Random Walk in 1D

Preliminaries

Clear all symbols from previous evaluations to avoid conflicts

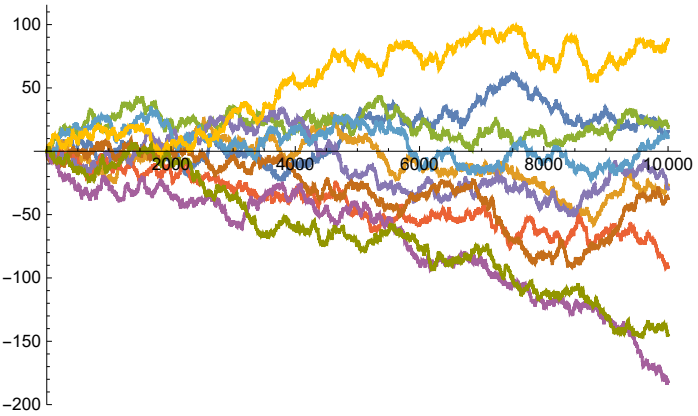
In[1]:=
```
Clear["Global`*"]
```

Simple walk in 1D - arbitrary step of maximal length *d*

Start at origin and make 10000 random steps:

In[2]:=
```
p = 10; (* number of walks *)
n = 10000;(* number of steps *)
d = 1.0;(* maximal length of each step *)
X = ConstantArray[0.0, {p, n + 1}];
(* SeedRandom[2021] *)
Do[
  Do[
   X[[ip, in + 1]] = X[[ip, in]] + d * RandomReal[{-1.0, 1.0}],
   {in, 1, n}
  ],
  {ip, 1, p}
 ];
ListPlot[X, Joined → {True}]
```

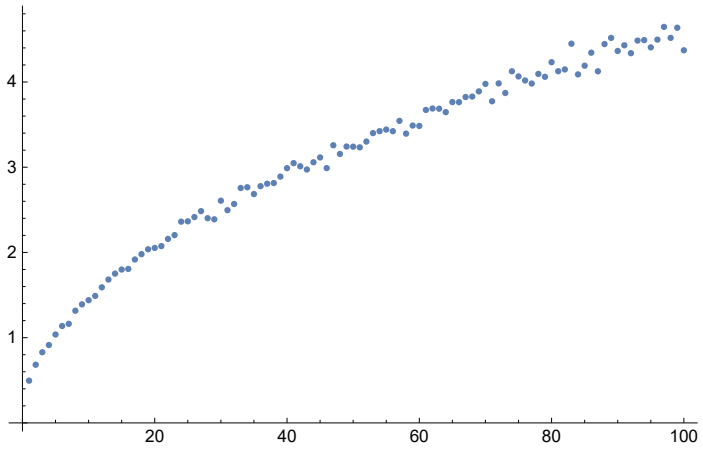
Out[7]=



Take p random walks for each number of steps n (up to nmax) and calculate the mean distance from the origin

In[8]:=
```
(* SeedRandom[2021] *)
nmax = 100;(* maximal number of steps *)
p = 1000;(* number of walks to average *)
d = 1.0;(* maximal length of each step *)
(* array for averaging distances from origin for each number of steps *)
Lc = ConstantArray[0.0, nmax];
(* array for calculating standard deviation
 of distances from origin for each number of steps *)
Lc2 = ConstantArray[0.0, nmax];
Do[
  Do[
   x = 0.0;
   Do[
    x += d * RandomReal[{-1.0, 1.0}],
    {in, 1, n}
   ];
   Lc[[n]] += Abs[x];
   Lc2[[n]] += x^2,
   {ip, 1, p}
  ];
  Lc[[n]] = Lc[[n]] / p;
  Lc2[[n]] = Sqrt[Lc2[[n]] / p - Lc[[n]]^2],
  {n, 1, nmax}
 ];
ListPlot[Lc, Joined → {False}]
```

