Maximálně symetrické prostory

Rovina E^2

translace a rotace

$$X_{x_0} : z \to X_{x_0} z$$

$$x(X_{x_0} z) = x(z) + x_0 \qquad y(X_{x_0} z) = y(z)$$

$$Y_{y_0} : z \to Y_{y_0} z$$

$$x(Y_{y_0} z) = x(z) \qquad y(Y_{y_0} z) = y(z) + y_0$$

$$R_{\varphi_0} : z \to R_{\varphi_0} z$$

$$x(R_{\varphi_0} z) = \cos\varphi_0 \, x(z) - \sin\varphi_0 \, y(z)$$

$$y(R_{\varphi_0} z) = \sin\varphi_0 \, x(z) + \cos\varphi_0 \, y(z)$$

generátory

$$\vec{X} = \frac{\vec{\partial}}{\partial x}$$

$$\vec{Y} = \frac{\vec{\partial}}{\partial y}$$

$$\vec{R} = -y \frac{\vec{\partial}}{\partial x} + x \frac{\vec{\partial}}{\partial y} = \frac{\vec{\partial}}{\partial \varphi}$$

algebra generátorů

$$[\vec{X}, \vec{R}] = \vec{Y} \qquad [\vec{Y}, \vec{R}] = -\vec{X} \qquad [\vec{X}, \vec{Y}] = 0$$

proof: $[\vec{X}, \vec{R}] = \left[\frac{\vec{\partial}}{\partial x}, -y\frac{\vec{\partial}}{\partial x} + x\frac{\vec{\partial}}{\partial y}\right] = \frac{\vec{\partial}}{\partial y} = \vec{Y}$

Lieova algebra $iso(E^2)$

obecně pro levou akci: $[\xi_M, \xi_N] = -c_{MN}{}^{K} \xi_K$

$\Rightarrow c_{XY}{}^{R} = 0 \qquad c_{XR}{}^{Y} = -1 \qquad c_{YR}{}^{X} = 1 \qquad$ ← netriviální složky

$k_{RR} = -\frac{1}{2} c_{RX}{}^{Y} c_{RY}{}^{X} - \frac{1}{2} c_{RY}{}^{X} c_{RX}{}^{Y} = 1 \qquad$ ostatní $= 0$

$$k_{rs} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Maximálně symetrické prostory

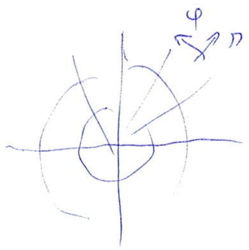
Lobačevského rovina

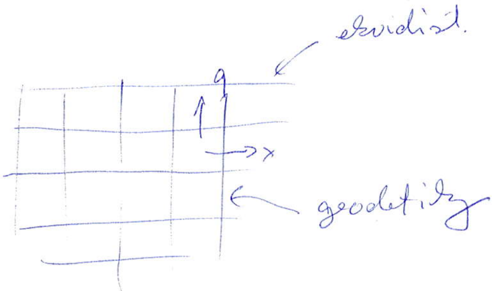
$$g = dr^2 + \mathrm{sh}^2 r \, d\varphi^2$$

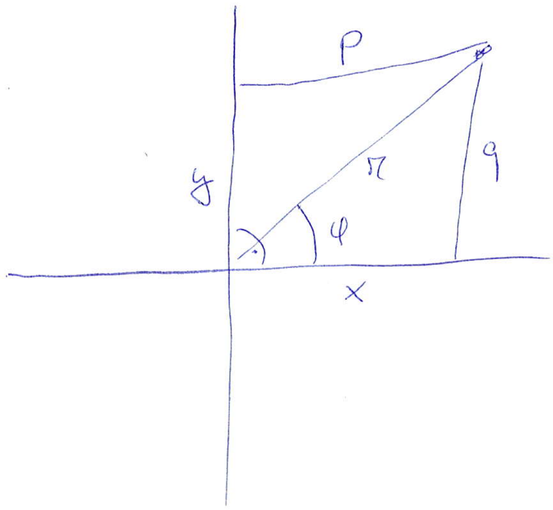
$$\mathrm{ch}\, r = \mathrm{ch}\, x \, \mathrm{ch}\, q$$

