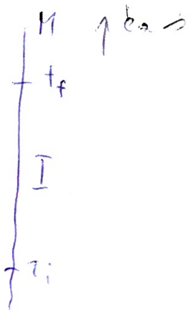
TEORIE POLE V 0+1 DIM
(harmonický oscilátor)



Akce a rovnice pohybu

$f : M \to \mathbb{R}$ vývoj "pole" v čase

akce

$$S[I](f) = +\frac{1}{2}\int_I (m\dot f^2 - k f^2)\,dt + \int_I f\, j\, dt$$

$$= -\frac{1}{2} f \bullet \mathcal{F}[I] \bullet f + f \bullet (dt\, j)$$

$$\mathcal{F}[I] = -\frac{\tilde d}{dt} \bullet (\chi[I]\, m\, dt\, \delta) \bullet \frac{\tilde d}{dt} + (\chi[I]\, k\, dt\, \delta)$$

$\dot m = 0$

pohybové rovnice

$$\tilde{\mathcal{F}} = m\, dt \frac{\tilde d^2}{dt} + k\, dt\, \delta$$

$$\tilde{\mathcal{F}} \bullet f = j\, dt \quad \Longrightarrow \quad \left[+\frac{d^2}{dt^2} + \omega^2\right] f = \frac{j}{m} \qquad \omega^2 = \frac{k}{m}$$

0+1 rozštěpení

$q(t) = q[t] \bullet f = f(t)$ hodnota v čase t

$p(t) = p[t] \bullet f = m \dot f(t)$ hybnost v čase t

$$L(q, \dot q) = \frac{1}{2} m \dot q^2 - \frac{1}{2} k q^2 + j q$$

$$H(q, \dot q) = \frac{1}{2m} p^2 + \frac{1}{2} k q^2 - j q$$

okrajové členy

$$d\tilde{\mathcal{F}}[t] = m\, dt\, \delta_t \frac{\tilde d}{dt} = q[t]\, p[t] \qquad f_1 \bullet d\tilde{\mathcal{F}}[t] \bullet f_2 = m f_1(t) \dot f_2(t)$$

$$\partial\mathcal{F}[t] = p[t]\, q[t] - q[t]\, p[t] \qquad f_1 \bullet \partial\mathcal{F}[t] \bullet f_2 = m(\dot f_1 f_2 - f_1 \dot f_2)|_t$$

Prostor řešení

nehomog. řešení

$$\bar{f} = q_0 \cos(\omega(t-t_0)) + \frac{p_0}{m\omega} \sin(\omega(t-t_0)) + \frac{J}{K}$$

homogenní řešení

$$\bar{f} = q_0 \cos(\omega(t-t_0)) + \frac{p_0}{m\omega} \sin(\omega(t-t_0))$$

$$\bar{f} = f_0[t_0](q_0, p_0)$$

sympl. forma

$$f_1 \cdot \partial f[t_0] \cdot f_2 = p_1 q_2 - q_1 p_2$$

$$\{q[t], p[t]\} = -1$$

$$\{f(t_2), f(t_1)\} = G_c(t_2 | t_1)$$

Pr:

$$f_i = q_i \cos(\omega(t-t_i)) + \frac{p_i}{m\omega} \sin(\omega(t-t_i))$$

$$f_1 \cdot \partial F[t_0] \cdot f_2 = m(\dot{f}_1 f_2 - f_1 \dot{f}_2)\big|_{t_0} =$$

$$= \left(-m q_1 \omega \sin(\omega(t_0-t_1)) + p_1 \cos(\omega(t_0-t_1))\right)$$
$$\left(q_2 \cos(\omega(t_0-t_2)) + \frac{p_2}{m\omega} \sin(\omega(t_0-t_2))\right)$$

$$- \; 1 \leftrightarrow 2$$

$$= -m\omega q_1 q_2 \left(\sin(\omega(t_0-t_1)) \cos(\omega(t_0-t_2)) - \cos(\omega(t_0-t_1)) \sin(\omega(t_0-t_2))\right)$$
$$+ \frac{p_1 p_2}{m\omega} \left(\cos(\omega(t_0-t_1)) \sin(\omega(t_0-t_2)) - \sin(\omega(t_0-t_1)) \cos(\omega(t_0-t_2))\right)$$
$$+ p_1 q_2 \left(\cos(\omega(t_0-t_1)) \cos(\omega(t_0-t_2)) + \sin(\omega(t_0-t_1)) \sin(\omega(t_0-t_2))\right)$$
$$- q_1 p_2 \left(\sin(\omega(t_0-t_1)) \sin(\omega(t_0-t_2)) + \cos(\omega(t_0-t_1)) \cos(\omega(t_0-t_2))\right)$$