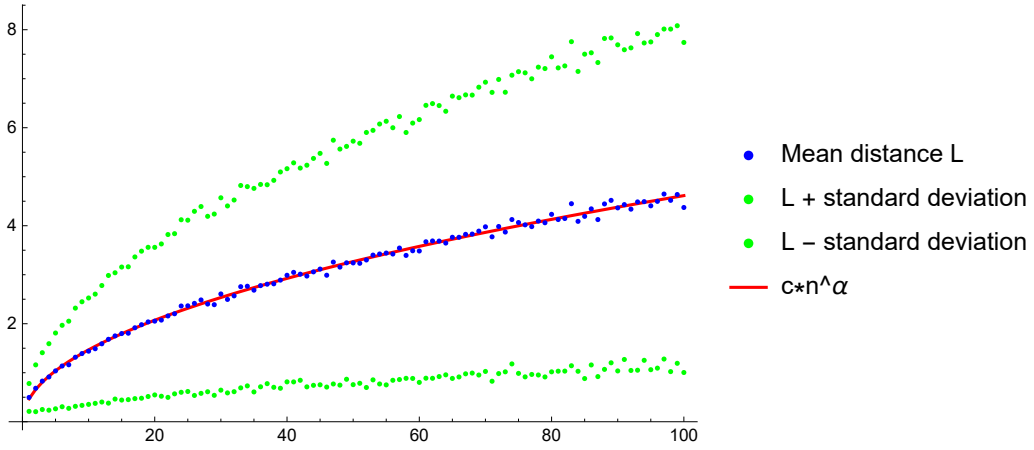
Out[14]=



In[15]:=
```
{cc, αc} = {a, b} /. FindFit[Lc, a * y^b, {a, b}, y];
Print["Parameters of fitting c*n^α: c = ", cc, ", α = ", αc]
fnc = Array[cc * #^αc &, nmax];
ListPlot[{Lc, Lc + Lc2, Lc - Lc2, fnc},
 Joined → {False, False, False, True},
 PlotStyle → {Blue, Green, Green, Red},
 PlotLegends →
  {"Mean distance L", "L + standard deviation", "L - standard deviation", "c*n^α"}]
```

```
Parameters of fitting c*n^α: c = 0.466666063029281, α = 0.497548656284058
```

Out[18]=

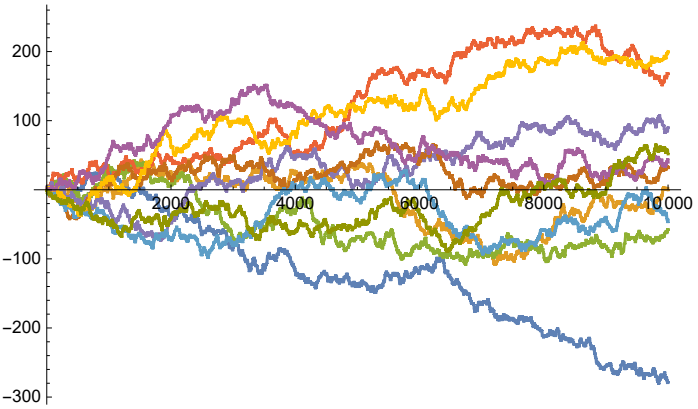


Simple walk in 1D - discrete steps ($d = 1$) of random direction

Start at origin and make 10000 random steps:

In[19]:=
```
p = 10; (* number of walks *)
n = 10000;(* number of steps *)
d = 1.0;(* maximal length of each step *)
X = ConstantArray[0.0, {p, n + 1}];
(* SeedRandom[2021] *)
Do[
  Do[
   X[[ip, in + 1]] = X[[ip, in]] + d * RandomChoice[{-1, 1}],
   {in, 1, n}
  ],
  {ip, 1, p}
 ];
ListPlot[X, Joined → {True}]
```

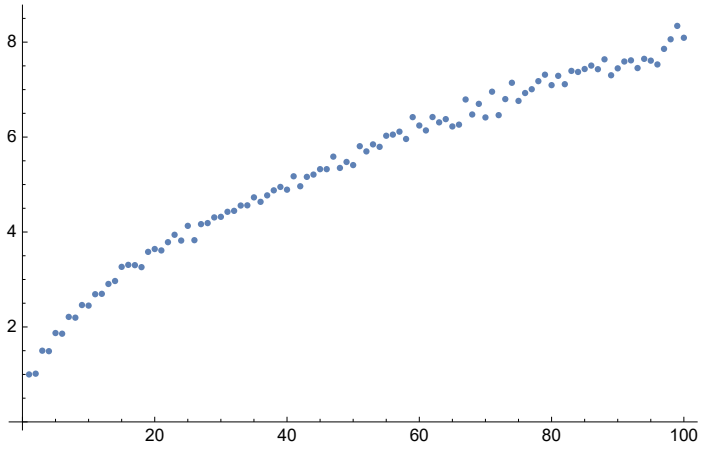
Out[24]=



Take p random walks for each number of steps n (up to nmax) and calculate the mean distance from the origin

In[25]:=
```
(* SeedRandom[2021] *)
nmax = 100;(* maximal number of steps *)
p = 1000;(* number of walks to average *)
d = 1.0;(* maximal length of each step *)
(* array for averaging distances from origin for each number of steps *)
L = ConstantArray[0.0, nmax];
(* array for calculating standard deviation
 of distances from origin for each number of steps *)
L2 = ConstantArray[0.0, nmax];
Do[
  Do[
   x = 0.0;
   Do[
    x += d * RandomChoice[{-1, 1}],
    {in, 1, n}
   ];
   L[[n]] += Abs[x];
   L2[[n]] += x^2,
   {ip, 1, p}
  ];
  L[[n]] = L[[n]] / p;
  L2[[n]] = Sqrt[L2[[n]] / p - L[[n]]^2],
  {n, 1, nmax}
 ];
ListPlot[L, Joined → {False}]
```

Out[31]=



In[32]:=
```
{c, α} = {a, b} /. FindFit[L, a * y^b, {a, b}, y];
Print["Parameters of fitting c*n^α: c = ", c, ", α = ", α]
fn = Array[c * #^α &, nmax];
ListPlot[{L, L + L2, L - L2, fn, fnc},
 Joined → {False, False, False, True, True},
 PlotStyle → {Blue, Green, Green, Red, Orange},
 PlotLegends → {"Mean distance L", "L + standard deviation",
   "L - standard deviation", "c*n^α", "non-discrete case"}]
```

```
Parameters of fitting c*n^α: c = 0.791077230523443, α = 0.501763709405183
```

Out[35]=