$$\tan\varphi = \frac{\mathrm{th}\, q}{\mathrm{sh}\, x}$$

$$g = \mathrm{ch}^2 q \, dx^2 + dq^2$$







$$\mathrm{th}\, x = \mathrm{th}\, r \cos\varphi$$

$$\mathrm{th}\, y = \mathrm{th}\, r \sin\varphi$$

$$\mathrm{sh}\, p = \mathrm{sh}\, r \cos\varphi$$

$$\mathrm{sh}\, q = \mathrm{sh}\, r \sin\varphi$$

$$\mathrm{th}^2 r = \mathrm{th}^2 x + \mathrm{th}^2 y$$

$$\mathrm{sh}^2 r = \mathrm{sh}^2 p + \mathrm{sh}^2 q$$

$$\mathrm{th}\, p = \mathrm{th}\, x \, \mathrm{ch}\, y$$

$$\mathrm{th}\, q = \mathrm{th}\, y \, \mathrm{ch}\, x$$

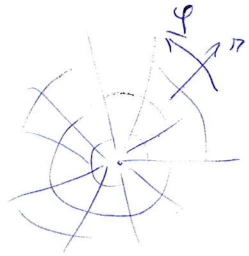
$$\frac{\mathrm{ch}\, x}{\mathrm{ch}\, y} = \frac{\mathrm{ch}\, p}{\mathrm{ch}\, q}$$

$$\mathrm{sh}\, x = \frac{\mathrm{sh}\, p}{\mathrm{ch}\, q}$$

$$\mathrm{sh}\, y = \frac{\mathrm{sh}\, q}{\mathrm{ch}\, p}$$

$$\frac{\mathrm{sh}\, p}{\mathrm{sh}\, q} = \frac{\mathrm{th}\, x}{\mathrm{th}\, y}$$

Isometrie Lobačevského roviny



$$g_{L^2} = dr^2 + \mathrm{sh}^2 r \, d\varphi^2$$

$$R = \frac{\partial}{\partial \varphi}$$

$$\mathcal{L}_R \, g_{L^2} = 0$$ R Killingův vektor

$$X = \cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{\mathrm{th}\, r} \frac{\partial}{\partial \varphi}$$

$$\mathcal{L}_X \, g_{L^2} = \mathcal{L}_X \left(dr^2 + \mathrm{sh}^2 r \, d\varphi^2 \right) =$$

$$= dr \vee \mathcal{L}_X dr + \left(\mathcal{L}_X \mathrm{sh}^2 r \right) d\varphi^2 + \mathrm{sh}^2 r \, d\varphi \vee \mathcal{L}_X d\varphi$$

$$= dr \vee d(X[r]) + 2 \,\mathrm{sh}\, r \,\mathrm{ch}\, r \, X[r] \, d\varphi^2 + \mathrm{sh}^2 r \, d\varphi \vee d X[\varphi]$$

$$= dr \vee d(\cos\varphi) + \mathrm{sh}\, 2r \cos\varphi \, d\varphi^2 - \mathrm{sh}^2 r \, d\varphi \vee d\left(\sin\varphi \frac{1}{\mathrm{th}\, r}\right)$$

$$= -\sin\varphi \, dr \vee d\varphi + \mathrm{sh}\, 2r \cos\varphi \, d\varphi^2 + \sin\varphi \frac{\mathrm{sh}^2 r}{\mathrm{th}^2 r \,\mathrm{ch}^2 r} d\varphi \vee dr - 2 \,\mathrm{sh}\, r \,\mathrm{ch}\, r \cos\varphi \, d\varphi^2$$

$$= 0$$

X je Killingův vektor

$$Y = \sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{\mathrm{th}\, r} \frac{\partial}{\partial \varphi}$$

$$\mathcal{L}_Y \, g_{L^2} = 0$$ obdobně X

Y je Killingův vektor

Lieove algebra symetrií

$$R = \frac{\partial}{\partial \varphi} = \operatorname{sh} x \frac{\partial}{\partial q} - \operatorname{ch} x \operatorname{th} q \frac{\partial}{\partial x}$$

$$X = \cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{\operatorname{th} r} \frac{\partial}{\partial \varphi} = \frac{\partial}{\partial x}$$

$$Y = \sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{\operatorname{th} r} \frac{\partial}{\partial \varphi} = \operatorname{ch} x \frac{\partial}{\partial q} - \operatorname{sh} x \operatorname{th} q \frac{\partial}{\partial x}$$

$$[X, Y] = R$$

$$[X, R] = Y$$

$$[R, Y] = X$$

lineární komb. s konst. koef Kill. v. je Kill. v.

$$V = V^R R + V^X X + V^Y Y \qquad V^R, V^X, V^Y \in \mathbb{R}$$

Lieova z. Kill v. je Kill. v.

