3+1 rozštěpení

t časová fce

$\vec{t}\cdot dt = 1$ časový tok

$\vec{t} = N\vec{n} + \vec{N}$ lapse a shift

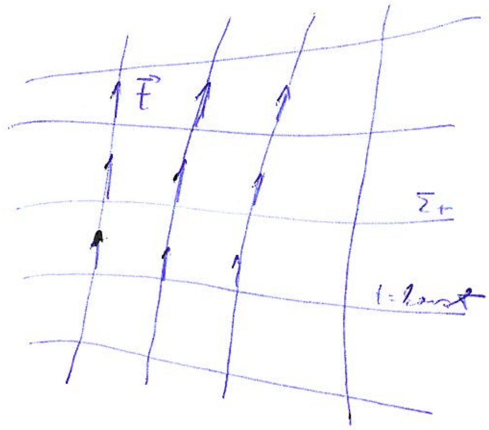
$N\,dt = n$ normála

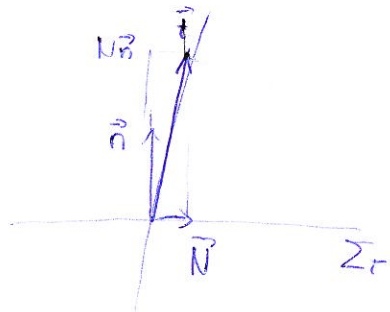
$\vec{n}\cdot g\cdot\vec{n} = -1 \qquad \vec{n}\cdot n = 1$

$q = g|_\Sigma$ prostorová metrika

$g = -nn + q \qquad g^{-1} = -\vec{n}\vec{n} + q^{-1}$

$g^{1/2} = N\,dt \wedge q^{1/2} \qquad q^{1/2} = (\mathrm{Det}\, q)^{1/2}$





Rozštěpení pro skalární pole

$$\varphi = \Phi|_\Sigma \qquad \text{hodnota pole na } \Sigma$$

$$\dot\varphi = \frac{d}{dt}\varphi = t^a d_a\Phi = N n^a d_a\Phi + N^a d_a\Phi$$

$$n^a d_a\Phi = \frac{1}{N}\left(\dot\varphi - N^a d_a\Phi\right)$$

$$J = \frac{g^{\frac12}}{\hat g^{\frac12}}\, j = N\,dt\, j$$

$$S[\langle\Sigma_f|\Sigma_i\rangle](\Phi) = \int_{\langle t_f|t_i\rangle} L(\varphi,\dot\varphi)\,dt$$

$$= -\frac12\int\Big(d_a\Phi \underbrace{g^{ab}}_{-n^a n^b + q^{ab}} d_b\Phi + V\Phi^2\Big)\underbrace{g^{\frac12}}_{N\hat g^{\frac12} dt} + \int \underbrace{J\Phi}_{N j\, dt}$$

Lagrangian

$$L(\varphi,\dot\varphi) = \int_\Sigma\Big(\frac12\frac1N\left(\dot\varphi - N^a d_a\varphi\right)^2\hat g^{\frac12} - \frac12 N\, d_a\varphi\, q^{ab} d_b\varphi\, \hat g^{\frac12} - \frac12 N V \hat g^{\frac12}\varphi^2 + N j\varphi\Big)$$

$$= \frac12\left(\dot\varphi - \mu\cdot\varphi\right)\cdot\tilde q\cdot\left(\dot\varphi - \mu\cdot\varphi\right) - \frac12\varphi\cdot\tilde v\cdot\varphi + \tilde j\cdot\varphi$$

kde (bi)distribuce na Σ $\tilde q, \mu, \tilde v, \tilde j$ jsou

$$\tilde q = \frac1N \hat g^{\frac12}\delta$$

$$\tilde v = \tilde d_a\cdot\left(N q^{ab}\hat g^{\frac12}\delta\right)\cdot\tilde d_b + N V \hat g^{\frac12}\delta$$

$$\tilde j = N j$$

$$\mu = N^a \overleftrightarrow{d}_a$$

Hybnost

$$\pi = \frac{\delta L}{\delta \dot\varphi} = \tilde{q}\cdot(\dot\varphi - \mu\cdot\varphi) = \frac{1}{N}(\dot\varphi - \vec{N}^a d_a\varphi)\, g^{\frac{1}{2}}$$

$$\dot\varphi = \pi\cdot\tilde{q}^{-1} + \mu\cdot\varphi = N\pi g^{-\frac{1}{2}} + \vec{N}^a d_a\varphi$$

