Clear all symbols from previous evaluations to avoid problems

In[1]:=
```
Clear["Global`*"]
```

Examples of simple Markov Chains

Problematic network

See Nezbeda, Kolafa and Kotrla: Introduction to Molecular Simulations, Prague 2002 (in Czech), page 46

Let there be a network which is not very reliable:
1) the probability the network will work tomorrow if it works today is 60%
2) and the probability the network will not work tomorrow if it does not work today is 70%.

Questions:
1) What is the probability the network will work in *k* days if it works today?
2) Does this probability depend on *k*? Does it depend on the initial state of the network?

In[2]:=
```
(* initial state - the network certainly works today *)
p = {1.0, 0.0};
(* vector p contains probabilities whether the network works or doesn't *)
(* transition matrix - probabilities of going from one state to another *)
W = {{0.6, 0.4}, {0.3, 0.7}} ;
MatrixForm[W]
kfin = 20; (* number of days - the length of the Markov chain *)
Do[
  p = N[p.W];
  Print[p],
  {k, 1, kfin}
 ];
```

Out[4]//MatrixForm=

$$\begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix}$$

```
{0.6, 0.4}
{0.48, 0.52}
{0.444, 0.556}
{0.4332, 0.5668}
{0.42996, 0.57004}
{0.428988, 0.571012}
{0.4286964, 0.5713036}
{0.42860892, 0.57139108}
{0.428582676, 0.571417324}
{0.4285748028, 0.5714251972}
{0.4285724408, 0.5714275592}
{0.4285717323, 0.5714282677}
{0.4285715197, 0.5714284803}
{0.4285714559, 0.5714285441}
{0.4285714368, 0.5714285632}
{0.428571431, 0.571428569}
{0.4285714293, 0.5714285707}
{0.4285714288, 0.5714285712}
{0.4285714286, 0.5714285714}
{0.4285714286, 0.5714285714}
```

```
In[7]:= (* left eigenvectors (in rows, normalized to be sum of elements = 1)
         and corresponding eigenvalues *)
        {λ, v} = Eigensystem[Transpose[W]];
        Do[
         If[
          Abs[Total[v[[i]]]] > 0.0, v[[i]] = v[[i]] / Total[v[[i]]]], (* normalization *)
         {i, 1, 2}
        ]
        {λ, v}
Out[9]= {{1., 0.3}, {{0.4285714286, 0.5714285714}, {-0.7071067812, 0.7071067812}}}
```

Random flipping of a die over an edge

Let us have a regular six-sided die. Put it on the table and start flipping each time over a random edge. Thus
1) the probability the vertical side will appear at the top is 1/4
2) and the probability the top or bottom side will appear at the top is 0.

Questions:
1) What is the probability a certain side will appear at the top after *k* flips if there is 1 at the beginning?
2) Again: Does this probability depend on *k*? Does it depend on the initial state?

In[10]:=
```
(* initial state - the side 1 is certaily up *)
p = {1.0, 0.0, 0.0, 0.0, 0.0, 0.0};
(* vector p contains probabilities of each side to be at the top *)
(* transition matrix - probabilities of going from one state to another *)
n = 6;
W = ConstantArray[1/4, {n, n}];
Do[
  W[[i, i]] = 0;
  W[[i, n - i + 1]] = 0,
  {i, 1, n}
 ];
MatrixForm[W]
kfin = 20; (* number of flips - the length of the Markov chain *)
Do[
  p = N[p.W];
  Print[p],
  {k, 1, kfin}
 ];
```

Out[14]//MatrixForm=

$$\begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$$

```
{0., 0.25, 0.25, 0.25, 0.25, 0.}
{0.25, 0.125, 0.125, 0.125, 0.125, 0.25}
{0.125, 0.1875, 0.1875, 0.1875, 0.1875, 0.125}
{0.1875, 0.15625, 0.15625, 0.15625, 0.15625, 0.1875}
{0.15625, 0.171875, 0.171875, 0.171875, 0.171875, 0.15625}
{0.171875, 0.1640625, 0.1640625, 0.1640625, 0.1640625, 0.171875}
{0.1640625, 0.16796875, 0.16796875, 0.16796875, 0.16796875, 0.1640625}
{0.16796875, 0.166015625, 0.166015625, 0.166015625, 0.166015625, 0.16796875}
{0.166015625, 0.1669921875, 0.1669921875, 0.1669921875, 0.1669921875, 0.166015625}
{0.1669921875, 0.1665039063, 0.1665039063, 0.1665039063, 0.1665039063, 0.1669921875}
{0.1665039063, 0.1667480469, 0.1667480469, 0.1667480469, 0.1667480469, 0.1665039063}
{0.1667480469, 0.1666259766, 0.1666259766, 0.1666259766, 0.1666259766, 0.1667480469}
{0.1666259766, 0.1666870117, 0.1666870117, 0.1666870117, 0.1666870117, 0.1666259766}
{0.1666870117, 0.1666564941, 0.1666564941, 0.1666564941, 0.1666564941, 0.1666870117}
{0.1666564941, 0.1666717529, 0.1666717529, 0.1666717529, 0.1666717529, 0.1666564941}
{0.1666717529, 0.1666641235, 0.1666641235, 0.1666641235, 0.1666641235, 0.1666717529}
{0.1666641235, 0.1666679382, 0.1666679382, 0.1666679382, 0.1666679382, 0.1666641235}
{0.1666679382, 0.1666660309, 0.1666660309, 0.1666660309, 0.1666660309, 0.1666679382}
{0.1666660309, 0.1666669846, 0.1666669846, 0.1666669846, 0.1666669846, 0.1666660309}
{0.1666669846, 0.1666665077, 0.1666665077, 0.1666665077, 0.1666665077, 0.1666669846}
```

In[17]:=

```
(* left eigenvectors (in rows, normalized to be sum of elements = 1)
 and corresponding eigenvalues *)
{λ, v} = N[Eigensystem[Transpose[W]]];
Do[
 If[
  Abs[Total[v[[i]]]] > 0.0, v[[i]] = v[[i]] / Total[v[[i]]]], (* normalization *)
 {i, 1, 2}
]
{λ, v}
```

Out[19]=

```
{{1., -0.5, -0.5, 0., 0., 0.},
 {{0.1666666667, 0.1666666667, 0.1666666667, 0.1666666667, 0.1666666667, 0.1666666667},
  {1., 0., -1., -1., 0., 1.}, {0., 1., -1., -1., 1., 0.},
  {-1., 0., 0., 0., 0., 1.}, {0., -1., 0., 0., 1., 0.}, {0., 0., -1., 1., 0., 0.}}}
```