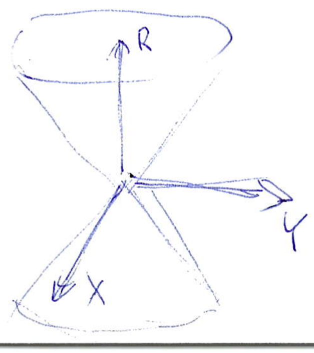
$$[\xi_\mu, \xi_\nu] = -C_{\mu\nu}{}^{\kappa} \xi_\kappa \qquad \xi_X = X \quad \xi_Y = Y \quad \xi_R = R$$

$$C_{XY}{}^R = -C_{YX}{}^R = -1 \qquad C_{XR}{}^Y = -C_{RX}{}^Y = -1 \qquad C_{RY}{}^X = -C_{YR}{}^X = -1$$

Killingova metrika

$$2K_{\alpha\beta} = -C_{\alpha\mu}{}^{\nu} C_{\beta\nu}{}^{\mu}$$

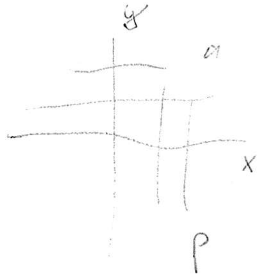
$$-K_{XX} = -K_{YY} = K_{RR} = 1 \qquad \operatorname{sign} K = (- - +)$$



LA alg. izometrií

proof 1

ξ υ ς



$$X = \cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi}$$

$$Y = +\sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi}$$

$$R = \frac{\partial}{\partial\varphi}$$

$\varphi = \frac{\pi}{2} \quad r = r_0$

$- \frac{1}{\operatorname{th} r_0}\frac{\partial}{\partial\varphi}$

$$[X,Y] = \left[\cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi},\ \sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi}\right]$$

$$= \left[\cos\varphi \frac{\partial}{\partial r},\ \cos\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi}\right] + \left[-\sin\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi},\ \sin\varphi \frac{\partial}{\partial r}\right]$$

$$\frac{1}{\operatorname{th}^2 r}\left[-\sin\varphi \frac{\partial}{\partial\varphi},\ \cos\varphi \frac{\partial}{\partial\varphi}\right]$$

$$= \cos^2\varphi \frac{\partial \operatorname{th}^{-1} r}{\partial r}\frac{\partial}{\partial\varphi} + \sin^2\varphi \frac{\partial \operatorname{th}^{-1} r}{\partial r}\frac{\partial}{\partial\varphi}$$

$$+ \frac{1}{\operatorname{th}^2 r}\sin^2\varphi \frac{\partial}{\partial\varphi} - \frac{\cos\varphi}{\operatorname{th}^2 r}\left[\sin\varphi \frac{\partial}{\partial\varphi}, \frac{\partial}{\partial\varphi}\right]$$

$$= -\frac{1}{\operatorname{th}^2 r}\frac{1}{\operatorname{ch}^2 r}\frac{\partial}{\partial\varphi} + \frac{1}{\operatorname{th}^2 r}\frac{\partial}{\partial\varphi}$$

$$= \frac{1}{\cancel{\operatorname{th}^2 r}}\left(\frac{\cancel{\operatorname{sh}^2 r}}{\operatorname{ch}^2 r}\right)\frac{\partial}{\partial\varphi} = R$$

$C_{XY}{}^{R} = 1$

$C_{XR}{}^{Y} = 1$

$C_{RY}{}^{X} = 1$

$$[X,Y] = R$$

$$[X,R] = \left[\cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi},\ \frac{\partial}{\partial\varphi}\right]$$

$$= \sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi} = Y$$

$$[Y,R] = \left[\sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi},\ \frac{\partial}{\partial\varphi}\right]$$

$$= -\left(\cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi}\right) \overset{\wedge}{=} -X$$

$$K_{\alpha\beta} = -\frac{1}{2} C_{\alpha\mu}{}^{\nu} C_{\beta\nu}{}^{\mu}$$

$$K_{XX} = -\frac{1}{2} 2 (C_{XY}{}^{R} C_{XR}{}^{Y} = -1$$

$$K_{YY} = -\frac{1}{2} 2 C_{YX}{}^{R} C_{YR}{}^{X} = -1$$

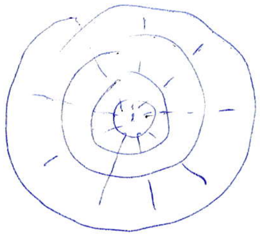
$$K_{RR} = -\frac{1}{2} 2 C_{RX}{}^{Y} C_{RY}{}^{X} = 1$$