$$= m\omega q_1 q_2 \sin(\omega(t_1-t_2)) + \frac{p_1 p_2}{m\omega} \sin(\omega(t_1-t_2))$$
$$+ (p_1 q_2 - q_1 p_2) \cos(\omega(t_1-t_2))$$

Greenovy funkce

$\omega = \text{konst}$ (tj. $m, \kappa = \text{konst}$)

$$G_{ret}^{t_2 t_1} = \frac{\sin(\omega(t_2 - t_1))}{m\omega}\, \Theta(t_2 - t_1)$$

$$G_{adv}^{t_2 t_1} = - \frac{\sin(\omega(t_2 - t_1))}{m\omega}\, \Theta(t_2 - t_1)$$

$$G_{PJ}^{t_2 t_1} = \frac{\sin(\omega(t_2 - t_1))}{m\omega}\, \operatorname{sign}(t_2 - t_1)$$

$$G_c^{t_2 t_1} = \frac{\sin(\omega(t_2 - t_1))}{m\omega}$$

Cauchyho propagátor

$$D_c[t] = - G_c \cdot \partial T[t]$$

$$(D_c[t_0] \cdot f)(t) = - m \frac{\partial G_c}{\partial t_1}(t|t_0)\, f(t_0) + m\, G_c(t|t_0) \frac{df}{dt}(t_0)$$

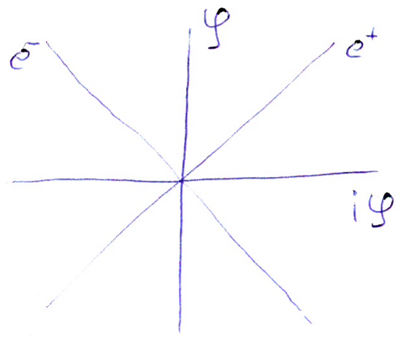
$$= \cos(\omega(t - t_0))\, f(t_0) + \frac{\sin(\omega(t - t_0))}{m\omega}\, m \dot{f}(t_0)$$

$$\Phi_c[t_0](q_0, p_0) = \cos(\omega(t - t_0))\, q_0 + \frac{\sin(\omega(t - t_0))}{m\omega}\, p_0$$

Poissonovy závorky

$$\{ f(t_2), f(t_1) \} = \frac{\sin(\omega(t_2 - t_1))}{m\omega}$$

Frekvenční rozštěpení



Komplexní řešení - prostor $\mathcal{S}^{\mathbb{C}}$

$$e^+(t) = \frac{1}{\sqrt{2\omega m}} \exp(-i\omega t) \qquad \omega > 0$$

$$e^-(t) = \frac{1}{\sqrt{2\omega m}} \exp(i\omega t)$$

$$e^\mp \cdot \partial F \cdot e^\pm = \frac{1}{2\omega m}\, m\,(\dot{e}^- e^+ - e^- e^+)\Big|_t = \pm i$$

$$e^\pm \cdot \partial F \cdot e^\pm = 0$$

umožňuje definovat "normu" na pos. def. řešeních

$$\langle \phi_1, \phi_2 \rangle_+ = -i\, \phi_1^* \cdot \partial F \cdot \phi_2 \qquad \phi_1, \phi_2 \sim e^+$$

v 0+1 dim "triviální" tvrzení

$\langle\ ,\ \rangle_+$ je nedeg. i pos. def.