Hamiltonián

$$H = \pi\cdot\dot\varphi - L(\varphi,\dot\varphi) \qquad \dot\varphi(\varphi,\pi)$$

$$= \frac{1}{2}\pi\cdot\tilde{q}^{-1}\cdot\pi + \frac{1}{2}\varphi\cdot\tilde{v}\cdot\varphi - \tilde{j}\cdot\varphi + \pi\cdot\mu\cdot\varphi$$

$$= \int_\Sigma \Big[N\Big(\frac{1}{2}\pi^2 g^{-\frac{1}{2}} + \frac{1}{2} d_a\varphi\cdot q^{ab} d_b\varphi\, g^{\frac{1}{2}} + \frac{1}{2}V\varphi^2 g^{\frac{1}{2}} - j\varphi\Big) + \vec{N}^a\pi\, d_a\varphi\Big]$$

lineární v N a $\vec{N}^a$ (vazby v plné teorii)

prostoročasový zápis

$$H[\Sigma] = \frac{1}{2}\phi\cdot\mathcal{H}[\Sigma]\cdot\phi - \mathcal{J}[\Sigma]\cdot\phi$$

$\mathcal{H}[\Sigma]$ a $\mathcal{J}[\Sigma]$ distribuce na M

Př.

$$\bar{\Phi}_c[\Sigma_1](\varphi_1\pi_1)\cdot\partial F\cdot\phi_c[\Sigma_2](\varphi_2,\pi_2) =$$

$$= (\varphi_1^x\, \underline{\Pi}[\Sigma]_x\cdot G_c - \pi_{1x}\,\varphi[\Sigma]^x\cdot G_c)\cdot\partial F\cdot(-G_c\cdot\underline{\Pi}[\Sigma_2]_y\varphi_2^y + G_c\cdot\varphi[\Sigma_2]^y\pi_{2y})$$

$$= \varphi_1^x\,\underline{\Pi}[\Sigma_1]_x\cdot G_c\cdot\underline{\Pi}[\Sigma_2]_y\,\varphi_2^y + \pi_{1x}\,\varphi[\Sigma_1]^x\cdot G_c\cdot\varphi[\Sigma_2]^y\,\pi_{2y}$$

$$- \varphi_1^x\,\underline{\Pi}[\Sigma_1]_x\cdot G_c\cdot\varphi[\Sigma_2]^y\,\pi_{2y} - \pi_{1x}\,\varphi[\Sigma_1]^x\cdot G_c\cdot\underline{\Pi}[\Sigma_2]_y\,\varphi_2^y$$

Pozorovatelné hodnoty a hybnosti

$\varphi[\Sigma] : P \to \mathcal{V}[\Sigma]$ prostor hodnot $\mathcal{V}[\Sigma] = \mathfrak{F}\Sigma$

$\pi[\Sigma] : P \to \tilde{\mathcal{V}}[\Sigma]$ prostor hybností $\tilde{\mathcal{V}}[\Sigma] = \tilde{\mathfrak{F}}\Sigma$

lineární pozorov. na P (č. φ)

$\varphi[\Sigma](\phi) = \underline{\varphi}[\Sigma]\cdot\phi$ $\qquad \underline{\varphi}[\Sigma_t]^x_y = \delta(t\,x|y)$

$\pi[\Sigma](\phi) = \underline{\Pi}[\Sigma]\cdot\phi$ $\qquad \underline{\Pi}[\Sigma_t]_{xy} = (g^{\frac{1}{2}}\vec{n}\cdot\overleftrightarrow{d}_c)(t\,x|y)$

mají prostorově Hamilt.