$$K_{XY} = K_{XR} = K_{YR} = 0$$

$$\text{sign } K = - - +$$

Orbity isometrií = cykly

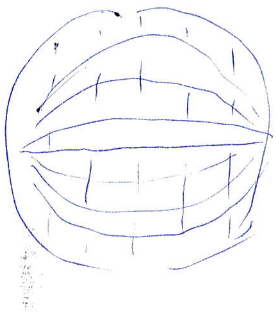
R



svazek skrze bod

cyklus = kružnice

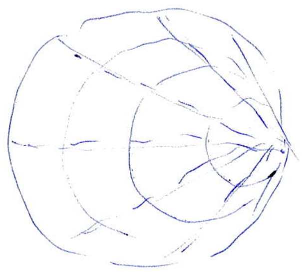
X, Y



svazek kolmý na př.

cyklus = eqidistanta
= exocyklus

R ± X



svazek rovnoběžek

cyklus = horocyklus

Akce isometrií na generátorech (KV)

komutační relace

$$[X,Y]=R \qquad [X,R]=Y \qquad [R,Y]=X$$

rotace generovaná R

$$[R,X]=-Y \qquad [R,Y]=X \quad \Rightarrow \quad C_R\cdot=\begin{bmatrix}0&-1\\1&0\end{bmatrix} \qquad C_R^2=-\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$\exp(\Delta\varphi\, C_R)=\cos\Delta\varphi\begin{bmatrix}1&0\\0&1\end{bmatrix}+\sin\Delta\varphi\begin{bmatrix}0&-1\\1&0\end{bmatrix}$$

$$\begin{bmatrix}V^X\\V^Y\end{bmatrix}=\exp(\Delta\varphi\, C_R)\begin{bmatrix}V_0^X\\V_0^Y\end{bmatrix}$$

$\Downarrow$

$$X_{\Delta\varphi}=R_{\Delta\varphi *}X=\cos\Delta\varphi\, X+\sin\Delta\varphi\, Y \qquad \Leftarrow V_0^X=1 \quad V_0^Y=0$$

$$Y_{\Delta\varphi}=R_{\Delta\varphi *}Y=-\sin\Delta\varphi\, X+\cos\Delta\varphi\, Y \qquad \Leftarrow V_0^X=0 \quad V_0^Y=1$$

boost generovaný X

$$[X,R]=Y \qquad [X,Y]=R \qquad C_X\cdot=-\begin{bmatrix}0&1\\1&0\end{bmatrix} \qquad C_X^2=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$\exp(\Delta x\, C_X)=\operatorname{ch}\Delta x\begin{bmatrix}1&0\\0&1\end{bmatrix}-\operatorname{sh}\Delta x\begin{bmatrix}0&1\\1&0\end{bmatrix}$$

$$\begin{bmatrix}V^R\\V^Y\end{bmatrix}=\exp(\Delta x\, C_X)\begin{bmatrix}V_0^R\\V_0^Y\end{bmatrix}$$

$\Downarrow$

$$R_{\Delta x}=X_{\Delta x *}R=\operatorname{ch}\Delta x\, R-\operatorname{sh}\Delta x\, Y \qquad \Leftarrow V_0^R=1 \quad V_0^Y=0$$

$$Y_{\Delta x}=X_{\Delta x *}Y=-\operatorname{sh}\Delta x\, R+\operatorname{ch}\Delta x\, Y \qquad \Leftarrow V_0^R=0 \quad V_0^Y=1$$

boost generovaný Y

$$[Y,R]=-X \qquad [Y,X]=-R \qquad C_Y=\begin{bmatrix}0&1\\1&0\end{bmatrix} \qquad C_Y^2=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$\exp(\Delta y\, C_Y)=\operatorname{ch}\Delta y\begin{bmatrix}1&0\\0&1\end{bmatrix}+\operatorname{sh}\Delta y\begin{bmatrix}0&1\\1&0\end{bmatrix}$$

$\Downarrow$

$$R_{\Delta y}=Y_{\Delta y *}R=\operatorname{ch}\Delta y\, R+\operatorname{sh}\Delta y\, X \qquad \Leftarrow V_0^R=1 \quad V_0^X=0$$

$$X_{\Delta y}=Y_{\Delta y *}X=\operatorname{sh}\Delta y\, R+\operatorname{ch}\Delta y\, X \qquad \Leftarrow V_0^R=0 \quad V_0^X=1$$