báze v $\mathcal{S}^{\mathbb{C}}$

e^+, e^- tvoří bázi v $\mathcal{S}^{\mathbb{C}}$

$f = a e^+ + a^* e^-$ řešení z $\mathcal{S}$

$$a = -i\, e^- \cdot \partial F \cdot f = \frac{1}{\sqrt{2}}\left(\sqrt{m\omega}\, q + \frac{i}{\sqrt{m\omega}}\, p\right) \qquad \text{nezávisí na } t$$

$$a^* = i\, e^+ \cdot \partial F \cdot f = \frac{1}{\sqrt{2}}\left(\sqrt{m\omega}\, q - \frac{i}{\sqrt{m\omega}}\, p\right)$$

$$\{a, a^*\} = -\, e^- \cdot \partial F \cdot G_c \cdot \partial F \cdot e^+ = i$$

Kvantování

$$q, p \longrightarrow \hat{q}, \hat{p}$$

$$\{q, p\} = -1 \longrightarrow \frac{i}{\hbar}[\hat{q}, \hat{p}] = -\hat{1} \qquad [\hat{q}, \hat{p}] = i\hbar\,\hat{1}$$

$$f \longrightarrow \hat{f}$$

$$\{f^{t_1}, f^{t_2}\} = G_c^{t_1 t_2} \longrightarrow \frac{i}{\hbar}[\hat{f}^{t_1}, \hat{f}^{t_2}] = G_c^{t_1 t_2}\,\hat{1} \qquad [\hat{f}, \hat{f}] = -i\hbar G_c\,\hat{1}$$

$$\frac{d^2}{dt^2} f + \omega^2 f = 0 \longrightarrow \frac{d^2}{dt^2}\hat{f} + \omega^2 \hat{f} = 0$$

kreační a anihilační oper.

$$\hat{f}(t) = \hat{a}\, e^+(t) + \hat{a}^+ e^-(t)$$

$$\hat{a} = -i\, e^- \cdot \partial f \cdot \hat{f} = \frac{1}{\sqrt{2}}\left(\sqrt{m\omega}\,\hat{q} + \frac{i}{\sqrt{m\omega}}\hat{p}\right)$$

$$\hat{a}^+ = i\, e^+ \cdot \partial f \cdot \hat{f} = \frac{1}{\sqrt{2}}\left(\sqrt{m\omega}\,\hat{q} - \frac{i}{\sqrt{m\omega}}\hat{p}\right)$$

$$[\hat{a}, \hat{a}^+] = -e^- \cdot \partial f \cdot [\hat{f}, \hat{f}] \cdot \partial f \cdot e^+ = -i\hbar\, e^- \cdot \partial f \cdot e^+ = \hbar\,\hat{1}$$

Hamiltonián

$$\hat{H} = \frac{\omega}{2}\left(\frac{\hat{p}^2}{m\omega} + m\omega\,\hat{q}^2\right) = \frac{\omega}{2}\left(\hat{a}\hat{a}^+ + \hat{a}^+\hat{a}\right)$$

Fockovské báze v 0+1

$$[\hat{a}, \hat{a}^+] = \hat{1}$$ $\hbar = 1$

$$\hat{n} = \hat{a}^+ \hat{a} = \frac{1}{2}\left(\frac{\hat{p}^2}{m\omega} + m\omega \hat{q}^2 - \hat{1}\right)$$ $\hat{n}^+ = \hat{n}$

hermit. po def.

vlastní vekt $\hat{n}$

$$\hat{n}\,|m\rangle = m\,|m\rangle$$

$$\hat{n}\,\hat{a}|m\rangle = (m-1)\,\hat{a}|m\rangle$$

$$\langle m|\hat{a}^+\hat{a}|m\rangle = m\,\langle m|m\rangle$$ $\Rightarrow$ m kladné

$$\hat{n}\,\hat{a}^+|m\rangle = (m+1)\,\hat{a}^+|m\rangle$$

$$|m-1\rangle = \frac{1}{\sqrt{m}}\,\hat{a}\,|m\rangle$$

$$|m+1\rangle = \frac{1}{\sqrt{m+1}}\,\hat{a}^+|m\rangle$$

pozitivita $\Rightarrow$

$$\exists\ |vac\rangle \qquad \hat{a}\,|vac\rangle = 0$$

$$|m\rangle = \frac{1}{\sqrt{m!}}\,\hat{a}^{+m}\,|vac\rangle$$ $|vac\rangle = |0\rangle$

báze kvant

$$|m\rangle \qquad m \in \mathbb{N}_0$$

$$|\psi\rangle = \sum_m \psi_m\,|m\rangle$$