From spinning tops to SUSY in the sky: List of exam questions

What to expect

You will be asked one theoretical and one computational question out of the following two sets. You will have about 20 minutes to prepare your answers. After that, we will discuss your answers and some (un)related problems. The discussion should last about 20 minutes, including my feedback to you.

1 Computational questions

1) Derive the Hamilton's canonical equations from the geometric perspective.

2) Calculate canonical projection of a Hamiltonian vector field of a (to be given) conserved quantity. Is it a hidden symmetry?

3) Show that a 'square' of a Killing–Yano tensor gives a Killing tensor.

4) Given the covariant Poisson bracket for the spinning particle, show that the system admits a generic supercharge.

5) Given the covariant Poisson bracket for the spinning particle, show that Killing–Yano tensors give rise to non-generic superinvariants of the spinning particle motion.

2 Theoretical questions

1) Discuss basic geometrical concepts of Hamiltonian mechanics. Namely, introduce the notions of symplectic 2-form, Hamiltonian vector fields, phase space flows, Hamiltonian, Poisson brackets.

2) Formulate Noether's theorem and provide its geometrical meaning in the phase space. Introduce the notion of hidden symmetries and discuss some examples.

3) Introduce Killing tensors. Discuss their basic properties, algebra, and applications to geodesic motion.

4) Explain the idea behind the Eisenhart lift – why is it useful?

5) Introduce Killing–Yano tensors. Why are they useful?

6) Discuss various descriptions of spinning particles in curved spacetime.