c^2\vec{B}\vec{B} - \frac{1}{2}(E^2 + c^2B^2)\overset{\leftrightarrow}{I}\Big)$$

zákon zachování energie a hybnosti

$$\nabla_{\nu}T^{\nu\mu}_{\text{EM}} = -\mathit{\Phi}^{\mu} \qquad \frac{\partial u}{\partial t} + \vec{\nabla}\cdot\vec{\mathcal{S}} = -w \qquad \frac{\partial\vec{g}}{\partial t} + \vec{\nabla}\cdot\overset{\leftrightarrow}{T} = -\vec{f}$$

invarianty

$$\mathscr{L} = -\frac{\varepsilon_{\text{o}}c^2}{4}\,F_{\mu\nu}F^{\mu\nu} \qquad \mathscr{L} = \frac{\varepsilon_{\text{o}}}{2}E^2 - \frac{\varepsilon_{\text{o}}c^2}{2}B^2$$

$$\tilde{\mathscr{L}} = \frac{\varepsilon_{\text{o}}c^2}{8}\,F^{\kappa\lambda}F^{\mu\nu}\,\varepsilon_{\kappa\lambda\mu\nu} \qquad \tilde{\mathscr{L}} = \varepsilon_{\text{o}}c\,\vec{E}\cdot\vec{B}$$

Lorentzovy transformace

$$a^{\mu'} = \Lambda^{\mu'}{}_{\nu} a^{\nu} \qquad a^{\mu} = \Lambda^{\mu}{}_{\nu'} a^{\nu'}$$

$$\eta_{\mu\nu} = \eta_{\alpha'\beta'}\,\Lambda^{\alpha'}{}_{\mu}\Lambda^{\beta'}{}_{\nu} \qquad \eta_{\mu'\nu'} = \eta_{\alpha\beta}\,\Lambda^{\alpha}{}_{\mu'}\Lambda^{\beta}{}_{\nu'}$$

$\Lambda^{\alpha}{}_{\mu'}$ a $\Lambda^{\alpha'}{}_{\mu}$ jsou ortonormální

$$\Lambda^{\mu}{}_{\alpha'}\Lambda^{\alpha'}{}_{\mu} = \delta^{\mu}_{\nu} \qquad \Lambda^{\mu'}{}_{\alpha}\Lambda^{\alpha}{}_{\mu'} = \delta^{\mu'}_{\nu'}$$

$\Lambda^{\alpha}{}_{\mu'}$ a $\Lambda^{\nu'}{}_{\beta}$ jsou navzájem inverzní

$$\Lambda_{\alpha'}{}^{\mu} = \Lambda^{\mu}{}_{\alpha'} \qquad \Lambda_{\alpha}{}^{\mu'} = \Lambda^{\mu'}{}_{\alpha}$$

ortonormalita $\Leftrightarrow$ transponovaná matice je inverzní

Rozštěpení Lorentzových transformací na paralelní a transverzální směr

Lorentzovat transformace je dána 3-rychlostí $\vec{v} = v\vec{e}_{\shortparallel}$. Ta určuje význačný ‘paralelní’ směr $\vec{e}_{\shortparallel}$.
4-veličiny budeme štěpit na časovou složku, složku ve směru $\vec{e}_{\shortparallel}$ a složky kolmé na $\vec{e}_{\shortparallel}$:

$$a^{\mu} \equiv \begin{bmatrix} a^0 \\ a_{\shortparallel} \\ \vec{a}_{\perp} \end{bmatrix} \qquad \vec{a} = a_{\shortparallel}\vec{e}_{\shortparallel} + \vec{a}_{\perp} \qquad a_{\shortparallel} = \vec{e}_{\shortparallel}\cdot\vec{a} \qquad \vec{e}_{\shortparallel}\cdot\vec{a}_{\perp} = 0$$

$$\Lambda^{\mu'}{}_{\nu} = \begin{bmatrix} \gamma & -\gamma\frac{v}{c} & 0 \\ -\gamma\frac{v}{c} & \gamma & 0 \\ 0 & 0 & \overset{\leftrightarrow}{I}_{\perp} \end{bmatrix} \qquad \Lambda^{\mu}{}_{\nu'} = \begin{bmatrix} \gamma & \gamma\frac{v}{c} & 0 \\ \gamma\frac{v}{c} & \gamma & 0 \\ 0 & 0 & \overset{\leftrightarrow}{I}_{\perp} \end{bmatrix}$$

$$x^{\mu'} = \Lambda^{\mu'}{}_{\nu} x^{\nu} \qquad t' = \gamma\Big(t - \frac{v}{c^2}x_{\shortparallel}\Big) \qquad x'_{\shortparallel} = \gamma\big(x_{\shortparallel} - vt\big) \qquad \vec{x}'_{\perp} = \vec{x}_{\perp}$$

$$\nabla_{\mu'} = \Lambda^{\nu}{}_{\mu'}\nabla_{\nu} \qquad \frac{\partial}{\partial t'} = \gamma\Big[\frac{\partial}{\partial t} + v\,\nabla_{\shortparallel}\Big] \qquad \nabla'_{\shortparallel} = \gamma\Big[\nabla_{\shortparallel} + \frac{v}{c^2}\frac{\partial}{\partial t}\Big] \qquad \vec{\nabla}'_{\perp} = \vec{\nabla}_{\perp}$$

$$j^{\mu'} = \Lambda^{\mu'}{}_{\nu} j^{\nu} \qquad \rho' = \gamma\Big(\rho - \frac{v}{c^2}j_{\shortparallel}\Big) \qquad j'_{\shortparallel} = \gamma\big(j_{\shortparallel} - \rho v\big) \qquad \vec{j}'_{\perp} = \vec{j}_{\perp}$$

$$A^{\mu'} = \Lambda^{\mu'}{}_{\nu} A^{\nu} \qquad \phi' = \gamma\big(\phi - vA_{\shortparallel}\big) \qquad A'_{\shortparallel} = \gamma\Big(A_{\shortparallel} - \frac{v}{c^2}\phi\Big) \qquad \vec{A}'_{\perp} = \vec{A}_{\perp}$$

$$F_{\mu'\nu'} = F_{\alpha\beta}\,\Lambda^{\alpha}{}_{\mu'}\Lambda^{\beta}{}_{\nu'} \qquad \begin{aligned} E'_{\shortparallel} &= E_{\shortparallel} & \vec{E}'_{\perp} &= \gamma\big(\vec{E}_{\perp} + \vec{v}\times\vec{B}_{\perp}\big) \\ B'_{\shortparallel} &= B_{\shortparallel} & \vec{B}'_{\perp} &= \gamma\Big(\vec{B}_{\perp} - \frac{1}{c^2}\vec{v}\times\vec{E}_{\perp}\Big) \end{aligned}$$