

Spectral Differentiation using FFT

Converted Trefethen's program p4.m from Matlab to Mathematica using claude.ai (partially modified)

In[257]:=

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(* Setup common parameters *)
n = 24;
h = 2 * Pi / n;
x = N[Table[h * i, {i, 1, n}]];

(* Define auxiliary plotting function *)
createPlot[data_, title_, yRange_] :=
  ListPlot[Transpose[{x, data}], PlotStyle -> {Blue, PointSize[0.03]},
    Joined -> True, Mesh -> All, PlotRange -> {{0, 2 * Pi}, yRange}, GridLines -> Automatic,
    Frame -> True, FrameLabel -> {{",", ""}, {"x", title}}, ImageSize -> Medium]

(* Part 1: Differentiation of a hat function *)
v = Table[Max[0, 1 - Abs[x[[i]] - Pi] / 2], {i, 1, n}];
vHat = Fourier[v, FourierParameters -> {1, -1}];

(* Create k vector for spectral
   differentiation and multiply the Fourier image by i*k *)
k = Join[Table[i, {i, 0, n / 2 - 1}], {0}, Table[i - n, {i, n / 2 + 1, n - 1}]];
wHat = I * k * vHat;

(* Compute spectral derivative *)
w = Re[InverseFourier[wHat, FourierParameters -> {1, -1}]];

(* Create plots for hat function *)
hatPlot = createPlot[v, "hat function", {-0.5, 1.5}];
hatDerivPlot = createPlot[w, "spectral derivative", {-1, 1}];

(* Part 2: Differentiation of exp(sin(x)) *)
v2 = Table[Exp[Sin[x[[i]]]], {i, 1, n}];
v2Prime = Table[Cos[x[[i]]] * Exp[Sin[x[[i]]]], {i, 1, n}];

(* Compute Fourier transform *)
v2Hat = Fourier[v2, FourierParameters -> {1, -1}];
w2Hat = I * k * v2Hat;

(* Compute spectral derivative *)
w2 = Re[InverseFourier[w2Hat, FourierParameters -> {1, -1}]];

(* Create plots for exp(sin(x)) *)
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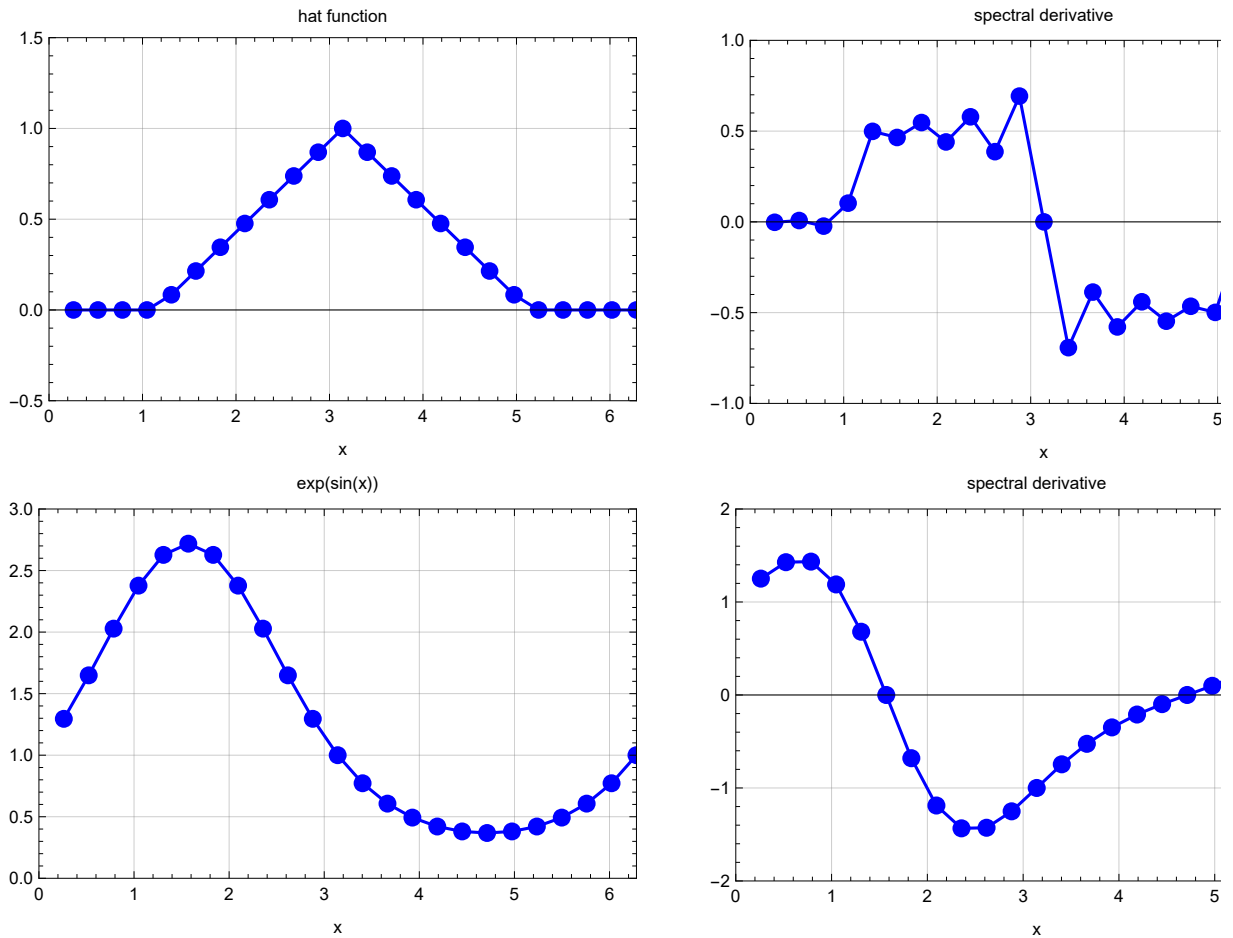
expSinPlot = createPlot[v2, "exp(sin(x))", {0, 3}];
expSinDerivPlot = createPlot[w2, "spectral derivative", {-2, 2}];

(* Display all plots together *)
Grid[{{hatPlot, hatDerivPlot}, {expSinPlot, expSinDerivPlot}}]

(* Calculate error *)
error = Max[Abs[w2 - v2Prime]];
Print["Error of the spectral derivative for the smooth function exp(sin(x)) is ", error]

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Out[275]=



Error of the spectral derivative for the smooth function $\exp(\sin(x))$ is $9.57234291832 \times 10^{-13}$