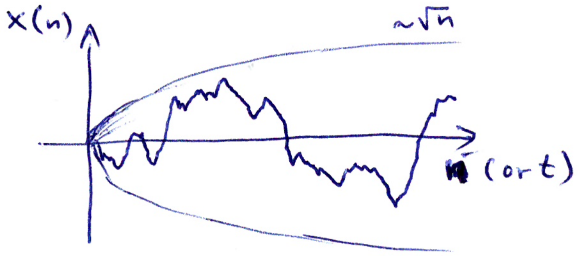
Random walks in 1D (see HW1 for 2D walks)



- the position is a random variable dependent on the previous step

$$x_{n+1} = x_n + d\,\xi_n$$

where d is a parameter characterizing the average length of the step

and ξ_n is a random variable with some given probability distribution (e.g. $\xi_n \in \langle -1, 1\rangle$ uniformly)

- after N steps, starting at the origin, we have

$$x_N = d \sum_{i=1}^{N} \xi_n,$$

and if $E(\xi_n) = z$ (often 0) and $D(\xi_n) = b^2$ for all n,

then, according to the CLT, we get the mean position

$$E(x_N) = d\,N\,E(\xi_n) = N d z \quad (=0 \text{ for } z=0)$$

with variance

$$D(x_N) = N d^2 b^2 = \sigma^2 \quad \text{or} \quad \sigma = d\,b\sqrt{N}$$

- particularly, for uniformly distributed ξ_i on $\langle -1, 1\rangle$

we have

$$E(x_N) = 0$$

$$D(x_N) = \frac{N d^2}{3} \quad \text{or} \quad \sigma = d\sqrt{\frac{N}{3}}$$

$$\left[\text{since } D_{\langle a,b\rangle}(\xi) = \frac{(b-a)^2}{12} \Rightarrow D_{\langle -1,1\rangle}(\xi) = \frac{(1-(-1))^2}{12} = \frac{1}{3}\right]$$

and the distribution of final positions after N steps will be approximately normal, i.e. $N\left(0, d\sqrt{\frac{N}{3}}\right)$

- for mean distance from the origin, we get for p random walks

$$E(\|x_N\|) = \frac{1}{p}\sum_{i=1}^{p} \|x_N^{(i)}\| \propto C\cdot\sqrt{N}$$

final position for a walk (i)

where the constant C will be a little bit different from $\frac{d}{\sqrt{3}}$