Grupa isometrií L^2

algebra generátorů isometrií na L^2

$[X,Y]=R \qquad [R,X]=-Y \qquad [R,Y]=X$

LA grupy isometrií $=\mathrm{iso}(L^2)$

$c_{XY}{}^{R}=1 \qquad c_{RX}{}^{Y}=-1 \qquad c_{RY}{}^{X}=1$

$-k_{XX}=-k_{YY}=k_{RR}=1$

akce na generátorech (tj. na LA)

$X_{\Delta\varphi}=R_{\Delta\varphi *}X=\cos\Delta\varphi\, X+\sin\Delta\varphi\, Y$
$Y_{\Delta\varphi}=R_{\Delta\varphi *}Y=-\sin\Delta\varphi\, X+\cos\Delta\varphi\, Y$

rotace v X-Y rovině vzhledem k metrice k

$R_{\Delta x}=\mathcal{X}_{\Delta x *}R=\operatorname{ch}\Delta x\, R-\operatorname{sh}\Delta x\, Y$
$Y_{\Delta x}=\mathcal{X}_{\Delta x *}Y=-\operatorname{sh}\Delta x\, R+\operatorname{ch}\Delta x\, Y$

boost v R-Y rovině vzhledem k metrice k

$R_{\Delta y}=\mathcal{Y}_{\Delta y *}R=\operatorname{ch}\Delta y\, R+\operatorname{sh}\Delta y\, X$
$X_{\Delta y}=\mathcal{Y}_{\Delta y *}X=\operatorname{sh}\Delta y\, R+\operatorname{ch}\Delta y\, X$

boost v R-X rovině vzhledem k metrice k

struktura grupy isometrií

isometrie L^2 $\iff$ isometrie Minkowského p. D=3

$R_{\Delta\varphi},\ \mathcal{X}_{\Delta x},\ \mathcal{Y}_{\Delta y}$ (postor generátorů s metrikou k)

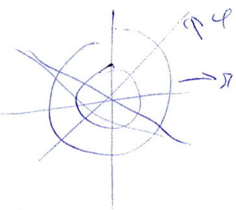
$\Downarrow$

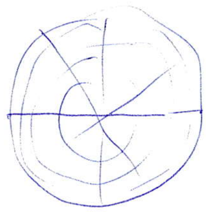
$\mathrm{Iso}(L^2)=SO(1,2)$
$\mathrm{iso}(L^2)=so(1,2)$

Přizpůsobené souřadnice

rotace R – polární souřadnice r, φ

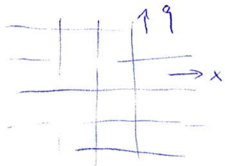
$$g = dr^2 + \mathrm{sh}^2 r \, d\varphi^2$$

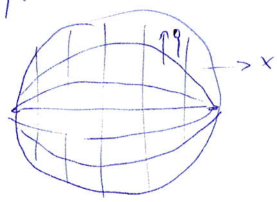




$$R = \frac{\partial}{\partial \varphi} \qquad r, \varphi$$

translace X – Lobač. souřadnice q, x





$$g = dq^2 + \mathrm{ch}^2 q \, dx^2$$

$$\mathrm{th}\, x = \mathrm{th}\, r \cos\varphi \qquad \mathrm{ch}\, r = \mathrm{ch}\, q \,\mathrm{ch}\, x \quad \text{(Pythag. v.)}$$

$$\mathrm{sh}\, q = \mathrm{sh}\, r \sin\varphi \qquad \tan\varphi = \frac{\mathrm{th}\, q}{\mathrm{sh}\, x}$$

$$\mathrm{sh}\, r \, dr = \mathrm{sh}\, q \,\mathrm{ch}\, x \, dq + \mathrm{ch}\, q \,\mathrm{sh}\, x \, dx$$

$$\mathrm{sh}^2 r \, d\varphi = \mathrm{sh}\, x \, dq - \mathrm{sh}\, q \,\mathrm{ch}\, q \,\mathrm{ch}\, x \, dx$$

kde $\mathrm{sh}^2 r = \mathrm{sh}^2 q + \mathrm{ch}^2 q \,\mathrm{sh}^2 x = \mathrm{sh}^2 q \,\mathrm{ch}^2 x + \mathrm{sh}^2 x = \mathrm{ch}^2 q \,\mathrm{ch}^2 x - 1$

$$\frac{1}{\mathrm{ch}\, x} \frac{\partial}{\partial q} = \sin\varphi \frac{\partial}{\partial r} + \cos\varphi \frac{1}{\mathrm{sh}\, r \,\mathrm{ch}\, r} \frac{\partial}{\partial \varphi}$$

