Character table for the symmetry group of cubane

Deadline: Monday, **December 2, 2024**

1. (4 points) Determine the point group of symmetry of the cubane molecule C_8H_8 in its equilibrium geometry (it has 48 elements). What are the conjugacy classes of this group? To justify the results, it is enough to indicate which symmetry operations can be used to convert individual elements of symmetry between themselves.

2. (10 points) Determine the character table of this group. The solution must include an explanation of how you get the individual characters of irreducible representations (it is not enough to copy the table from the web). For example, you can take advantage of the fact that it is a direct product of two subgroups, determine some representations by the action of the group on general vectors or pseudovectors in space, calculate other characters using orthogonality relations, etc. It might also help to consider how the functions $2z^2 - x^2 - y^2$ and $x^2 - y^2$ transform.

3. (6 points) Consider a vector space generated by the functions $x^2, y^2, z^2, xy, xz, yz$. Determine the irreducible invariant subspaces of this vector space under the action of the cubane symmetry group and add to the character table on the appropriate row, according to which irreducible representations these functions (more precisely, their certain linear combinations) transform.