$\mathcal{H}[\Sigma] = \frac{1}{2}\underline{\Pi}[\Sigma]\cdot\tilde{g}^{-1}\cdot\underline{\Pi}[\Sigma] + \underline{\varphi}[\Sigma]\cdot v\cdot\underline{\varphi}[\Sigma] + \underline{\varphi}[\Sigma]\cdot\mu\cdot\underline{\Pi}[\Sigma]$

$J[\Sigma] = \tilde{j}\cdot\underline{\varphi}[\Sigma]$

symplektická forma

$\overrightarrow{d}\mathfrak{F}[\Sigma]_{xy} = \underline{\varphi}^z_x[\Sigma]\,\underline{\Pi}[\Sigma]_{yz}$ $\qquad \overleftarrow{d}\mathfrak{F}[\Sigma]_{xy} = \underline{\Pi}[\Sigma]_{xz}\,\underline{\varphi}[\Sigma]^z_y$

$\partial\mathfrak{F}[\Sigma]_{xy} = \underline{\Pi}[\Sigma]_{xz}\,\underline{\varphi}[\Sigma]^z_y - \underline{\varphi}[\Sigma]^z_x\,\underline{\Pi}[\Sigma]_{zy}$

Cauchyho propagátor

$D_c[\Sigma] = -G_c\cdot\partial\mathfrak{F}[\Sigma] = -G_c\cdot\underline{\Pi}[\Sigma]_x\,\underline{\varphi}[\Sigma]^x + G_c\cdot\underline{\varphi}[\Sigma]^x\,\underline{\Pi}[\Sigma]_x$

$\Phi_c[\Sigma](\varphi,\pi) = -G_c\cdot\underline{\Pi}[\Sigma]_x\,\varphi^x + G_c\cdot\underline{\varphi}[\Sigma]^x\,\pi_x$

propagace hodnot $\qquad$ propagace hybností

$\varphi^x = \underline{\varphi}[\Sigma]^x\cdot\Phi_c[\Sigma](\varphi,\pi) = -\underline{\varphi}[\Sigma]^x\cdot G_c\cdot\underline{\Pi}[\Sigma]_y\,\varphi^y + \underline{\varphi}[\Sigma]^x\cdot G_c\cdot\underline{\varphi}[\Sigma]^y\,\pi_y$

$\Rightarrow \underline{\varphi}[\Sigma]^x\cdot G_c\cdot\underline{\Pi}[\Sigma]_y = -\delta^x_y \qquad \underline{\varphi}[\Sigma]^x\cdot G_c\cdot\underline{\varphi}[\Sigma]^y = 0$

$\pi_x = \underline{\Pi}[\Sigma]_x\cdot\Phi_c[\Sigma](\varphi,\pi) = -\underline{\Pi}[\Sigma]_x\cdot G_c\cdot\underline{\Pi}[\Sigma]_y\,\varphi^y + \underline{\Pi}[\Sigma]_x\cdot G_c\cdot\underline{\varphi}[\Sigma]^y\,\pi_y$

$\Rightarrow \underline{\Pi}[\Sigma]_x\cdot G_c\cdot\underline{\varphi}[\Sigma]^y = \delta^y_x \qquad \underline{\Pi}[\Sigma]_x\cdot G_c\cdot\underline{\Pi}[\Sigma]_y = 0$

Poissonovy závorky

$\{\varphi[\Sigma]^x, \pi[\Sigma]_y\} = \underline{\varphi}[\Sigma]^x\cdot G_c\cdot\underline{\Pi}[\Sigma]_y = -\delta^x_y$

$\Rightarrow$ kanonicky sdruž. souř.

$\{A, B\} = \left(\frac{\delta A}{\delta\varphi^x}\underline{\varphi}^x + \frac{\delta A}{\delta\pi_x}\underline{\Pi}_x\right)\cdot G_c\cdot\left(\frac{\delta B}{\delta\varphi^y}\underline{\varphi}^y + \underline{\Pi}_y\frac{\delta B}{\delta\pi_y}\right)\Big|_\Sigma$

$= \frac{\delta A}{\delta\pi_x}\frac{\delta B}{\delta\varphi^x} - \frac{\delta A}{\delta\varphi^x}\frac{\delta B}{\delta\pi_x}$