Problem: 2D Ising model

The goal

To simulate the 2D Ising model using the Monte Carlo method and to determine temperature dependencies of thermodynamic quantities and the critical temperature.

Details

Write a program which simulates the 2D Ising model with zero magnetic field $B = 0$ on the square lattice with $n \times n$ spins. The Hamiltonian is thus given by

$$H(\sigma) = -J \sum_{<\alpha\beta>} s_\alpha s_\beta \,,$$

where spins s_α can have values ± 1 and the sum runs over the nearest neighbors. For simplicity, set the coupling constant $J = 1$ and also the Boltzmann constant $k_B = 1$.

Use the Monte Carlo method and the Metropolis-Hastings algorithm as described in the lecture notes and demonstrated in the *Mathematica* notebook for the 1D Ising model to generate a sufficient number of configurations to get mean values of magnetization, susceptibility, internal energy and specific heat as functions of temperature in the interval $1.5 < T < 3.0$. Try to run the simulation for several numbers of spin in the interval $20 < n < 100$ and from the maximums of the specific heat estimate the critical temperature T_c for which the phase transition appears.

As a starting configuration use the one with all spins up ($s_alpha = +1$) to obtain approximately the upper curve of the spontaneous magnetization. Do the averaging over many independent runs (100 or more) to get reasonably smooth dependencies and keep in mind that it is necessary first to thermalise the system (to skip 100 or more configurations) before averaging is done in each run. Do at least 500 *measurements* after each complete sweep (one sweep consists of trying to change every spin row by row).

Output

To fulfill the task, provide your own code which simulates the 2D Ising model together with an output file containing dependence of thermodynamic quantities on temperature.