$$\frac{\partial}{\partial x} = \cos\varphi \frac{\partial}{\partial r} - \sin\varphi \frac{1}{\mathrm{th}\, r} \frac{\partial}{\partial \varphi} \qquad = X$$

$$\mathrm{th}\, r \frac{\partial}{\partial r} = \mathrm{th}\, q \frac{\partial}{\partial q} + \frac{\mathrm{th}\, x}{\mathrm{ch}^2 q} \frac{\partial}{\partial x}$$

$$\frac{\partial}{\partial \varphi} = \mathrm{sh}\, x \frac{\partial}{\partial q} - \mathrm{th}\, q \,\mathrm{ch}\, x \frac{\partial}{\partial x} \qquad = R$$

$$R = \mathrm{sh}\, x \frac{\partial}{\partial q} - \mathrm{ch}\, x \,\mathrm{th}\, q \frac{\partial}{\partial x}$$

$$X = \frac{\partial}{\partial x}$$

$$Y = \mathrm{ch}\, x \frac{\partial}{\partial q} - \mathrm{sh}\, x \,\mathrm{th}\, q \frac{\partial}{\partial x}$$

Isometrie

Rotace

$$R_{\Delta\varphi} : x \longrightarrow x_{\Delta\varphi} = R_{\Delta\varphi}\, x$$

$$r_{\Delta\varphi} = r \qquad\qquad \text{kde } r_{\Delta\varphi} = r(x_{\Delta\varphi}) \text{ atd.}$$
$$\varphi_{\Delta\varphi} = \varphi + \Delta\varphi \qquad\qquad r = r(x)$$

$$\frac{D x_{\Delta\varphi}}{d\Delta\varphi} = \frac{d r_{\Delta\varphi}}{d\Delta\varphi}\frac{\partial}{\partial r} + \frac{d\varphi_{\Delta\varphi}}{d\Delta\varphi}\frac{\partial}{\partial\varphi} = \frac{\partial}{\partial\varphi} = R$$

$$DR_{\Delta\varphi}\big|_x = \frac{\partial X^a_{\Delta\varphi}}{\partial X^b}\bigg|_x dX^b\big|_x \frac{\partial}{\partial X^a}\bigg|_{x_{\Delta\varphi}} = dr\big|_x \frac{\partial}{\partial r}\bigg|_{x_{\Delta\varphi}} + d\varphi\big|_x \frac{\partial}{\partial\varphi}\bigg|_{x_{\Delta\varphi}}$$

rotace KV X

$$X_{\Delta\varphi}\big|_{x_{\Delta\varphi}} = R_{\Delta\varphi *}\left(X\big|_x\right) = X\big|_x \cdot DR_{\Delta\varphi}\big|_x =$$
$$= \cos\varphi \frac{\partial}{\partial r}\bigg|_{x_{\Delta\varphi}} - \sin\varphi \frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi}\bigg|_{x_{\Delta\varphi}}$$

rotace KV R

$$R_{\Delta\varphi}\big|_{x_{\Delta\varphi}} = R_{\Delta\varphi *}\, R\big|_x = \frac{\partial}{\partial\varphi}\bigg|_{x_{\Delta\varphi}} = R\big|_{x_{\Delta\varphi}}$$

$\Downarrow$

$$R_{\Delta\varphi} = R$$

platí

$$\frac{\partial}{\partial r} = \cos\varphi\, X + \sin\varphi\, Y$$
$$\frac{1}{\operatorname{th} r}\frac{\partial}{\partial\varphi} = -\sin\varphi\, X + \cos\varphi\, X$$

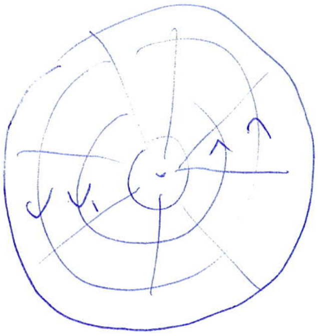
$\Rightarrow$

$$X_{\Delta\varphi}\big|_{x_{\Delta\varphi}} = \cos\varphi\cos(\varphi+\Delta\varphi)\, X\big|_{x_{\Delta\varphi}} + \cos\varphi\sin(\varphi+\Delta\varphi)\, Y\big|_{x_{\Delta\varphi}}$$
$$+ \sin\varphi\sin(\varphi+\Delta\varphi)\, X\big|_{x_{\Delta\varphi}} - \sin\varphi\cos(\varphi+\Delta\varphi)\, Y\big|_{x_{\Delta\varphi}}$$
$$= \cos\Delta\varphi\, X\big|_{x_{\Delta\varphi}} + \sin\Delta\varphi\, Y\big|_{x_{\Delta\varphi}}$$

