Problem: Diffusion-limited aggregation (DLA)

The goal

To simulate diffusion-limited aggregation in the square lattice and to determine the fractal dimension of the resulting *Brownian tree*.

Details

Write a program which simulates diffusion-limited aggregation in the square lattice as explained during the lecture on the DLA problem. Using the lattice $n \times n$ with $n > 500$, place the seed in the middle of the lattice and aggregate randomly walking particles as soon as their nearest neighbor is occupied. Continue aggregation until the mass m (number of occupied sites in the square lattice) of the generated Brownian tree reaches at least 10 000.

Start each walk randomly on a circle with the radius $R_{\text{start}} = R_{\text{max}} + 1$ where R_{max} is the current largest distance of an occupied site from the seed. If the distance R of a randomly walking particle from the seed is greater than R_{kill}, start another random walk. For more efficient simulation you can also consider to implement jumps for $R > R_{\text{jump}}$ which must be adjusted to be in the interval $R_{\text{start}} < R < R_{\text{kill}}$. Here, a jump means to move immediately to a randomly chosen site on a circle around the current position with the radius $R - R_{\text{start}}$.

After a sufficiently large Brownian tree is generated, calculate the number $N(a)$ of occupied sites in a box of size $a \times a$ with the initial seed in the middle as a function of a. From this dependence plotted in the log-log scale, estimate the fractal dimension as its slope, that is the fractal dimension can be approximated as

$$D_f \doteq \frac{\ln N(a)}{\ln a}\,.$$

Be careful to use only box sizes a for which the data are not biased with boundaries, for both small and large values of a the function $N(a)$ has a different behavior from a^{D_f}.

Output

To fulfill the task, provide your own code for DLA in the square lattice together with an output file containing $N(a)$ as a function of a, a log-log plot of this function showing also approximation of $N(a)$ with a^{D_f} where D_f was determined by the least-square fitting.