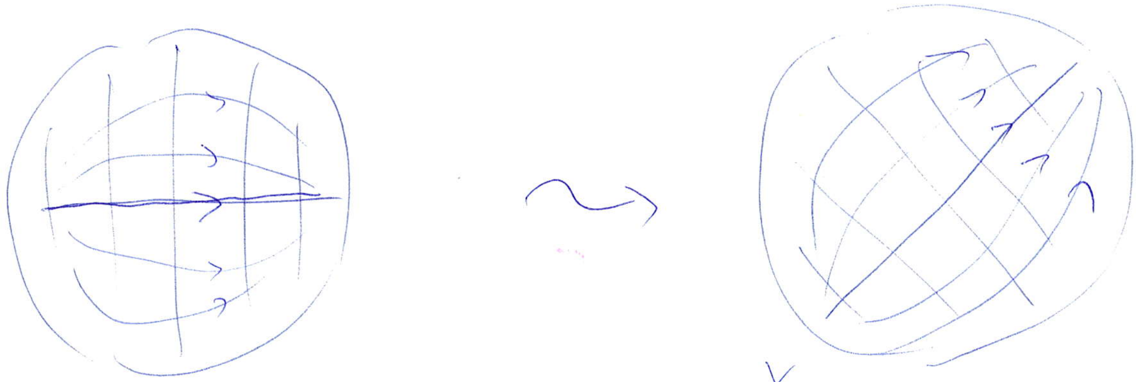
$$X_{\Delta\varphi} = \cos\Delta\varphi\, X + \sin\Delta\varphi\, Y$$

obdobně

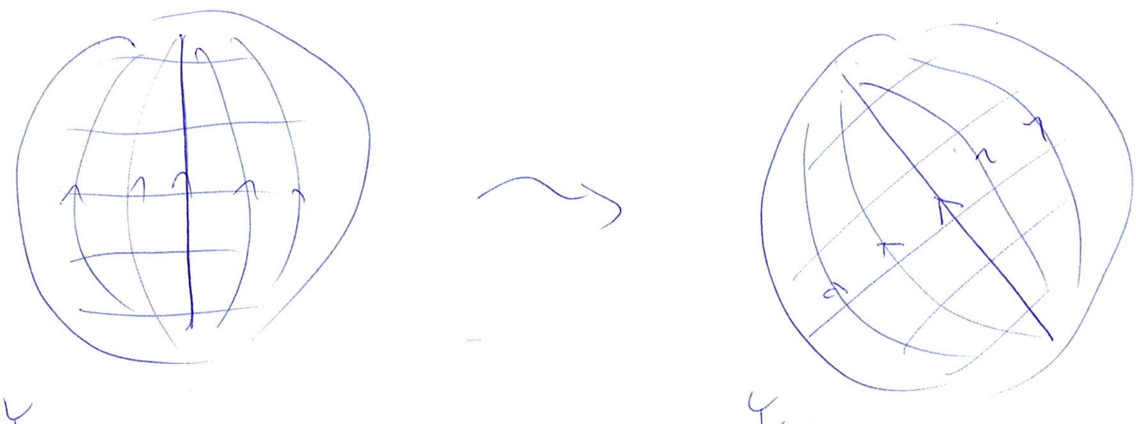
$$Y_{\Delta\varphi} = -\sin\Delta\varphi\, X + \cos\Delta\varphi\, Y$$



R



X $X_{\Delta\varphi}$



Y $Y_{\Delta\varphi}$

Isometrie

translace ve směru X

$$\mathcal{X}_{\Delta x} : x \longrightarrow x_{\Delta x} = \mathcal{X}_{\Delta x}\, x$$

$$q_{\Delta x} = q \qquad \text{kde } q_{\Delta x} = q(x_{\Delta x}) \text{ atd.}$$
$$x_{\Delta x} = x + \Delta x \qquad q = q(x)$$

$$\frac{D x_{\Delta x}}{d\Delta x} = \frac{d q_{\Delta x}}{d\Delta x}\frac{\partial}{\partial q} + \frac{\partial x_{\Delta x}}{\partial \Delta x}\frac{\partial}{\partial x} = \frac{\partial}{\partial x} = X$$

$$D\mathcal{X}_{\Delta x}|_x = \dots = dq_x \frac{\partial}{\partial q}\Big|_{x_{\Delta x}} + dx|_x \frac{\partial}{\partial x}\Big|_{x_{\Delta x}}$$

translace KV R

$$R_{\Delta x}|_{x_{\Delta x}} = \mathcal{X}_{\Delta x *}(R|_x) = R|_x \cdot D\mathcal{X}_{\Delta x}|_x =$$

$$= \operatorname{sh} x \frac{\partial}{\partial q}\Big|_{x_{\Delta x}} - \operatorname{ch} x \operatorname{th} q \frac{\partial}{\partial x}\Big|_{x_{\Delta x}}$$

platí

$$-\operatorname{th} q \frac{\partial}{\partial x} = \operatorname{ch} x\, R - \operatorname{sh} x\, Y$$

$$\frac{\partial}{\partial q} = -\operatorname{sh} x\, R + \operatorname{ch} x\, Y$$

$$R_{\Delta x}|_{x_{\Delta x}} = -\operatorname{sh} x \operatorname{sh}(x+\Delta x)\, R + \operatorname{sh} x \operatorname{ch}(x+\Delta x)\, Y$$
$$+ \operatorname{ch} x \operatorname{ch}(x+\Delta x)\, R - \operatorname{ch} x \operatorname{sh}(x+\Delta x)\, Y \Big|_{x_{\Delta x}}$$

$$= \operatorname{ch} \Delta x\, R|_{x_{\Delta x}} - \operatorname{sh} \Delta x\, Y|_{x_{\Delta x}}$$

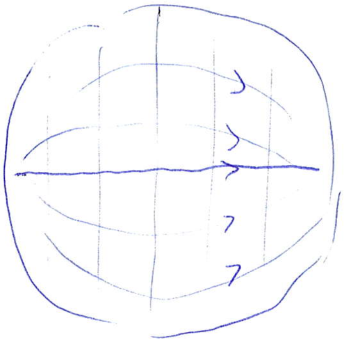
$$R_{\Delta x} = \operatorname{ch}\Delta x\, R - \operatorname{sh}\Delta x\, Y$$

obdobně translace KV Y

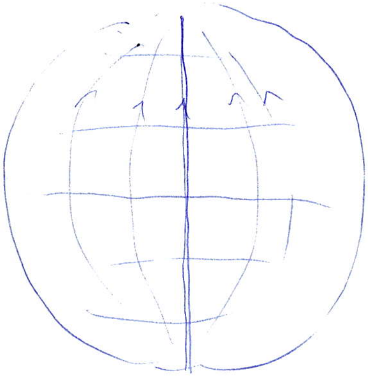
$$Y_{\Delta x} = -\operatorname{sh}\Delta x\, R + \operatorname{ch}\Delta x\, Y$$

translace KV X

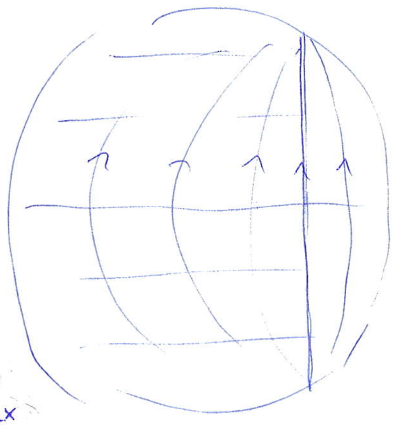
$$X_{\Delta x}|_{x_{\Delta x}} = \mathcal{X}_{\Delta x *} X|_x$$
$$= \frac{\partial}{\partial x}\Big|_{x_{\Delta x}} = X|_{x_{\Delta x}}$$
$$X_{\Delta x} = X$$



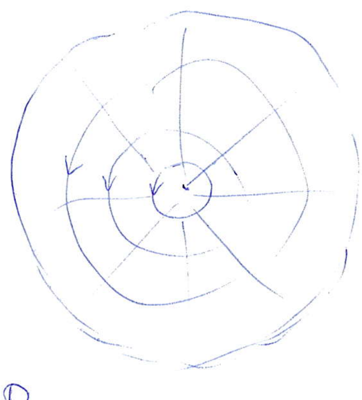
X



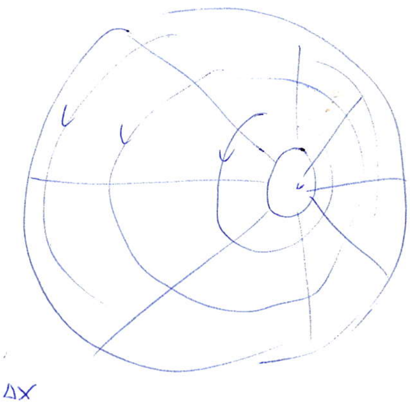
Y



$Y_{\Delta x}$



R



$R_{\Delta x}$